JOINT SOURCE/CHANNEL CODING FOR TRANSMISSION OF MULTIPLE SOURCES

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Abstract

A practical joint source/channel coding algorithm is proposed for the transmission of multiple images and videos to reduce the overall reconstructed source distortion at the receiver within a given total bit rate. It is demonstrated that by joint coding of multiple sources with such an objective, both improved distortion performance as well as reduced quality variation can be achieved at the same time. Experimental results based on multiple images and video sequences justify our conclusion.

Keywords
joint source/channel coding, unequal error protection, JPEG2000, motion JPEG2000.

1. Introduction

Today's multimedia applications often require images and videos to be transmitted over noisy channels. To transmit them efficiently, source coding is needed to remove as much redundancy as possible. On the other hand, in order to combat errors introduced by noisy channels, channel coding is often employed to add controlled redundancy. Shannon’s separation theorem states that these two steps can be done separately, with the assumptions that infinite coding complexity etc. are allowed. In practice, however, such assumptions cannot be fulfilled. Therefore, joint source/channel coding (JSCC) has attracted a lot of research effort recently. Many schemes have been proposed for the efficient transmission of different multimedia sources. These schemes can be generally divided into two categories: equal error protection (EEP) and unequal error protection (UEP). In EEP, a coded source receives the same amount of protection from channel coding, while in UEP, different amount of protection is applied to different sections of a coded source according to their relative importance in reconstructing the source.

JSCC for scalable image sources with UEP was studied in [2]. A dynamic programming method was used to perform joint rate allocation based on the optimization criteria of mean squared error (MSE), peak signal/noise ratio (PSNR) and available source rate. The last criterion was suggested as the preferred approach since it reduces the complexity, eliminates the need to transmit the rate schedule and allows optimal transmission at intermediate rates. A UEP scheme was proposed for protecting JPEG2000 bitstreams with turbo codes in [3]. Its joint rate allocation is based on the Viterbi algorithm (VA) for highly scalable JPEG2000 bitstreams and turbo codes. It provides UEP gain and gives a good approximation to the optimal performance obtained by brute force search. Dependence between video coding units was taken into consideration for joint source/channel coding by [4]. Models for both channel coding and video coding performance subject to channel errors were developed for additive white Gaussian noise (AWGN) channels. The scheme is able to provide unequal error protection for different frames in a video sequence.

All the above schemes consider the coding for one image or one video sequence. However, with current state-of-the-art data compression techniques, communication channels are now capable of delivering several compressed images or video sequences concurrently. One solution is to code each image or video sequence independently, while keeping the aggregate bit rate below the channel capacity. However, as pointed out in [5], there are several shortcomings with independent coding over a constant bit rate (CBR) channel. For example, such shortcomings include inefficient use of channel capacity and large variations in picture quality among video sequences. The problem of multiplexing multiple videos has been studied. For example, the authors of [5] proposed a joint coding scheme based on MPEG-2 to dynamically distribute a total bit rate among several video sequences and achieve a more uniform picture quality. However, these schemes do not take channel coding into account.

In [1], a practical joint source/channel coding algorithm based on scalable source coders was proposed for the transmission of multiple images sharing a common channel. It effectively minimizes the expected distortion over all reconstructed images at the receiver end within a given total bit rate. In this paper, we demonstrate that joint coding of multiple sources with such an objective can not only achieve improved end-to-end quality, but also reduce quality variation among reconstructed sources at the same time, where the latter property is attractive for video applications. Experimental results for both multiple images coded by JPEG2000 and HDTV video sources coded by motion JPEG2000 show the efficacy of the algorithm.

This paper is organized as follows: In Section 2, the potential gains provided by joint coding of multiple sources are demonstrated. In Section 3, a specific algorithm that achieves those gains is described. Experimental results are provided in Section 4 and Section 5 draws conclusions.

2. Joint Coding Gains

Consider that multiple sources are to be transmitted over a noisy channel within a given total bit rate. At the transmitter, each source is first coded by a scalable source coder. For each source, its resulting bitstream is partitioned into several segments and each segment is then channel encoded to form a fixed-length channel packet to be sent over the channel. Each packet is assigned a specific channel code rate and the code rate in turn determines the number of source and parity bytes in the packet. At the receiver end, for each source, channel decoding is first performed to recover source bytes from its received channel packets. Channel decoding stops for a given source whenever a channel decoding failure occurs or all the channel packets for the source have been correctly decoded. This practice prevents an otherwise possibly catastrophic effect due to loss of synchronization during the source decoding. The recovered source bytes are then decoded by a source decoder to reconstruct the source.

We consider a joint coding scheme that dynamically distributes a total bit rate among the multiple sources in order to minimize the overall expected distortion for the reconstructed sources at the receiver. In the remainder of this section, we prove that by joint coding multiple sources in such a way, it is possible to achieve both reduced overall expected distortion and a more uniform quality across the reconstructed sources at the receiver.

To begin, we define $\overline{R}_i$ as the expected effective source decoding rate for source *i* at the receiver, which can be written as

$$\overline{R}_i = c_i^1 R_i^s = c_i^1 c_i^2 R_i = c_i R_i \qquad (1)$$

In this expression, R_i is the total rate assigned to source *i* at the transmitter. This rate accounts for both source and parity data. Define the channel coding rate as c_i^2, the portion allocated to just source data is $R_i^s = c_i^2 R_i$. Furthermore, we define c_i^1 as the expected fraction of R_i^s that is decodable by a source decoder at the receiver, considering the effect of residual errors after channel decoding. Finally $c_i = c_i^1 c_i^2$ represents the expected fraction of the assigned rate R_i that can be effectively used by a source decoder to reconstruct the source at the receiver.

With (1), the expected distortion (MSE) for a reconstructed source *i* at the receiver can be modeled as [6]

$$ED_i(R_i) \approx \varepsilon^2 \sigma_i^2 2^{-2\overline{R}_i} = \varepsilon^2 \sigma_i^2 2^{-2c_i R_i} \qquad (2)$$

with the condition that $\overline{R}_i = c_i R_i$ is large enough. In (2), σ_i^2 is the variance of source *i* and ε^2 is a constant related to the performance of a practical compression algorithm.

Let *M* be the number of sources to be coded jointly, and let *R* be the average bit rate over all sources. The objective of a joint coding scheme is

$$\min \sum_{i=1}^{M} ED_i(R_i) \quad s.t. \quad \sum_{i=1}^{M} R_i \le MR \qquad (3)$$

To find the optimal rate for each source, we may use the Lagrangian method and set

$$\frac{\partial}{\partial R_i}\left\{ \sum_{i=1}^{M} ED_i(R_i) + \lambda \sum_{i=1}^{M} R_i \right\} = 0 \qquad (4)$$

This gives

$$\lambda = \varepsilon^2 \sigma_i^2 (c_i + c_i^{'} R_i) 2^{-2c_i R_i} \approx \varepsilon^2 c_i \sigma_i^2 2^{-2c_i R_i} \qquad (5)$$

with the assumption that $c_i >> \left| c_i^{'} \right| R_i$, where $c_i^{'}$ is the derivative of c_i with respect to R_i. This assumption simplifies the derivation and is clearly true when the coefficient c_i is insensitive to changes of R_i.

From (5), bit rate R_i for source *i* is

$$R_i = \frac{1}{2c_i}\log_2 \frac{\varepsilon^2 (c_i\sigma_i^2)}{\lambda} \quad (6)$$

By imposing the average bit rate constraint, λ can be re-written as

$$\lambda = \varepsilon^2 \left[\prod_{i=1}^{M} (c_i\sigma_i^2)^{1/c_i}\right]^c 2^{-2cMR} \quad (7)$$

where c is defined as $(\sum_{i=1}^{M} \frac{1}{c_i})^{-1}$.

Together with (2) and (6), the optimal rate assigned to source i is given by

$$R_i^* = \frac{cM}{c_i} R + \frac{1}{2c_i}\log_2 \frac{c_i\sigma_i^2}{\left[\prod_{i=1}^{M} (c_i\sigma_i^2)^{1/c_i}\right]^c} \quad (8)$$

and the optimal distortion corresponding to rate R_i^* is

$$ED_i(R_i^*) = \frac{\varepsilon^2}{c_i}\left[\prod_{i=1}^{M} (c_i\sigma_i^2)^{1/c_i}\right]^c 2^{-2cMR} \quad (9)$$

According to (2), when the same rate R is assigned to each source (separate coding), the ratio between the expected distortion of two arbitrary sources i and j is

$$\frac{ED_i(R)}{ED_j(R)} = \frac{\sigma_i^2}{\sigma_j^2} 2^{-(c_j - c_i)R} \quad (10)$$

and from (9), the corresponding ratio with joint coding is

$$\frac{ED_i(R_i^*)}{ED_j(R_j^*)} = \frac{c_j}{c_i} \quad (11)$$

Suppose that c_i remains constant independent of a specific source i and is insensitive to changes of R_i. With these two assumptions, we see from (10) and (11) that joint coding provides constant quality regardless of the variances of the original sources. We call the gain in terms of quality variance reduction among reconstructed sources at the receiver "variance multiplexing" gain hereafter and define it as

$$\text{Variance Multiplexing Gain} = \left(1 - \frac{Var(MSE_{joint})}{Var(MSE_{separate})}\right) \times 100\% \quad (12)$$

From previous results in the literature (also verified below), for effective JSCC schemes, $\overline{R}_i$ varies in a small range close to R_i^s for different sources and total rates. This implies that the coefficient c_i^1 also varies in a small range close to 1. Based on this observation, it suffices to analyze c_i^2 to determine if our two assumptions are satisfied.

As discussed above, the optimization criterion is expected distortion. Solutions to such problems are called "distortion-based optimal solutions" in [7]. Meanwhile, as proposed in [2], for quality scalable source coders, maximizing the expected number of correctly received source bits can be an alternative. Solutions of such problems are called "rate-based solutions" in [7]. One unique feature of the rate-based criterion is that its solution is independent of specific source statistics or source coding performance. The rate-based criterion is proven to yield suboptimal solutions to the distortion-based problem [7]. However, in the context of fixed length channel packets and a source coder with nonincreasing and convex operational distortion-rate function, an error bound between the MSE achieved under the two criteria was derived, and the gap was shown to be small [7]. Therefore, a distortion-based scheme has solutions close to those based on the rate-optimal criterion, which is independent of a specific source. This implies that c_i^2 is approximately independent of a specific source, which satisfies our first assumption (it is also verified by our experiments below.)

For a rate-based optimal solution, it has been shown in [8] that an optimal rate allocation typically has a long run of channel packets assigned the same code rate. This is due to the fact that the number of available code rates is usually much smaller than the number of transmitted packets, and also to the property that the code rates are nonincreasing for an optimal solution [8]. Even at the points where the code rate changes, these changes are usually small. This is due to the fact that the suitable rates inside a channel code family for a specific channel are not far apart. In Fig.1, we plot c_i^2 as functions of R_i for the rate-based optimal solution based on the experimental results in [7], with the total bit rate ranging up to 4.00 bpp. These results were obtained for the rate compatible punctured turbo (RCPT) and convolutional (RCPC) codes, for binary symmetric channel (BSC) with bit error rates (BER) of 0.05 and 0.1 respectively. From the figure, except at very low rates for RCPC, c_i^2 remains almost constant. Among these curves, the maximum of $\left|(c_i^2)^{'}\right|$ occurs at $R_i = 0.0078$ bpp with a value of 0.0143 and $c_i^2 = 0.5714$ bpp. Obviously $c_i^2 >> \left|(c_u^2)^{'}\right| R_i$. Therefore, it can be concluded that c_i^2 (and so c_i) is weakly dependent on R_i, which satisfies our second assumption.

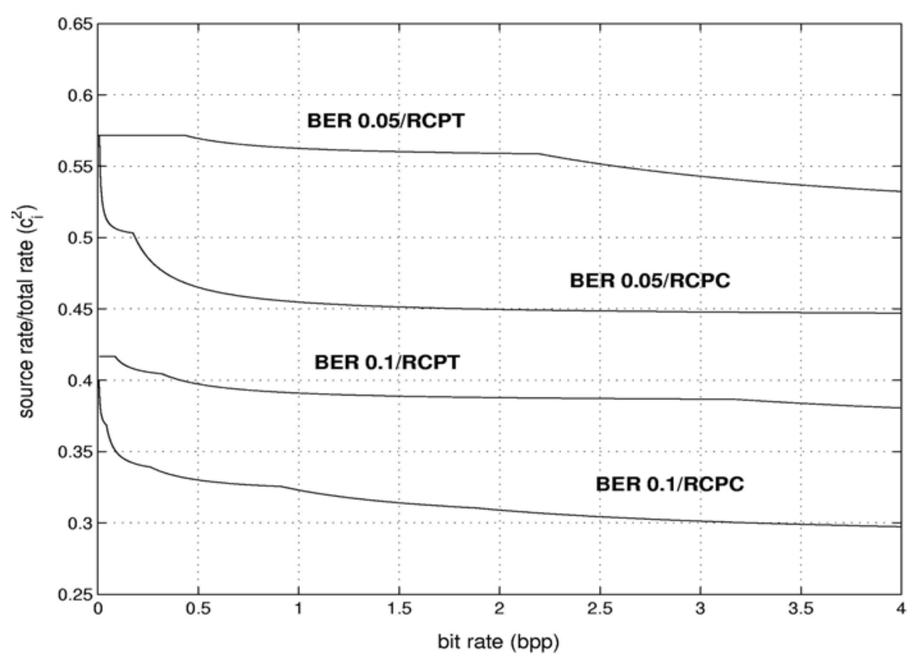

Figure 1: The coefficient c_i^2 as a function of total rate R_i.

From (2) and (9), together with the two assumptions made in this section, the ratio between the overall expected distortion for *M* sources coded separately and jointly is

$$\frac{\sum_{i=1}^{M} ED_i(R)}{\sum_{i=1}^{M} ED_i(R_i^*)} = \frac{\frac{1}{M}(\sum_{i=1}^{M}\sigma_i^2)}{\left(\prod_{i=1}^{M}\sigma_i^2\right)^{1/M}} \tag{13}$$

which is the ratio between the arithmetic mean and the geometric mean of the source variances. Since the arithmetic mean is always equal to or larger than the geometric mean, the overall expected distortion can be reduced if multiple sources are coded jointly. This gain is called "quality multiplexing" gain hereafter and it is defined as

Quality Multiplexing Gain = PSNR$_{joint}$ – PSNR$_{sepearte}$

$$=10\left[\lg\frac{1}{M}(\sum_{i=1}^{M}\sigma_i^2) - \frac{1}{M}\lg\left(\prod_{i=1}^{M}\sigma_i^2\right)\right] \tag{14}$$

Based on (11) and (13), it is clear that by jointly coding multiple sources with a goal to minimize the overall expected distortion, it is possible to obtain both improved and more uniform quality of reconstructed frames at the receiver.

3. Joint Source/Channel Coding System

A specific joint coding algorithm was proposed in [1] to jointly code multiple sources that can minimize the overall expected distortion. It is briefly described in this section. For more details, the reader is referred to [1].

Let L_i be the bitstream length allocated to frame *i* (corresponding to its bit rate R_i) by the joint coding scheme and resulting in N_i channel packets of a fixed length L_p. Let $V_i = [r_i^1 r_i^2 \cdots r_i^{N_i}]$ be a vector representing the channel code rates assigned to the N_i packets by the JSCC scheme. Therefore, V_i determines the optimal rate allocation between the source and channel codes, and the protection level each source segment receives from the channel codes. (3) can be re-written as

$$\min\sum_{i=1}^{M}(D_{i,0} - E[\Delta D_i(V_i)]) \quad s.t. \quad \sum_{i=1}^{M} L_i(V_i) \le L_T \tag{15}$$

where $D_{i,0}$ is the zero rate distortion of frame *i*, $E[\Delta D_i(V_i)]$ is the expected distortion reduction when V_i is employed. L_T is the aggregated bitstream length corresponding to the total bit rate *MR* to be shared by the *M* frames. (15) can be converted into an unconstrainted version by the Lagrangian multiplier method

$$\min\left\{-\sum_{i=1}^{M} E[\Delta D_i(V_i)] + \lambda\sum_{i=1}^{M} L_i(V_i)\right\} \tag{16}$$

For a given λ, the optimal solution can be obtained by solving each term (corresponding to each frame) independently. And λ is adjusted such that the sum of the bitstream lengths from all the frames is less than or equal to the aggregated bitstream length.

At the receiver, for each frame, the channel decoding stops whenever a decoding failure occurs or all the channel packets for the frame have been correctly decoded. Only the source bytes recovered from the error free channel packets are decoded by a source decoder to reconstruct the original source. Therefore, together with (16), the objective function to be minimized for frame *i* is

$$\begin{aligned} f(V_i) &= -E[\Delta D(V_i)] + \lambda L_i(V_i) \\ &= -\sum_{m=1}^{N_i} \left(\sum_{n=1}^{m} d_i^m(r_i^n)\right)\left(\prod_{n=1}^{m}\left[1 - Pe(r_i^n)\right]\right) Pe(r_i^{m+1}) + \lambda L_p N_i \end{aligned} \quad (17)$$

where $d_i^n(r_i^n)$ is the distortion reduction brought by the source bytes included in the *n*th channel packet with code rate r_i^n. And $Pe(r_i^n)$ denotes the probability of channel packet decoding failure when the code rate r_i^n is used to protect the nth packet (note that $Pe(r_i^{N_i+1})$ is defined to be 1 to indicate the end of the bitstream.) A Viterbi based scheme was proposed to solve (17) efficiently.

4. Experimental Results

In the first experiment, the assumptions made in Section 2 about the JSCC scheme are verified. The JPEG2000 based Kakadu software is used as the source coder (the layering functionality is employed during the encoding to generate quality scalable bitstreams.) And the RCPT codes from [9] are used as the channel coder. Two images Lenna and Whitehouse (each is a 512 $\times$512, 8-bit gray-level image) are chosen as our test images, which have very different distortion-rate characteristics. JSCC is performed separately for the two images for binary symmetric channel (BSC) ε = 0.01 at total bit rates of 1.00, 0.50 and 0.25 bpp. The channel code rate set $\left\{\frac{8}{9}, \frac{4}{5}, \frac{8}{11}, \frac{2}{3}\right\}$ is chosen for the channel, and each channel packet has a fixed length of 500 bytes. The coefficients c_i^1, c_i^2 and c_i are obtained for both images and are listed in Table 1. The coefficient c_i^2 is obtained directly from the JSCC scheme, while c_i^1 and c_i are obtained by extensive channel simulations with at least 2000 trials for each case. It can be seen that the coefficient c_i^1 is varying in a small range (an interval of about 0.04) close to 1. Additionally, c_i^2 and c_i are seen as insensitive to the different images and total bit rates. A maximum ratio of only 1.04 between the values of c_i is observed for the two images. This should be compared to the ratio of image variances, which is 1.82. This shows the potential of obtaining the variance multiplexing gain as derived in Section 2

In the following experiment, five images Lenna, Peppers, Goldhill, Baboon and Whitehouse are coded both separately and jointly for BSC ε = 0.01 to demonstrate the quality and variance multiplexing gains. For each case, at least 2000 trials are conducted and the results (in MSE) are listed in Table 2. Quality multiplexing gains are achieved between 0.58 and 1.66 dB per image (in terms of PSNR converted from the corresponding MSE values). Variance multiplexing gains between 67% to 90% are obtained.

The rest of the experiments are conducted for HDTV sequences coded by motion JPEG2000. Three progressive HDTV sequences Blue_sky, River_bed and Tractor are chosen as our test sequences. Only the luminance components of the first 200 frames from each sequence are considered, where each luminance component (frame) is a 1920 × 1080 8-bit gray-level image. The following multiplexing dimensions are considered and listed along with their acronyms:

1) (s,s): separate coding of each frame from the 3 sequences (600 encodings)
2) (j,s): joint coding of 3 frames, one from each sequence (200 encodings)
3) (s,j): joint coding of 200 frames from one sequence (3 encodings)
4) (j,j): joint coding of all 600 frames from all sequences (1 encoding).

It is clear that (j,s) exploits the diversity between the sequences one frame at a time, (s,j) exploits

Table 1:The coefficients for different images at different total bit rates.

Images	1.00 bpp			0.50 bpp			0.25 bpp		
	c_i^1	c_i^2	c_i	c_i^1	c_i^2	c_i	c_i^1	c_i^2	c_i
Lenna	0.94	0.71	0.67	0.96	0.72	0.69	0.94	0.71	0.67
Whitehouse	0.94	0.71	0.67	0.92	0.73	0.67	0.93	0.72	0.67

Table 2: Comparison between separate and joint coding in MSE.

Images	1.00 bpp		0.50 bpp		0.25 bpp	
	Separate	Joint	Separate	Joint	Separate	Joint
Lenna	9.09	25.71	17.91	40.34	38.72	73.45
Peppers	14.04	29.50	22.38	43.81	40.94	77.07
Goldhill	22.78	41.91	43.69	74.15	74.00	137.25
Baboon	131.31	53.74	253.37	145.68	383.68	254.90
Whitehouse	164.98	82.42	301.17	181.27	485.49	352.53
Ave. MSE	68.33	46.66	125.04	97.05	204.57	179.04
PSNR (dB)	29.78	31.44	27.16	28.26	25.02	25.60
Var. MSE	5.46×10^3	5.21×10^2	1.90×10^4	4.01×10^3	4.54×10^4	1.48×10^4

the diversity within one sequence and (j,j) exploits the diversity across both sequences and frames.

Because of the vast amount of data involved with HDTV sequences, real channel simulation is prohibitive. However, from the previous observation that the coefficient c_i^1 changes only in a small range close to 1, it is reasonable to use the error free performance based on R_i^s at the transmitter as an indicator of the noisy performance that would be achieved at the receiver.

The three HDTV sequences were coded at 25 and 50 Mbps, 30 frame per second (fps) over BSC $\varepsilon = 0.1$ with the four multiplexing strategies. The RCPT code rates $\left\{\frac{4}{9},\frac{8}{10},\frac{2}{5},\frac{8}{21},\frac{4}{11},\frac{1}{3}\right\}$ were chosen for the channel, and each channel packet had a fixed length of 500 bytes. The reported PSNR values are calculated by converting from the corresponding average MSE values. As shown in Tables 3 and 4, compared to the non-multiplexed (s,s) case, about 0.32 to 0.79 dB per frame quality multiplexing gain and about 68% to 83% variance multiplexing gain can be achieved at 25 Mbps. At 50 Mbps, about 0.52 to 0.76 dB per frame quality multiplexing gain and about 87% to 92% variance multiplexing gain can be obtained. These quality multiplexing gains are in addition to the UEP gains given by the usual (s,s) JSCC scheme. Although not shown in the tables, the UEP gains are around 0.3 to 0.4 dB per frame for these sequences. Therefore, the quality multiplexing gains obtained are significant compared to the UEP gains.

Notice that different multiplexing strategies have different buffer size and delay requirements. Among them, (j,j) requires the largest buffer size (and the longest delay) but gives the best performance. While (s,j) buffers all the frames from a sequence, because the strong similarities

Table 3: Performance comparison among different multiplexing dimensions at 25 Mbps

25 Mbps	(s,s)		(s,j)		(j,s)		(j,j)	
	MSE	var MSE	MSE	var MSE	MSE	var MSE	MSE	var MSE
Blue_sky	40.99	962.96	36.88	285.41	25.79	174.55	25.07	101.19
River_bed	20.07	2.97	20.19	6.93	22.50	2.14	21.77	2.70
Tractor	14.09	67.30	12.82	31.92	14.82	45.57	15.92	68.69
Average	25.05	344.41	23.30	108.09	21.04	74.09	20.92	57.53
PSNR(dB)	34.14	-	34.46	-	34.90	-	34.93	-

Table 4: Performance comparison among different multiplexing dimensions at 50 Mbps

50 Mbps	(s,s)		(s,j)		(j,s)		(j,j)	
	MSE	var MSE	MSE	var MSE	MSE	var MSE	MSE	var MSE
Blue_sky	15.59	172.87	12.82	20.34	8.49	17.74	8.57	11.69
River_bed	9.50	38.35	9.44	0.73	10.58	0.63	10.44	0.47
Tractor	6.28	40.15	5.58	2.86	7.30	2.75	7.29	2.57
Average	10.45	61.65	9.28	7.98	8.79	7.04	8.76	4.91
PSNR(dB)	37.94	-	38.46	-	38.69	-	38.70	-

among the frames within one sequence, it can only provide very modest gains. On the other hand, while (j,s) only requires a 3-frame buffer (one frame for each sequence), it obtains most of the gains compared to the best case (j,j). This is due to the large differences in terms of the rate-distortion characteristics among the HDTV sequences.

Finally, the noise free performance for each frame of the Blue_sky sequence is illustrated in Figure 2. All four multiplexing strategies shown are at 50 Mbps. The original sequence has hard-to-compress frames at the end, which causes a large MSE variation at the receiver when each frame is coded separately. However, by joint coding with the three different multiplexing strategies, the MSE variation is significantly reduced, resulting in more uniform quality across the reconstructed frames at the receiver.

5. Conclusion

In this paper, we first demonstrate that by coding multiple sources jointly, quality improvement and a reduction in quality variation are achievable at the same time. A practical joint source/channel coding algorithm for multiple sources is then briefly described. Experimental results show the advantage of the proposed algorithm with multiple image and video sources coded by JPEG2000 and motion JPEG2000, respectively.

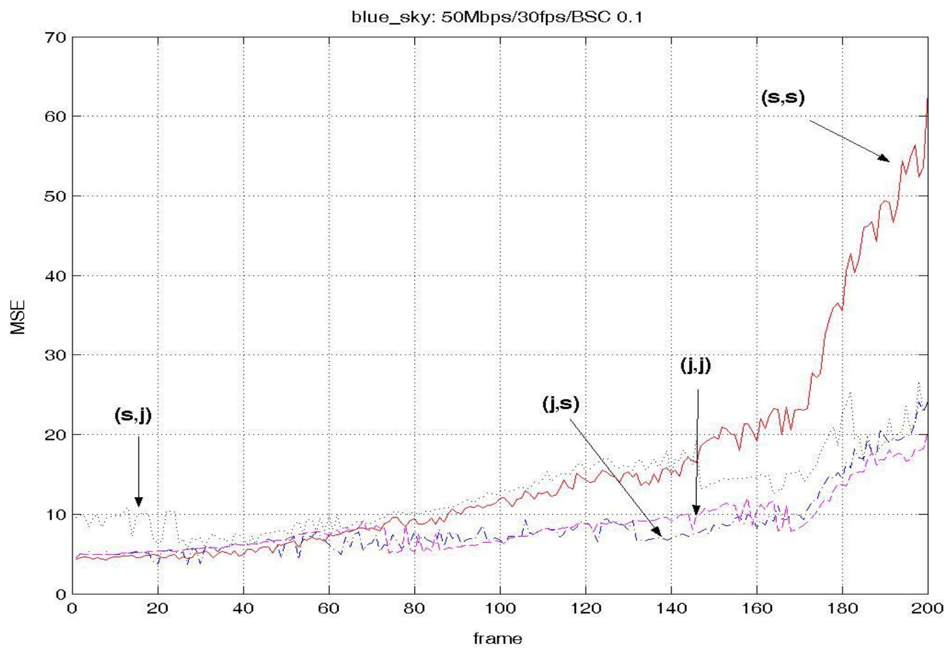

Figure 2: Blue_sky performance with different multiplexing strategies at 50 Mbps.

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