IMPROVEMENTS IN DEEP-SPACE TRACKING BY USE OF THIRD-ORDER LOOPS

R. C. TAUSWORTHE and R. S. CROW
Telecommunications Division
Jet Propulsion Laboratory
Pasadena, California

Summary Third-order phase-locked receivers have not yet found wide application in deep-space communications systems because the second-order systems now used have performed adequately on past spacecraft missions. However, a survey of the doppler profiles for future missions shows that an unaided second-order loop may be unable to perform within reasonable error bounds. This article discusses the characteristics of a simple third-order extension to present second-order systems that not only extends doppler -tracking capability, but widens the pull-in range and decreases pull-in time as well.

Introduction Second-order phase-locked receivers, used both in spacecraft and in ground tracking stations, have performed their function so satisfactorily that, up until now, there has been little or no reason to consider the installation of a more complicated system. Their performance characteristics have become well understood, analyzable, and easily optimized relative to almost any criterion in a straightforward, well-defined way. Their capability to track incoming signals over a great range of signal levels and doppler profiles and to maintain lock and coherence at very low signal-to-noise ratios has become an accepted engineering fact.

As the more difficult deep space missions come into being, however, there is a corresponding stringency of requirements placed on the tracking instrument, as well as corresponding need to re-evaluate the best ways of performing the tracking function technically, economically, and operationally. The Mariner Mars 1971 orbital missions and the Pioneer F jupiter flyby mission, for example, have doppler rate profiles that may cause up to 30-deg stead y-state phase error in the unaided second-order loops now implemented. Such stress in receivers decreases the efficiency with which command or telemetry data are detected (by 1.25 dB at 30 deg), makes acquisition of lock difficult and faulty, and increases the likelihood of cycle slipping and loss of lock.

The way to avoid these problems is clear: eliminate or diminish the offending loop stress. This can be done by widening the loop bandwidth, by programming the uplink and downlink frequencies to correct these effects, or by increasing the order of the tracking loop. Widening the loop bandwidth increases loop noises; hence, it cannot be accepted as a general solution for the diminution of loop stress. Programming the uplink frequency and ground station local oscillator in accordance with the predicted doppler profile is potentially a complete solution for this problem, in addition to being required for automated station operations, but it may introduce difficulty in reducing the two-way doppler data for navigation purposes, and it may also require accurate predicts during critical phases of a mission where an a priori doppler profile is uncertain.

A third-order loop will track a uniform doppler rate with no error, without the need for accurate predicts. Raising the order of the loop to three would thus seem to be a potential, even if only partial, solution because of its simplicity and deserves additional analysis and experimental work.

The basic characteristics of third-order phase-tracking systems have been known since the first works of Jaffee and Rechtin (Reference 1), but these systems have not found wide application in the past due to what seemed to be poor acquisition and stability characteristics. Design seemed more complicated and was not well-understood. However, these potential problems have been overcome to the extent that a loop of the third order can now out-perform a second-order loop not only in its ability to track a frequency ramp with practically zero phase error, but also in its ability to acquire lock more quickly and from greater initial frequency offsets. Even when synthesized with imperfect integrators within the loop filter, the thirdorder system will out-perform a perfect second-order system by orders-of -magnitude improvement in steady-state phase error, lock-in time, and pull-in range. One further advantage to the third-order system is that requirements for the long time constants dictated by the high loop gains needed in the second-order loop to maintain small tracking errors are greatly relaxed.

Other advantages are that the loop filter configuration is an extension of presently mechanized loops, so required modification is simple; the role of the receiver operator subsequent to lock is essentially eliminated; several bandwidths are not needed to acquire rapidly; and frequency drifts in the loop voltage-controlled-oscillator (VCO) cause essentially no degradation in performance. This last advantage may remove the need to have the VCO’s in ovens and thereby further extend the usefulness of the system.

Third-Order Loop Filter When minimizing the total transient distortion plus noise variance by the Wiener filtering technique (Reference 1), one is mathematically led to the following optimum loop filter for tracking an input phase acceleration $\theta(t) = \Lambda_0 t^2/2$:

$$F(s) = \frac{1 + \tau_2 s}{\tau_1 s} + \frac{1}{2\tau_1 \tau_2 s^2} \qquad (1)$$

The first part of this filter resembles that now used in the second-order loop (Reference 2).

Perfect integrators are not practical, so modifications are necessary. The loop filter to be considered in the remainder of this paper is

$$F(s) = \frac{1 + \tau_2 s}{1 + \tau_1 s} + \frac{1}{(1 + \tau_1 s)(\delta + \tau_3 s)} \tag{2}$$

which approximates the F(s) above when τ_1 and τ_3 are large, except in the region near the origin. The filter may be synthesized in many ways. Circuit I in Figure 1 is a parallel configuration capable of synthesizing an F(s) having complex zeros. Whether the zeros or F(s) are real or complex is determined by the parameter $k = \tau_1 / \tau_2$. The zeros of the overall loop transfer function are additionally determined by the parameter $r = AK\tau_2^2 / \tau_1$, in which A is the rms input signal level and K is the loop dc gain external to F (S).

Experiment has shown that complex zeros in F(s) are not desirable, and that the simpler filter, circuit II in Figure 1, is actually preferable. Its transfer function is

$$F(s) = K_1 K_2 \frac{(1 + T_2 s)(1 + T_4 s)}{(1 + T_1 s)(1 + T_3 s)} \tag{3}$$

The relationships between the two sets of filter parameters are

$$\begin{aligned}
T_1 &= \tau_1 \\
T_3 &= \tau_3 / \delta \\
T_2 T_4 &= \frac{\tau_2 \tau_3}{1 + \delta} \approx \tau_2 \tau_3 \\
T_2 + T_4 &= \frac{\delta \tau_2 + \tau_3}{1 + \delta} \simeq \tau_3 \\
\frac{T_2 T_4}{(T_2 + T_4)^2} &= \frac{\tau_2}{\tau_3} = k \\
K_1 K_2 &= 1 + \frac{1}{\delta}
\end{aligned} \tag{4}$$

The loop two-sided noise bandwidth in the simplified, but usual, case $\epsilon = \tau_2 / \tau_1 << 1$ and $\delta << 1$ is given by

$$w_L = \frac{r (r - k + 1)}{2\tau_2 (r - k)} \tag{5}$$

The steady-state error due to an initial radian frequency offset $\Omega_0 = 2\pi\Delta f$ and frequency rate $\Lambda_0 = 2\pi f$ is, under the same assumptions,

$$\phi_{ss} = \frac{\Omega_0 + \Lambda_0 t}{AK}(\delta) + \frac{\Lambda_0 \tau_1}{AK}\left(\frac{\epsilon}{k} + \delta\right) \tag{6}$$

The figures in parentheses are the factors by which the corresponding terms of the second-order loop error are reduced. For small ϵ and δ, the improvement is obvious.

Acquisition and Lock-In Behavior The phase-plane technique, which is useful in visualizing the lock-in behavior of second-order loops (Reference 5), does not readily extend the same advantage to third-order systems, partly because there are three initial conditions -- phase, frequency, and frequency rate -- needed to specify a unique trajectory, and partly because this trajectory is difficult to visualize, as it lies in a three-dimensional hyperplane. By analogy, however, one still can visualize that, if there is a beat frequency between the incoming sinusoid and the VCO, there will be a small dc voltage at the detector output tending to force the loop toward lock. The extra integration in the loop accumulates this force, accelerating the loop toward lock. There is thus an understandable reduction in the time required to reach the zero-beat lock-in region, and there is a corresponding increase in the loop pull-in-frequency range, as compared to that with a second-order system.

If loop damping is not set properly, however, the great velocity acquired by the loop phase and the momentum associated with the two integrations of the error may carry the loop frequency error past the lock region, perhaps out of lock so rapidly that recovery is not possible. Proper location of the system roots can reduce this velocity through the zero-beat region enough to prevent frequency overshoot or irrecoverable loss of lock. In fact, it has been experimentally determined that if the loop has no underdamped roots, there is no transit past the zero-beat region.

The theory developed for computing the pull-in range of a second-order loop is easily extended to account for effects in the third-order loop; there is an enhancement in the acquisition range by approximately the square root of the added integrator dc gain. In fact, experimental evidence (Figure 2) verifies this formula exceedingly well all the way out to the point where IF filtering or minute equipment bias imperfections begin to limit the loop pull-in. The amount of bias imperfection that can be tolerated in the phase detector/operational amplifier combination for a given initial loop frequency detuning is illustrated in Figure 3.

Root Loci For a given set of loop constants, the loop gain K or signal level A may be varied and the position of the poles of the overall loop transfer function

$$L(s) = \frac{rk + r\tau_2 s + r(\tau_2 s)^2}{rk + r\tau_2 s + r(\tau_2 s)^2 + (\tau_2 s)^3} \qquad (\epsilon \ll 1,\ \delta \ll 1) \tag{7}$$

plotted. Since $r = AK\tau_2^2/\tau_1$ is proportional to both signal level A and loop gain K, it may be used as the independent variable. The system roots start at the poles of F(s) at r = 0 and finally terminate at the zeros of F(s). For the generalized F(s) given in (2), these loci take on different characteristics, depending on the value of $k = \tau_2/\tau_3$.

The six loci illustrated in Figure 4 show, for various increasing k:

(a) When k > 1/3, there are two underdamped (complex) roots and one overdamped (real) root for all r > k.

(b) When k = 1/3, there are two underdamped roots and one overdamped root for all r > k except at r = 3, at which point all three roots become equal.

(c) When 1/4 < k < 1/3, there is a region where two roots pass from underdamped to critically damped, to overdamped, to critically damped, and finally to underdamped.

(d) At k = 1/4, the system roots are always critically damped or overdamped for r > 3.375.

(e) The k < 1/4 case is similar to the k = 1/4 case, except there is a root nearer the origin, indicating a more sluggish response.

(f) When k = 0, the zero cancels the pole near the origin, producing a second-order loop.

The cases illustrated in Figure 4b and 4d are of special interest. Figure 4b depicts the maximum value of k (i. e. , 1/3) that can be used when no underdamped roots are desired. In such a design, there is only one fixed operating Signal level (i.e., r = 3). Figure 4d shows the maximum value of k (i.e., 1/4) that can be used if no underdamped roots are desired at any signal level above a design point producing $r_0 = 3.375$.

To minimize the possibility that acquisition is faulty, it is necessary that damping be critical or beyond; to minimize steady-state error (see Eq. 6) once lock is achieved, k should be as large as possible. These two conditions are met in slightly different ways according to the type of signals to be tracked. If the design is to be for signals of a fixed level, then k could be set to 1/3 and r should then be set to produce critical damping at this level, r = 3 (Figure 4b). If the design is to be for signals of various intensities, then k should be made equal to 0.25 and r should then be set for critical damping, r = 3.375 (Figure 4d) at the weakest expected signal level, to ensure that the roots are never underdamped.

The gain margin is the ratio r/k, which is about 22 dB for variable signal level designs and about 19 dB for fixed signal level designs.

Effect of Internal and VCO Noise The effect of VCO and other noises internal to the loop can be modeled as an equivalent noise voltage, $n_v(t)$, appearing at the VCO input. The form of the phase-error it causes greatly resembles the corresponding equation for second-order loops, but the phase error variance is about 10 to 18 percent higher than that for the second-order case. Hence, there is no relaxation in the requirement for spectral purity in the VCO to be used. However, there are other effects in the VCO not well modeled spectrally; one such effect is a steady drift in rest frequency due to some change in the oscillator operating condition, such as temperature or bias voltage. These appear to the loop error detector as slight alterations to the frequency offset or rate of the incoming sinusoid. Such effects can be analyzed as part of the loop overall transient. Because the third-order loop minimizes the effect of such transients, the drift requirement on VCO's may be greatly relaxed.

Noise-Detuning of the VCO (Out of Lock) When acquisition begins with the loop filter capacitors having the initial random charges as deposited in them by input noise prior to application of signal, there are random voltages which deviate the VCO from its rest frequency by perhaps K_1K_2 times as far as that in a second-order loop with the same loop gain external to the filter. This comparison is somewhat unfair, as it fails to recognize the increased tracking capability of the third-order system. To judge performance between second- and third-order systems more fairly, it is necessary to raise the gain of the second-order loop by this same K_1K_2 to equate the static phase errors due to detuning. (There will be little change in the seond-order loop's ability to track an acceleration, however.) The frequency wanderings of the second-order system will then always exceed those of the third-order system by at least a factor $1 + (\epsilon/k\delta)$. When $k\delta$ is smaller than $\epsilon = \tau_2/\tau_1$, the advantage is obvious. For the cascade implementation with $T_2 = T_4$ and $T_1 = T_3$, this advantage is a factor of 2.

Concluding Remarks This paper has summarized the tracking performance gains that can be obtained by simple modifications or redesign of existing receivers. The interested reader desiring more detail may consult Reference 6, which develops the theoretical performance of practical third-order tracking systems; presents design methods and procedures by which they may be synthesized; discusses hardware configuration and implementation factors; and indicates, by actual observed data, that the design goals and performance measures are as specified by the theory.

References

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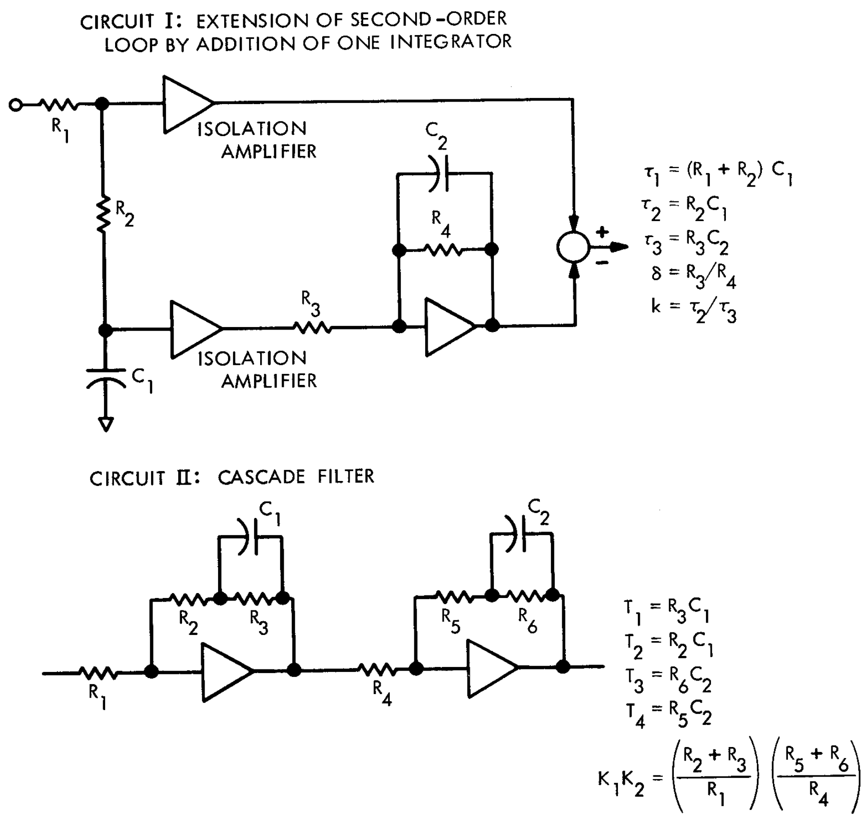
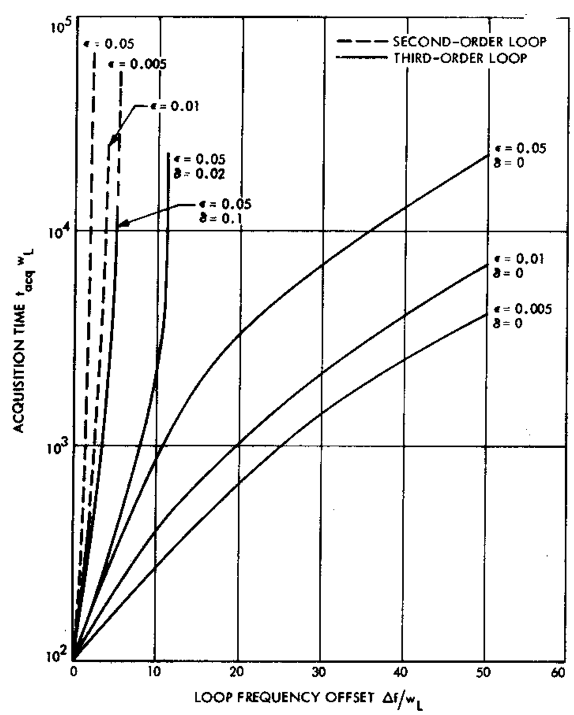


Fig. 1 - Third-order loop filters



Measured acquisition time as a function of loop frequency offset (normalized axis)

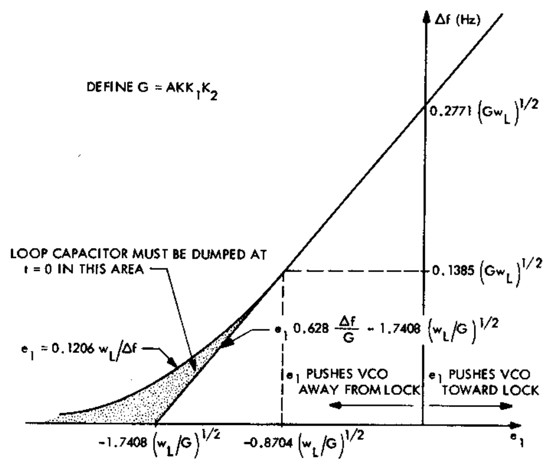


Fig. 3 - Maximum initial frequency detuning Δf(Hz) for which a third-order loop pulls to zero-beat, as a function of the phase-detector/operational amplifier bias imperfection e_1 (normalized by the peak S-curve value)

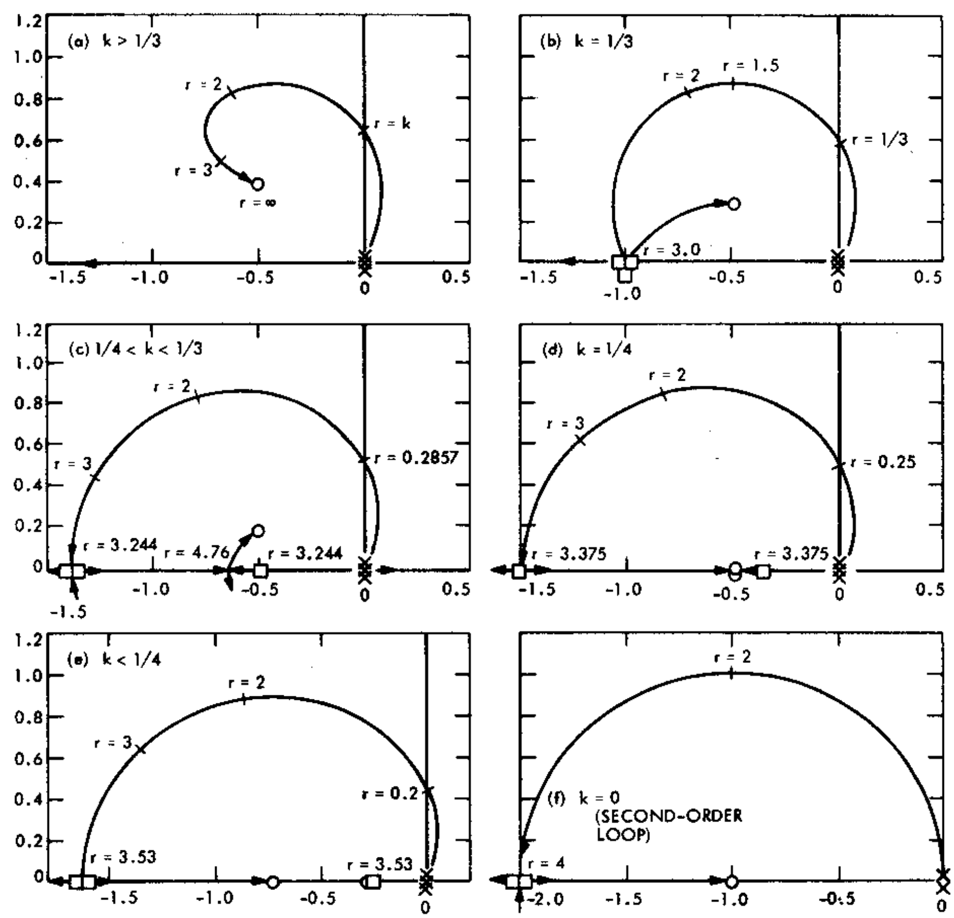


Fig. 4 - Root loci of the third-order as a function of $r = AK\tau_2^2/\tau_1$ for various values of the parameter $K = \tau_2/\tau_3$