A MODIFIED FOUR-QUADRANT FREQUENCY DISCRIMINATOR FOR CARRIER FREQUENCY ACQUISITION OF GPS RECEIVERS

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ABSTRACT

The four-quadrant frequency discriminator (FQFD) [2] plays an important role in GPS receivers for carrier synchronization. This paper presents a detailed study of the operating principle of the FQFD, and the acquisition performance degradation due to the gain fluctuation of the FQFD is discussed. A modified FQFD called the enveloped-four-quadrant frequency discriminator (Enveloped-FQFD) is proposed, which introduces an envelope calculator on the basis of the FQFD. Performance comparison of the FQFD and the Enveloped-FQFD is given through theoretical analysis and computer simulation. Simulation results show that by employing the Enveloped-FQFD, a quicker pull-in process and a wider threshold[1] than the FQFD can be achieved, while the additional hardware costs are trivial.

KEY WORDS

GPS, Acquisition, Carrier synchronization, Four-quadrant frequency discriminator

I. INTRODUCTION

Carrier synchronization algorithms keep seeking the solution with a short acquisition period and a low residual tracking error. Unfortunately this is an antinomy in the PLL design. Traditional PLLs can hardly meet the needs of carrier synchronization in the high dynamic circumstances such as high dynamic GPS receivers. The quick and accurate synchronization of carrier frequency becomes one of the key problems. Designers usually adopt a narrow bandwidth loop to assure the low residual tracking error while relying on some auxiliary methods such as new types of frequency discriminators, loop filter switching or loop gain switching techniques to shorten the pull-in period. GPS receivers for the dynamic applications usually adopt a combined solution of carrier synchronization. In these applications, after the PN code phase and the Doppler frequency are coarsely aligned, a FQFD loop pulls the doppler frequency from hundreds Hertz into the tracking range of the following automatic frequency control (AFC)[1] loop of about 10 Hertz. Then the AFC will quickly lock-in the carrier frequency [2]. In this way the carrier synchronization is achieved both quickly and accurately.

In section II the carrier frequency pull-in process of the FQFP is discussed in detail, the degradation caused by its gain fluctuation is also discussed. In section III the Enveloped-FQFP algorithm based on the FQFP is presented. Comparison between the FQFD and the Enveloped-FQFD is made by theoretical analysis and computer simulation in section IV. Conclusions are made in the last section of this paper.

II. THE OPERATING PRINCIPLE OF THE FQFD

The FQFP algorithm is somewhat different from the AFC used in tracking loops. The AFC loop usually employs a cross-product discriminator[3], which has a wider lock-in range and a lower SNR threshold than the traditional PLLs. The cross-product discriminator is restricted to its linear section in order to guarantee the frequency tracking accuracy, while the FQFD can work in the nonlinear section with a wider lock-in range than the cross-product discriminator.

In the high dynamic GPS receivers, the in-phase and quadrature outputs of the correlator are of the following forms[2]:

$$I(k) = A\cdot R[\varepsilon(k)]\mathrm{sinc}(f_d \pi T)\cdot\cos(2\pi f_d kT + \phi_k) + n_I(k) \quad (2.1)$$

$$Q(k) = A\cdot R[\varepsilon(k)]\mathrm{sinc}(f_d \pi T)\cdot\sin(2\pi f_d kT + \phi_k) + n_Q(k) \quad (2.2)$$

Where $R[\varepsilon(k)]$ is the correlation function, T is the correlation interval, ϕ_k is the initial phase error, and f_d is the input frequency error. The discriminator correction term of the FQFD is derived by comparing I and Q correlations between successive readings[2]:

$$I(k) - I(k-1) = -2A\cdot R[\varepsilon(k)]\cdot\mathrm{sinc}(f_d \pi T)\cdot\sin[(2k-1)\pi f_d T + \phi_k]\cdot\sin(\pi f_d T) \quad (2.3)$$

$$Q(k) - Q(k-1) = 2A\cdot R[\varepsilon(k)]\cdot\mathrm{sinc}(f_d \pi T)\cdot\cos[(2k-1)\pi f_d T + \phi_k]\cdot\sin(\pi f_d T) \quad (2.4)$$

The choice of the correction term and its sign is derived from the current correlations and their respective magnitudes. Assume $\theta_k = 2\pi f_d kT + \phi_k$, and we divide θ_k into four quadrant according to the sign of (2.5)[2]:

$$|I(k)| - |Q(k)| = A\cdot R[\varepsilon(k)]\mathrm{sinc}(f_d \pi T)\cdot\{|\cos\theta_k| - |\sin\theta_k|\} \quad (2.5)$$

Given that the PN code phase is coarsely aligned within one chip, we get $R[\varepsilon(k)] > 0$. Let β denotes the correction term expressed as[2]:

$$For\ |I(k)| \ge |Q(k)|: \beta = sign[I(k)] \cdot \Delta Q \quad (2.6)$$

$$For\ |I(k)| \le |Q(k)|: \beta = -sign[Q(k)] \cdot \Delta I \quad (2.7)$$

Where $\Delta I = I(k) - I(k-1), \Delta Q = Q(k) - Q(k-1)$. The term $sign[I(k)]$ and $-sign[Q(k)]$ in (2.6) and (2.7) are used to eliminate the influence of the sign of product terms before $\sin(\pi f_d T)$ in (2.3) and (2.4). As we can see, there is a phase difference $\varphi = \pi f_d T$ between the carrier phase of (2.1) and (2.2) and that of (2.3) and (2.4). Only when $|\pi f_d T| \le \pi/4$, can the sign correction in (2.6) and (2.7) work correctly. That's why the FQFP is assumed to be working within the range of $|f_d| \le 1/4T$. As $|\pi f_d T|$ grows larger than $\pi/4$, there will be some portion of correction value in (2.6) and (2.7) goes in the opposite direction, which results in frequency discriminator errors. But if we dig further into the problem, it can be seen that under some given SNR, because of the integration term of the NCO following the frequency discriminator, the FQFP can still pull-in frequency in the right direction as long as the average correction term has the correct sign, which extents the theoretical noise-free working range of the FQFP to $|f_d| \le 1/2T$.

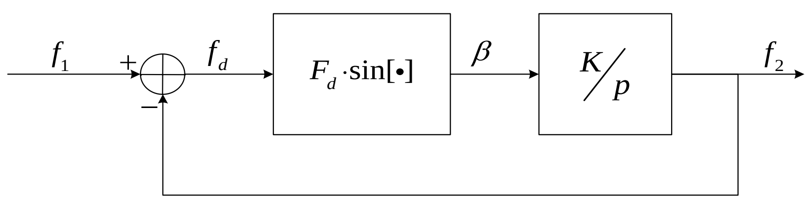

Fig.1 A simplified structure of the pull-in process of the FQFD

The FQFP loop can be approximated to the structure in Fig.1, where F_d denotes the discriminator gain:

$$F_d = |A \cdot R[\varepsilon(k)] \cdot \mathrm{sinc}(\pi f_d T)| \cdot \omega_d \quad (2.8)$$

and ω_d denotes the carrier phase:

$$\omega_d = \begin{cases} |\cos(2\pi f_d kT + \varphi + \phi_k)|, & while \quad |I(k)| \ge |Q(k)| \\ |\sin(2\pi f_d kT + \varphi + \phi_k)|, & while \quad |I(k)| \le |Q(k)| \end{cases} \quad (2.9)$$

The $1/p$ in Fig. 1 represents the integration term in the NCO, and K denotes the gain of the NCO. As can be seen in (2.8) and (2.9), F_d varies with time. Therefore, elimination of the fluctuation of F_d promises the loop a constant gain, enhances the pull-in capacity of the FQFP, and consequently shortens the carrier acquisition period.

III. THE ENVELOPED-FQFD

As discussed in section II, the choice of the discriminator correction term is determined by equation (2.5), while its magnitude fluctuates due to ω_d. If we consider only the envelope of ω_d, we can get a constant gain $F_d^{'} = A \cdot R[\varepsilon(k)] \cdot \mathrm{sinc}(\pi f_d T)$, which will improve the acquisition performance of the FQFD. Replace F_d with $F_d^{'}$ and we get the Enveloped-FQFD. Both the Enveloped-FQFD and the FQFD structures are shown in Fig.2.

In the Enveloped-FQFD, sign of the correction is derived from the traditional FQFD output, and its magnitude is derived from the envelope of ΔI and ΔQ, so the correction of the Enveloped-FQFD is of the form:

$$\beta^{'} = sign(\beta) \cdot \sqrt{\Delta I^2 + \Delta Q^2} \qquad (2.10)$$

In a digital implementation, $\beta^{'}$ can be calculated by using the following approximation:

$$\beta^{'} \approx sign(\beta) \cdot \{\max[|\Delta I|, |\Delta Q|] + \tfrac{1}{2}\min[|\Delta I|, |\Delta Q|]\} \qquad (2.11)$$

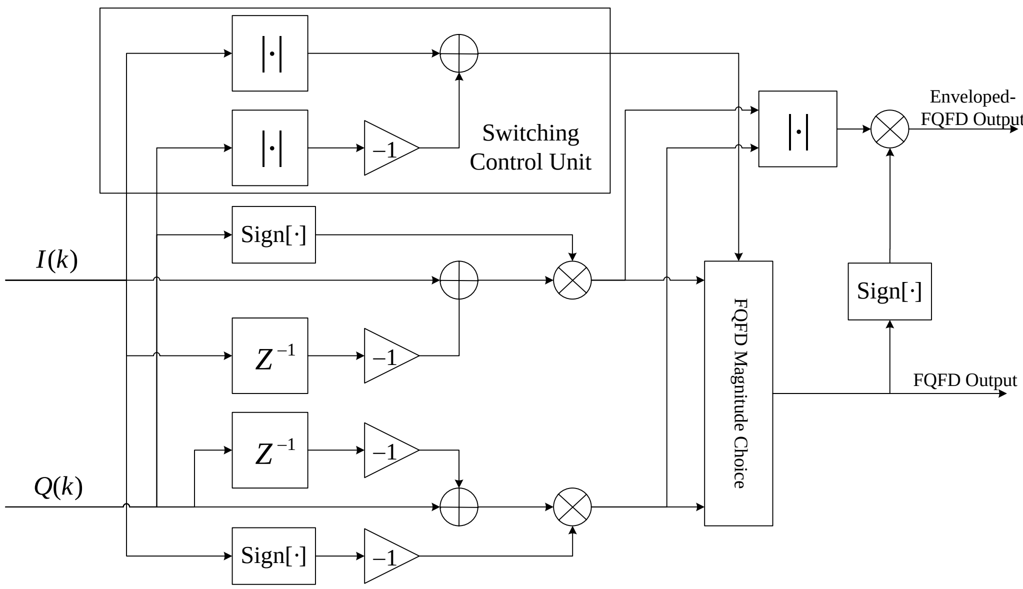

Fig.2 The structures of the Enveloped-FQFD and the FQFD

IV. THE PERFORMANCE COMPARISON

The frequency pull-in process of the FQFD can be expressed as:

$$f_2 = F_d \cdot K \cdot \sin(\pi f_e T) \cdot 1/p \qquad (3.1)$$

Where $f_e = f_1 - f_2$, f_1 is the initial frequency error and f_2 is output frequency of the acquisition loop. An alternative of (4.1) can be derived as:

$$f_e^{'} = f_1^{'} - F_d \cdot K \cdot \sin(\pi f_e T) \qquad (3.2)$$

Where $f_e^{'}, f_1^{'}, f_2^{'}$ are the derivatives of f_e, f_1, f_2 respectively. If f_1 is a constant, then $f_1^{'} = 0$. Equation (4.2) shows the relationship between the change rate of the input frequency and the change rate of the discriminator error frequency.

As denoted in (2.8), the gain fluctuation of F_d is determined by ω_d. Fig. 3 shows how the maximum value, the minimal value and the average value of ω_d vary with different input frequency errors. Fig.3 shows that as $|f_d T|$ goes beyond 0.25, the minimal value of ω_d goes negative, and the correction of the FQFD goes in a wrong direction. But the average value of ω_d is positive until $|f_d T|$ grows beyond 0.5. As we mentioned in section II, due to the integration term of the NCO, the FQFD can still pull-in the input frequency as long as $|f_d T| < 0.5$, and the estimated threshold[1] will be $\sigma_{\Delta f_d} T = 1/3 \cdot |f_{d\max}| = 0.167$, which is wider than that of the FQFD of 0.07. This feature plays an important role in the carrier acquisition, for it allows the acquisition algorithms to employ a wider frequency bin during the carrier searching so as to shorten the carrier frequency searching period.

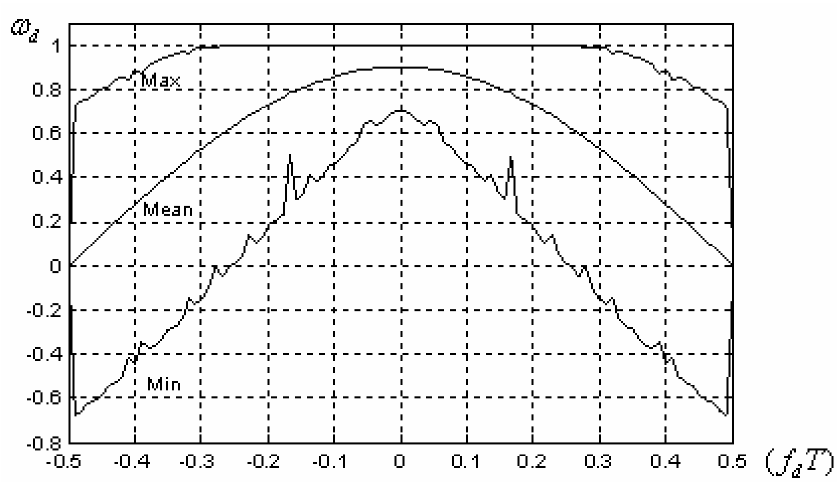

Fig. 3 ω_d fluctuation with. f_d

Fig. 3 shows clearly that the noise-free working range of the FQFD is $|f_d| < 1/2T$. And when $|f_d T| < 0.25$, the average value of ω_d fluctuates in the range of $[0.64, 0.9]$, which means the loss of gain in F_d. With respect to $F_d^{'}$ in the Enveloped-FQFD, the average gain degrades by 23%, and the degradation leads to a weakened pull-in process. A comparison of the phase plane curves between the FQFD and the Enveloped-FQFD is shown in Fig. 4, which shows that the Enveloped-FQFD has a more powerful pull-in capacity than the FQFD. In Fig. 5, the computer simulation result shows that the Enveloped-FQFD has a quicker pull-in process than the FQFD, where the input frequency error is a frequency step of 350Hz, the gain of NCO is set to 5, and the SNR is set to 21dB (the typical value of the correlator output SNR in GPS receivers).

V. CONCLUSION

Through a detailed study of the operating principle of the FQFD, a gain loss due to the fluctuation of the discriminator in the traditional FQFD loop is discovered. So an Enveloped-FQFD is proposed for the better acquisition performance, which can eliminate the discriminator gain fluctuation in the FQFD. The theoretical analysis and the computer simulation results show that the Enveloped-FQFD improves the gain of the FQFD discriminator by 23% with only little computational overhead and shortens the pull-in process period effectively.

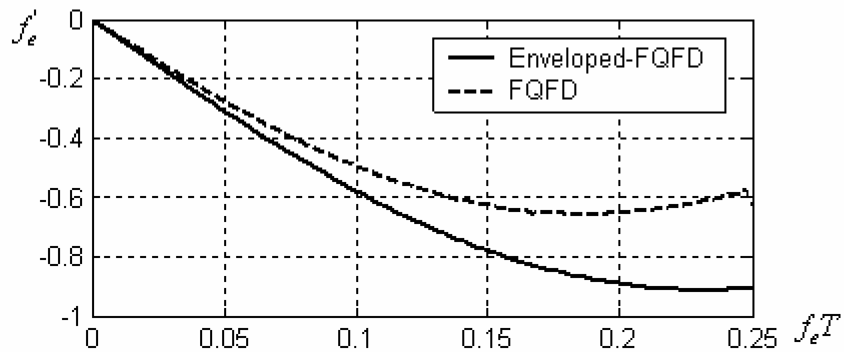

Fig. 4 Comparison of the phase plane characteristics

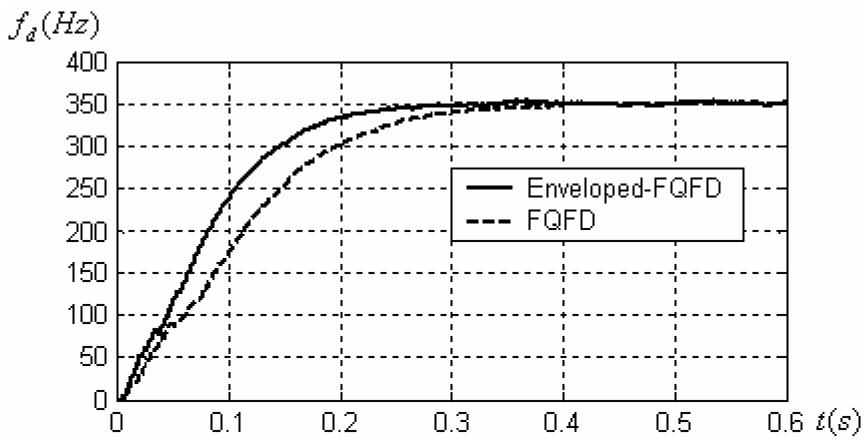

Fig. 5 Comparison of the frequency pull-in processes

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