ON REDUCED COMPLEXITY TECHNIQUES FOR BANDWIDTH EFFICIENT CONTINUOUS PHASE MODULATIONS IN SERIALLY CONCATENATED CODED SYSTEMS

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ABSTRACT

Serially concatenated coded (SCC) systems with continuous phase modulations (CPMs) as recursive inner codes have been known to give very high coding gains at low operative signal to noise ratios (SNRs). Moreover, concatenated coded systems with iterative decoding approach the bit error rate (BER) bounds given by the maximum likelihood (ML) criterion. Although SCC systems by themselves are reduced complexity systems when compared to the ML decoding, when very highly bandwidth efficient CPMs such as pulse code modulation /frequency modulation (PCM/FM) is used [1], they present a problem of extremely high decoding complexity at the receiver. The complexity of a CPM is described by the size of its trellis which is a function of the modulation index, the cardinality of the source alphabet and the length of the frequency pulse used. The surveyed complexity reduction techniques adopt approximations which will reduce the size of the trellis with *minimal* expense of power. In this paper, we present reduced complexity approaches to *sub-optimally* decode SCC PCM/FM by mainly two approaches — 1) Frequency pulse truncation. 2) Decision feedback.

1. INTRODUCTION

Continuous phase modulation belongs to a class of non-linear digital modulations. The phase of the CPM is constrained to be continuous by the use of frequency pulses, often times more than one symbol time, which then avoid instantaneous change of phase and thereby introduce memory into the system. This inherent memory of CPM allows it to be represented by a trellis with finite number of states (assuming a *rational* modulation index) [2]. CPMs get more spectrum efficient with longer frequency pulses (partial response CPMs) and higher order (M-ary) signalling. While the transmitter of a CPM, has to do a relatively simple task of obtaining the phase by linear filtering of data [2], the task of demodulating the CPM at the receiver gets rather extremely complex. The complexity of the system results from increased number of matched filtering operations due to a larger signal set (representation) of the CPM. Further, algorithms used to decode CPMs by the ML criterion, are required to keep track of the state metrics at every signalling interval [2, 3]. SCC systems use the soft-input soft-output (SISO) algorithm, a *max-log* version of [3], which is more complex than the maximum likelihood sequence detection algorithm, especially in an iterative decoding scheme, where the data block is repeatedly parsed for a number of times, before determining the underlying information symbols. The number of parallel computations increase with the size of trellis and the number of iterations, and the ML decoding [1] soon becomes unaffordable due to computational costs, latency due to processing and memory requirements, particularly with error control coding when information is transmitted in large blocks. This motivates us to search for complexity reduction techniques. Several methods which have been suggested in literature attempt to reduce the size of trellis as seen by the receiver [4]. These complexity reduction techniques use approximations to reduce the size of trellis, and yet approach the bit error rate (BER) bounds given by the ML criterion. The ultimate aim of all the techniques is to approach the optimal decoding performance with reduced complexity and minimal loss of power. In this paper, we look for *sub-optimal* decoding schemes applicable to SCC systems which use PCM/FM as inner code.

The organization of this paper is as follows. In Section 2, we present the signal model for CPM. In Section 3, we present an overview of SCC CPM system and the SISO algorithm. In Section 4, we discuss some reduced complexity approaches *applicable to* PCM/FM. And in Section 5, we present the simulation results and give our conclusions.

2. SIGNAL MODEL FOR CPM

The signal representation for a complex baseband CPM is of the form [2].

$$s(t;\boldsymbol{\alpha}) = e^{j\phi(t;\boldsymbol{\alpha})}, \tag{1}$$

where $\phi(t;\boldsymbol{\alpha})$ represents the phase of the above CPM [2]. In the most generic form,

$$\phi(t;\boldsymbol{\alpha}) = 2\pi \sum_{i=-\infty}^{\infty} h_{\underline{i}}\alpha_i q(t - iT_s), \tag{2}$$

[1]We are referring to the ML decoding of the CPM, which is different from the ML decoding of the CC. The outer decoder operates on the APP of the code symbols to produce the APP of the input bits to the system. Since the two decoders *talk* to each other in an iterative process, there is a sharp improvement in the performance of the system. However, the SCC system as a whole is not based on the ML criterion.

where $h_{\underline{i}}$ is the *modulation index* associated with the symbol α_i in the i-th symbol interval and T_s is the symbol duration. The modulation index changes cyclically through a finite set of N_h modulation indices ($\underline{i} \triangleq i \bmod N_h$). The *value* of the modulation indices indicate the amount of phase change introduced at the occurrence of a symbol. The phase pulse $q(t)$ defines the phase trajectory in the CPM.

$$q(t) = \begin{cases} 0, & t \leq 0 \\ \displaystyle\int_0^t g(\tau)\, d\tau, & 0 \leq t \leq LT_s \\ \frac{1}{2}, & t \geq LT_s, \end{cases} \tag{3}$$

where $g(t)$ is the *frequency pulse* of duration LT_s. The shape of the frequency pulse is an important parameter which affects the spectral properties of the CPM. PCM/FM uses a raised cosine (RC) pulse [1] given by

$$g(t) = \begin{cases} \frac{1}{2LT_s}\left[1 - \cos\left(\frac{2\pi t}{LT_s}\right)\right], & 0 \leq t \leq LT_s \\ 0, & \textit{otherwise}. \end{cases} \tag{4}$$

Due to the constraints on the *causal* phase pulse $q(t)$ in Eq. (3), Eq. (2) can be written as

$$\phi(t;\boldsymbol{\alpha}) = \underbrace{2\pi \sum_{i=n-L+1}^{n} h_{\underline{i}}\alpha_i q(t - iT_s)}_{\theta(t)} + \underbrace{\pi \sum_{i=0}^{n-L} h_{\underline{i}}\alpha_i}_{\vartheta_{n-L}}, \qquad nT_s \leq t \leq (n+1)T_s. \tag{5}$$

The L-tuple *correlative state vector*

$$\boldsymbol{\alpha}_n = \alpha_{n-L+1}, \ldots, \alpha_n, \tag{6}$$

in $\theta(t)$ contains the L most recent symbols modulated by the *time-varying* part of the phase pulse $q(t)$, which contribute to the phase trajectory of the CPM in the current signalling interval. The state of a CPM is specified in

$$\sigma' = \left[\vartheta_{n-L}, \alpha_{n-L+1}, \ldots, \alpha_{n-1}\right]. \tag{7}$$

On the assumption that the modulation index is a *rational* quantity [2], we can write

$$h_{\underline{i}} = \frac{2K_{\underline{i}}}{P'}, \tag{8}$$

where $K_{\underline{i}}$ and P' are relatively prime. The *cumulative phase state* ϑ_{n-L} in Eq. (5) now becomes

$$\vartheta_{n-L} = \frac{2\pi}{P'} \sum_{i=0}^{n-L} K_{\underline{i}}\alpha_i, \tag{9}$$

which can take on P' distinct values. ϑ_{n-L} defines the starting phase of the CPM at the beginning of the symbol interval (at time n), into which all the previous symbols have been absorbed, $\vartheta_{n-L} \in \left\{\frac{0\cdot 2\pi}{P'}, \frac{1\cdot 2\pi}{P'}, \frac{2\cdot 2\pi}{P'}, \ldots, \frac{(P'-1)\cdot 2\pi}{P'}\right\}$.

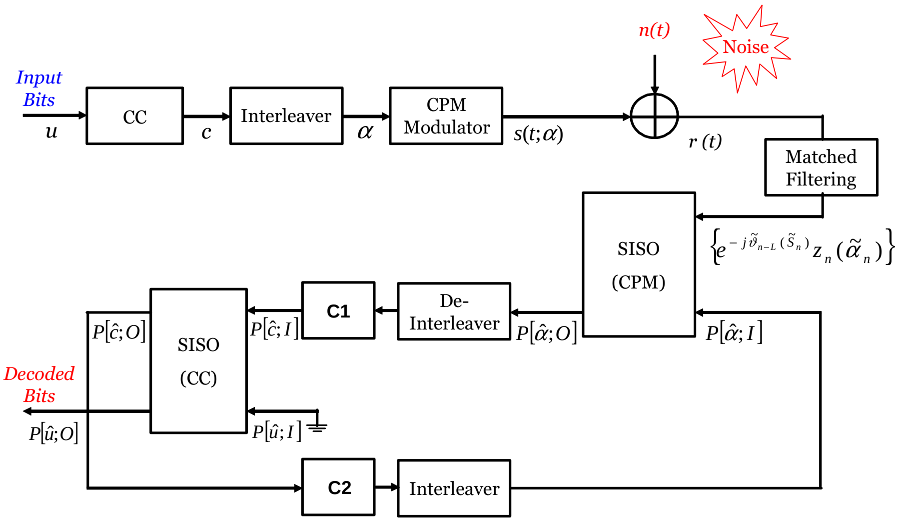

Figure 1: Serially Concatenated Coded CPM.

3. SERIAL CONCATENATION OF CPM WITH CONVOLUTIONAL CODE

The block diagram of a SCC CPM system is as shown in Fig. 1. SCC systems require the inner code to be recursive [5], for which CPMs evolve as a preferred choice. The inner modulation along with the outer code separated by a *pseudo random* interleaver forms the basic structure of the SCC system. It consists of an inner modulation and an outer code, separated by an interleaver. At the transmitter end, we have input bits, possibly from a source encoder. The bit stream is encoded by a convolutional code (CC). The encoded bits are mapped into symbols for CPMs with higher order signalling (quaternary, octal, etc) using *natural* or *gray* mapping. The system model assumes an AWGN channel. The SISO algorithm uses the matched filtered output in the metric computations. Since the CPM modulator operates on the coded (and interleaved) symbols of the input bits, the inner decoder (SISO decoder for CPM) uses the APP of the code symbols produced by the outer decoder (SISO decoder for CC). The outer decoder operates on the APP of the code symbols to produce APP of the input bits to the system. Although the two SISO devices are each based on ML decoding criterion, the overall decoding is *not* ML based since the burden of jointly decoding the inner and outer codes is decoupled [3, 6]. Thus the SCC systems are *reduced complexity* systems when compared to the ML decoding. The SISO algorithm uses the matched filtered output [7] in the metric computations as shown in Fig. 2. Optimal decoding requires M^L matched filters combined with P' phase states. PCM/FM has a 40 state trellis and requires 4 matched filters for optimal decoding. The branch metric for optimal decoding is given by

$$F_n(\widetilde{S}_n, \widetilde{E}_n) = \text{Re}\left\{e^{-j\widetilde{\vartheta}_{n-L}(\widetilde{S}_n)} z_n(\widetilde{\boldsymbol{\alpha}}_n)\right\}, \tag{10}$$

where

$$z_n(\widetilde{\boldsymbol{\alpha}}_n) = \int_{nT_s}^{(n+1)T_s} r(t)\; e^{-j2\pi \sum_{i=n-L+1}^{n} h_{\underline{i}}\widetilde{\alpha}_i q(t-iT_s)} dt, \tag{11}$$

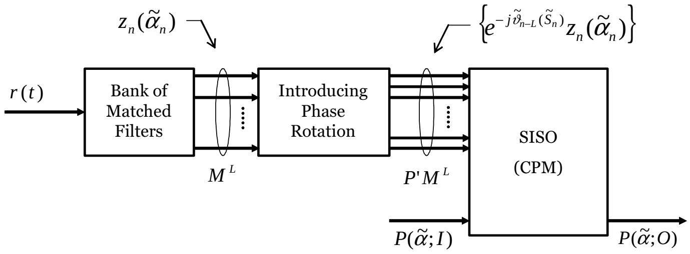

Figure 2: Matched Filtering for the SISO algorithm.

and the metric increment as we step through the trellis is given by

$$\lambda_n(\widetilde{E}_n) = \lambda_{n-1}(\widetilde{S}_n) + \digamma_n(\widetilde{S}_n, \widetilde{E}_n), \tag{12}$$

where $\widetilde{S}_n$ is the starting state for the *hypothesized* trellis branch considered to which the phase state $\widetilde{\vartheta}_{n-L}$ is associated and $\widetilde{E}_n$ is the ending state[2] and $h_{\underline{i}}$ are the *respective* modulation indices associated with $\widetilde{\alpha}_i$.

4. REDUCED COMPLEXITY TECHNIQUES

A. *Rimoldi's Approach*

Using the *tilted phase* approach [8], Rimoldi identified that during any signalling interval, the the number of cumulative states can be reduced by half. This means that the optimal decoding itself requires PM^L states against $P'M^L$, where $h_{\underline{i}} = \frac{K_i}{P}$ (i.e., P=$P'/2$). To realize this, we use the *pseudo* data symbols $u_i = \frac{(\alpha_i+M-1)}{2}$ in the description of cumulative phase tilt ϑ_{n-L} [3]. This transformation *decomposes* ϑ_{n-L} in Eq. (9) into deterministic *data independent* phase tilt ν_{n-L} and *data dependent* phase state θ_{n-L}, given by

$$\vartheta_{n-L} = \frac{2\pi}{P'} \sum_{i=0}^{n-L} K_{\underline{i}} \alpha_i = \frac{\pi}{P} \sum_{i=0}^{n-L} K_{\underline{i}} \alpha_i, \tag{13}$$

which can be written as

$$\vartheta_{n-L} = \theta_{n-L} + \nu_{n-L}, \tag{14}$$

where

$$\theta_{n-L} = \frac{2\pi}{P} \sum_{i=0}^{n-L} K_{\underline{i}} u_i, \tag{15}$$

and

$$\nu_{n-L} = -\frac{(M-1)\pi}{P} \sum_{i=0}^{n-L} K_{\underline{i}}. \tag{16}$$

[2] $(\widetilde{\vartheta}_{n-L}, \widetilde{\boldsymbol{\alpha}}_n)$ and also $(\widetilde{\vartheta}_{n-L}, \widetilde{S}_n)$ can be used to refer to the same trellis branch with the branch metric $(\widetilde{S}_n, \widetilde{E}_n)$ in Eq. (10).
[3] Note that the correlative state vector remains the same.

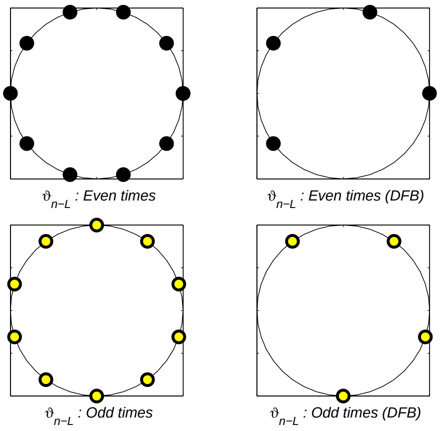

Figure 3: Phase State Reduction.

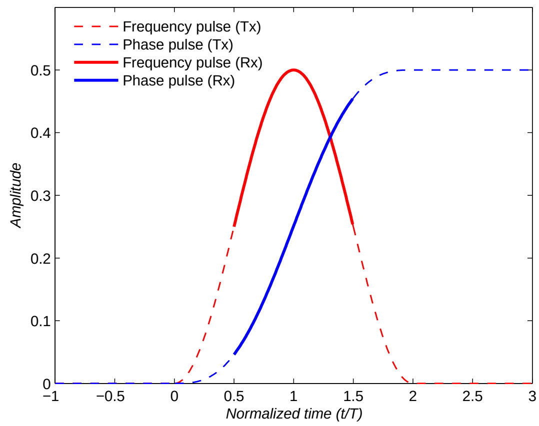

Figure 4: Pulse Truncation.

The phase tilt ν_{n-L} can be recursively obtained through

$$\nu_{n-L} = \nu_{n-L-1} - h_{\underline{n-L}}(M-1)\pi, \tag{17}$$

where $\theta_{n-L} \in \{\frac{0\cdot 2\pi}{P}, \frac{1\cdot 2\pi}{P}, \frac{2\cdot 2\pi}{P}, \ldots, \frac{(P-1)\cdot 2\pi}{P}\}$ and $\nu_{n-L} \in \{\frac{0\cdot 2\pi}{P'}, \frac{1\cdot 2\pi}{P'}, \frac{2\cdot 2\pi}{P'}, \ldots, \frac{(P'-1)\cdot 2\pi}{P'}\}$. The number states (and branches) in the trellis reduces by *half* compared to the classical treatment in [2]. So the metric update equation in Eq. (12) becomes

$$\lambda_n(\widetilde{E}_n) = \lambda_{n-1}(\widetilde{S}_n) + \daleth_n(\widetilde{S}_n, \widetilde{E}_n), \tag{18}$$

where[4]

$$\daleth_n(\widetilde{S}_n, \widetilde{E}_n) = \text{Re}\left\{e^{-j\nu_{n-L}} e^{-j\,\widetilde{\theta}_{n-L}(\widetilde{S}_n)} z_n(\widetilde{\boldsymbol{\alpha}}_n)\right\}. \tag{19}$$

All the further analyses from now on are presented as an improvement over Rimoldi's technique for optimal decoding.

B. *Decision Feedback*

Decision Feedback is a method of reducing the number of phase states via the state space partitioning approach [9, 10]. This method exploits the fact among the several branch metric computations, only a few of the metrics are competitive. Hence branch metrics are computed with fewer phase state multiplications in comparison with optimal decoding. The phase states which need to combined with the outputs of the matched filter bank are determined *at run time* using Eq. (21). The number of phase states used is $P_r < P$ as shown in Fig. 3, which give a complexity reduction. The implementation equations are given by

$$\daleth_n(\widetilde{S}_n^f, \widetilde{E}_n^f) = \text{Re}\left\{e^{-j\nu_{n-L}} e^{-j\,\hat{\theta}_{n-L}(\widetilde{S}_n^f)} z_n(\widetilde{\boldsymbol{\alpha}}_n)\right\}, \tag{20}$$

where $\widetilde{S}_n^f$ and $\widetilde{E}_n^f$ represent the states in the reduced trellis in the usual sense, and

$$\hat{\theta}_{n-L+1}(\widetilde{E}_n^f) = \hat{\theta}_{n-L}(\widetilde{S}_n^f) + \pi h_{\underline{n-L+1}} \hat{u}_{n-L+1}, \tag{21}$$

[4]Note that the correlative state vector remains the same as before.

where $\hat{u}_{n-L+1}$ represent the merging symbols (absorbed into the CPM state) for the states $\widetilde{S}_n^f$ at time n and $h_{\underline{n-L+1}}$ is modulation index *associated* with the merging symbols at time n. Both the phase tilt and cumulative phase updates Eq. (17) and Eq. (21) are performed using the *merging* symbols from the *survivor* branches in the forward recursion which maximize the new state metric at time n. The branch metrics computed during the forward recursion are *saved* for the reverse recursion metric computations.

C. Pulse Truncation

Pulse truncation [11] is motivated by the fact that the frequency pulse has low frequency content at the ends (see Fig. 4). This method is very effective in *correlative state reduction* ($\boldsymbol{\alpha}_n^t$), but gives a negligible loss of performance ($L_r < L$) for PCM/FM. The implementation equations for pulse truncated PCM/FM ($L_r = 1$) are given by

$$z_n(\widetilde{\boldsymbol{\alpha}}_n^t) = \int_{nT_s}^{(n+1)T_s} r(t - T_s/2)\ e^{-j2\pi \sum_{i=n-L_r+1}^{n} h_{\underline{i}}^t \widetilde{\alpha}_i^t q_{PT}(t-iT_s)} dt, \qquad \text{and} \tag{22}$$

$$\lambda_{n+1}(\widetilde{E}_n^t) = \lambda_n(\widetilde{S}_n^t) + \text{Re}\left[e^{-j\nu_{n-L}} e^{-j\widetilde{\theta}_{n-L}(\widetilde{S}_n^t)} z_n(\widetilde{\boldsymbol{\alpha}}_n^t)\right], \tag{23}$$

where again $\widetilde{S}_n^t$ and $\widetilde{E}_n^t$ represent the states in the reduced trellis.

D. Decision Feedback with Pulse Truncation

While decision feedback helps reduce the number of *phase states* pulse truncation reduces the number of *complex matched filters*. A *combination* of the above two techniques gives *both* the advantages (although the loss depends on the overall approximation).

Another way of trellis reduction is by the use of L_r at the receiver where $L_r > L$. This method can bring the number of phase states down more than the decision feedback method. However, the number of matched filters remains the same as in the full state CPM. But they seem to be good only in the uncoded case, where they approach the union bound at high SNR. The implementation equations for $L_r = 3$ and $L_r = 4$ are specified by the following equations (which *assume* PCM/FM as an example):

$$\lambda_{n+1}(\widetilde{E}_n^{f,L_r=3}) = \lambda_n(\widetilde{S}_n^{f,L_r=3}) + \text{Re}\left[e^{-j\nu_{n-3}} e^{-j\hat{\theta}_{n-3}(\widetilde{S}_n^{f,L_r=3})} e^{-j\pi h_{n-2}\hat{u}_{n-2}} z_n(\widetilde{\boldsymbol{\alpha}}_n)\right], \tag{24}$$

$$\hat{\theta}_{n-3+1}(\widetilde{E}_n^{f,L_r=3}) = \hat{\theta}_{n-4+1}(\widetilde{S}_n^{f,L_r=3}) + \pi h_{n-2}\hat{u}_{n-2}, \tag{25}$$

where $\hat{u}_{n-2}$ is now the merging symbol, and

$$\lambda_{n+1}(\widetilde{E}_n^{f,L_r=4}) = \lambda_n(\widetilde{S}_n^{f,L_r=4}) + \text{Re}\left[e^{-j\nu_{n-4}} e^{-j\hat{\theta}_{n-4}(\widetilde{S}_n^{f,L_r=4})} e^{-j\pi h_{n-3}\hat{u}_{n-3}} e^{-j\pi h_{n-2}\hat{u}_{n-2}} z_n(\widetilde{\boldsymbol{\alpha}}_n)\right], \tag{26}$$

$$\hat{\theta}_{n-4+1}(\widetilde{E}_n^{f,L_r=3}) = \hat{\theta}_{n-5+1}(\widetilde{S}_n^{f,L_r=3}) + \pi h_{n-3}\hat{u}_{n-3}, \tag{27}$$

where $\hat{u}_{n-3}$ is now the merging symbol. The initial condition for the data dependent θ_{n-L} is given by

$$I_{n-L} = \left[\underbrace{0, 0, \ldots, 0}_{M^{L_r-1}}, \underbrace{1, 1, \ldots, 1}_{M^{L_r-1}}, \ \ldots\ , \underbrace{P_r-1, P_r-1, \ldots, P_r-1}_{M^{L_r-1}}\right], \tag{28}$$

where $\theta_{n-L} = \frac{\pi}{P} I_{n-L}$ and $I_{n-L} \in \{0, 1, \ldots, P_r, \ldots, P\}$.

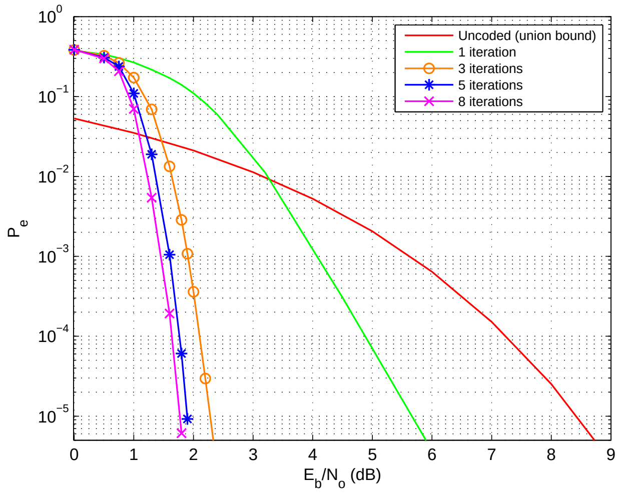

Figure 5: BER with 2048 interleaver.

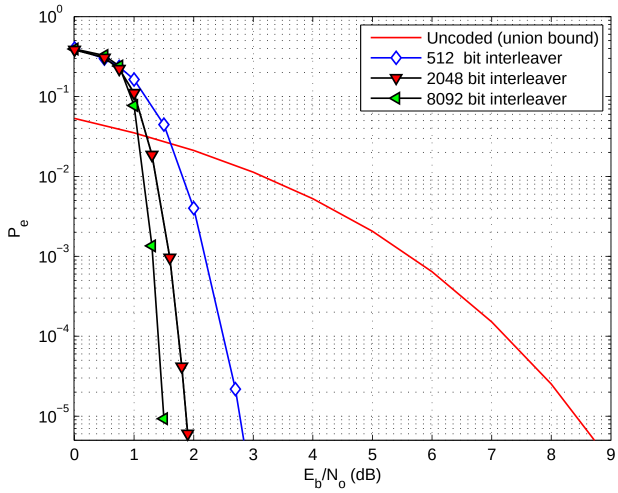

Figure 6: BER with 5 iterations.

Case	P_r	L_r	*Number of States*	*Number of MFs*	*Loss in dB Uncoded*	*Loss in dB Coded*	*Comments*
1	10	2	20	4	0	0	*Optimal*
2	10	1	10	2	0.01	0.015	*Recommended*
3	8	1	8	2	0.07	0.17	*Good*
4	4	1	4	2	> 1	1.01	*Not recommended*

Table 1: Comparison of Reduced Complexity Techniques for PCM/FM.

5. SIMULATION RESULTS AND CONCLUSIONS

We have developed a SCC PCM/FM system. The performance of the system for varying interleaver sizes and iterations is shown in Fig. 5 and Fig. 6. To draw a line between performance compromise and latency of the system, we *handpick* the size of the interleaver to be 2048 and the number of iterations to be 5. All the simulations assume C_1=C_2=0.65, which are the best scale factors determined on a *trial and error* basis. Also, the receiver assumes perfect synchronization to carrier phase (coherent detection). The coded PCM/FM system gives a coding gain (relative to the uncoded system) of 6.55 dB using an optimal 20 state SISO detector for the CPM. This is much larger than coding gains possible with Viterbi algorithm based $(5, 7)$ convolutional coded system.

Among the simulated effective complexity reduction techniques, only a few approximate the uncoded optimal full (20) state detection at low $\frac{E_b}{N_0}$. This is reflected in the BER curves for the coded case. For the coded system, we have built a highly efficient 10 state detector which performs within 0.02 dB of the optimal detector and an 8 state detector which is only slightly worse, with < 0.2 dB loss with respect to the optimal detector.

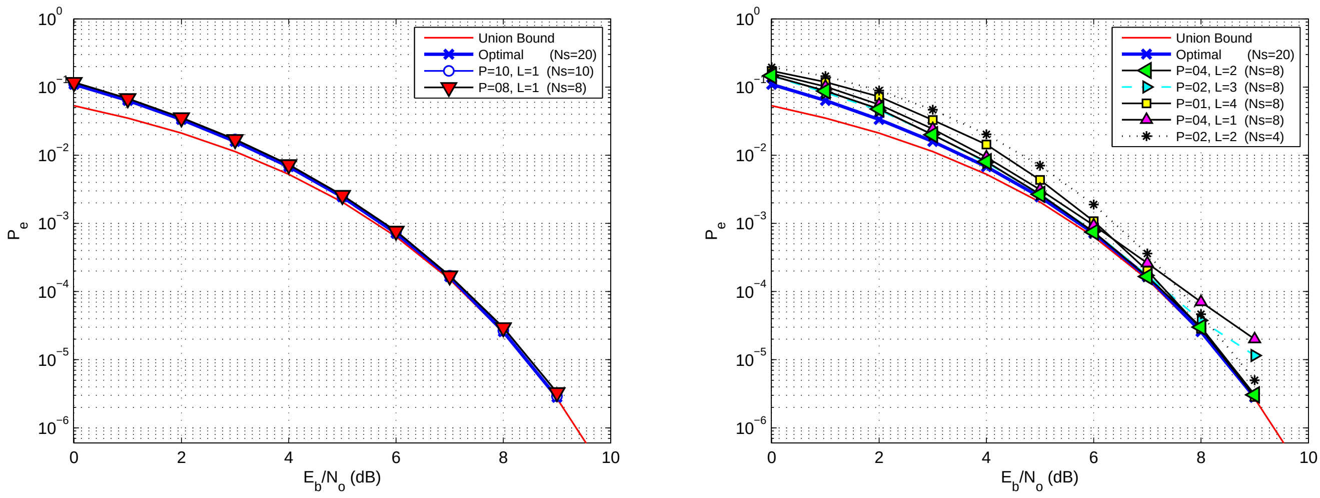

Figure 7: Results: Uncoded PCM/FM.

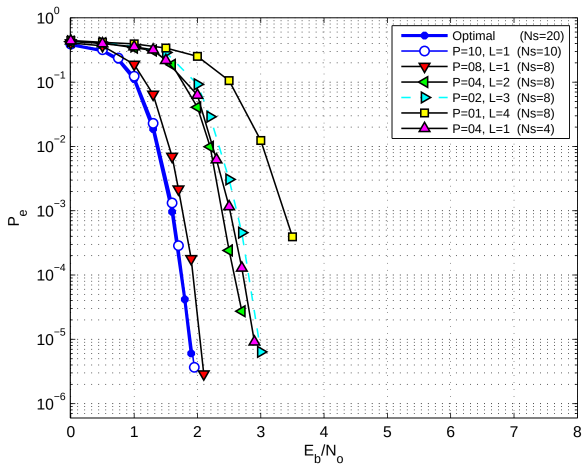

Figure 8: Results: Coded PCM/FM.

Case	L_r	*Initial condition for* ν_{n-L}
a	1	$h(M-1)\pi$
b	2	0
c	3	$h(M-1)\pi$
d	4	0

Case	K	P'	P	$h=\frac{K}{P}$	M	L	*Pulse Type*
Full state PCM/FM	7	20	10	$\frac{7}{10}$	2	2	RC

Table 2: PCM/FM Parameters.

6. ACKNOWLEDGEMENTS

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