THE USE OF MATCHED FILTERS FOR SYNCHRONIZATION

PETER H. HALPERN
General Dynamics Corporation
Orlando, Florida

Summary Matched filters are derived for best extracting synchronization. This is done for a somewhat general class of signaling systems as well as for NRZ and SØ PCM. The approach is based on the synthesis of finite time duration trigonometric pulses. The solution of simple calculus of variations problems yield a description of the matched filters.

Introduction The view we take here is that the synchronizer can be broken in two parts. The first part is the detector which should be designed to give the best possible estimate of the time of events. The second part, normally a phase locked loop of some sort, should optimally process the independent estimates of the detector.

The basic unit event for the purposes of synchronization is a transition or a change of state of the data. We can in fact detect the time of occurrence of these events in some optimal manner. It will be convenient to characterize the transition event over the same size time interval τ as the basic baud, but shifted $\tau/2$ seconds from the baud (see Figure 1). Thus if the Data is Non Return to Zero (NRZ) the transition event is a Split Phase (SØ) pulse; if the data is Split Phase, the transition event is NRZ for τ seconds.

For systems where there is an alphabet of possible signals in each baud, there will exist a somewhat larger alphabet of transition signals. We derive here synchronizers for these systems as well.

Once having detected the occurrence of transition events it is easy to derive agc and offset control signals as well as the control signal for phase lock loops.

We show here the synthesis of some of the matched filters for detecting transitions, these are drawn from a technique developed for the accurate approximation of finite time duration trigonometric pulses described in Reference (1).

Detection of the Time of Occurrence of an Event Suppose we wish to detect the time of occurrence of f(t) as accurately as possible in a white noise background, then the matched filter should maximize the derivative to noise ratio at ($t=\tau$) as the output crosses some threshold.

This is modified by the constraint of holding the probability of false threshold crossings to some value.

We therefore have a simple calculus of variations problem for the time reversed impulse response of the matched filter $h(\tau-t)$ we maximize:

$$J = \int_0^T h(\tau-t)\, f'(t)\, dt + \lambda_1 \int_0^T h(\tau-t) f(t)\, dt + \lambda_2 \int_0^T h^2(\tau-t)\,dt \tag{1}$$

where λ_1 and λ_2 are Lagrangian multipliers. This has the solution

$$h(\tau-t) = C_1 f(t) + C_2 f'(t) \tag{2}$$

where the relative values of C_1 and C_2 are determined by the cost of false crossings. In most systems the cost of false crossings overwhelms the small perturbation due to additive noise, in this case example (2) reduces to the classical matched filter

$$h(\tau-t) = C\, f(t) \tag{3}$$

To make these ideas concrete we first consider the important cases of NRZ and SØ systems.

Matched Filters for NRZ and SØ Systems Figure 2 shows the idealized outputs from NRZ and SØ matched filters with NRZ input. Figure 3 shows the same matched filters outputs for SØ input. Thresholds set on the triangular outputs generate signals which can be used for agc, offset controls and synchronization.

Using the techniques described in Reference (1) the approximation to any degree of accuracy is straightforward.

We note that the matched filter for NRZ has the transfer function

$$H_1(S) = \frac{1 - e^{-\tau S}}{S} \tag{4}$$

while that for SØ is given by

$$H_2(S) = \frac{1 - 2e^{\frac{-\tau S}{2}} + e^{-\tau S}}{S} \tag{5}$$

These can be written in a form more suitable for reactance function approximation of hyperbolic functions

$$H_1(S) = \frac{2}{S} \frac{1}{1 + \mathrm{Coth}(\tau S/2)} \tag{6}$$

$$H_2(S) = \frac{4}{S} \frac{\sinh^2(\frac{\tau S}{4}) \,/\, \sinh(\frac{\tau S}{2})}{1 + \mathrm{Coth}(\frac{\tau S}{2})} \tag{7}$$

We now can approximate,

$$\mathrm{Coth}\,(\tau S/2)$$

and

$$\sinh^2(\frac{\tau S}{4}) \;/\; \sinh(\frac{\tau S}{2})$$

in a pole-residue expansion with end point correction as described in Reference 1 and Reference 2. We obtain

$$\mathrm{Coth}\,(\frac{\tau S}{2}) \cong \frac{2}{\tau S} + \sum_{k\ \mathrm{even}} \frac{\frac{4}{\tau}\, S}{S^2 + k\frac{2\pi 2}{\tau^2}} + C_1 S \tag{8}$$

and

$$\sinh^2(\frac{\tau S}{4}) \,/\, \sinh(\frac{\tau S}{2}) \cong \sum_{k=2,6,10\ldots} \frac{\frac{4}{\tau}\, S}{S^2 + k\frac{2\pi 2}{\tau^2}} + C_2 S\,. \tag{9}$$

Here C_1 and C_2 are chosen to correct the approximation at $\tau S = \pi j$. The interesting feature about the approximation is that both H_1 and H_2 in (6) and (7) have the same denominator and can be realized with the same network.

A two and three pole pair realization is shown in Figure 4, the responses are shown in the photographs in Figure 5 and 6.

The realizations in Figure 3 have active RC equivalents by any of several techniques. For the two Pole Pair realization Eq. 8 and 9 become

$$\mathrm{Coth}\,\frac{\tau S}{2} \cong \frac{2}{\tau S} + \frac{\frac{4}{\tau}\,S}{S^2+\frac{4\pi^2}{\tau^2}} + \frac{2}{3}\,\frac{\tau}{\pi}\,\frac{S}{2} \tag{10}$$

and

$$\sinh^2\left(\frac{\tau S}{4}\right) / \sinh\left(\frac{\tau S}{2}\right) \cong \frac{\frac{4}{\tau}\,S}{S^2+\frac{4\pi^2}{\tau^2}} + \frac{.71}{3}\,\frac{\tau}{\pi}\,\frac{S}{2}\ . \tag{11}$$

For the three pole pair approximation we have

$$\mathrm{Coth}\left(\frac{\tau S}{2}\right) \cong \frac{2}{S} + \frac{\frac{4}{\tau}\,S}{S^2+\frac{4\pi^2}{\tau^2}} + \frac{\frac{4}{\tau}\,S}{S^2+\frac{16\pi^2}{\tau^2}} + \frac{.4\,\tau\,S}{\pi^2} \tag{12}$$

and Eq. (11) remains the same.

Synchronizers for More General Modulation Schemes Of growing interest are systems which transmit several bits per baud and use larger bandwidth-time-products in their signal design.

The advantages of such a system are manifold.

Larger BT signal designs:

1) Combat impulse noise,
2) More accurately concentrate spectral occupancy,
3) Combat intersymbol cross talk,
4) Decrease bit error rates directly due to larger energy of difference signals,
5) Combat various channel distortions.

For a discussion of some of the aspects of signal designs we refer the reader to Reference (3) for example.

As an example we consider a system whose signaling in each baud is composed of a linear sum of finite sine pulses:

$$E(t) = \sum_{1}^{n} a_k \sin \frac{2k\,\pi\,t}{\tau} \tag{13}$$

the possible n tuple s (a_1, a_2,—aa_n) determine the alphabet. If each of M possible n tuples is equally likely to be transmitted then $\log_2 M$ is the number of bits per baud. The matched filters for distinguishing these signals have similar expansions.

Now a synchronizer matched filter set can be designed to detect a change in any coefficient a_k from baud to baud. Just as the case for NRZ and SØ the synchronization filters are orthogonal to the data signals.

We can in fact draw our synchronizing filters from a set of harmonics not included in the signaling alphabet.

The synchronizing filters will be restricted to be of the form

$$h(\tau - t) = \sum_{n+1}^{m} b_k \cos(2k\pi t/\tau) \qquad 0 \le t \le \tau \tag{14}$$

the filters of Eq. (14) are flat orthogonal to the signals of Eq. (13) that is they will always have flat zero outputs at the end of each baud; furthermore they have notches at the frequencies in (13) so there must be a change of the a_k before there is any output from a filter of the form of Eq. (14). Refer to the NRZ and SØ case.

For $m \ge 2n$, it is possible to match to a change of any selected coefficient a_k and be orthogonal to the rest of the a's. We can build matched filters to each a_k separately. From the symmetry we need only consider the signals of Eq. (13) going over half the interval in the synchronizers matched filters refer to Figure 1.

In order to design a filter of the form of Eq. (14) matched to detect a change in a_k, and constrained to be orthogonal to changes in the other a's we maximize

$$\int_0^{\tau/2} \sin \frac{2k\pi t}{\tau} \sum_{n+1}^{m} b_j \cos \frac{2j\pi t}{\tau} \, dt \quad . \tag{15}$$

Under the constraints for $\{i: i \le n, i \ne k)$

$$\int_0^{\tau/2} \sin 2 i \frac{\pi t}{\tau} \sum_{n+1}^{m} b_j \operatorname{Cos} \frac{2 j \pi t}{\tau} \, dt = 0 \tag{16}$$

and $\Sigma\, b_j^2 = K$. Because the even (odd) harmonics are orthogonal to the odd (even) harmonics over the half interval, we wind up with the following expression for the coefficients b_j of the matched filters:

$$b_j = \frac{k}{j^2 - k^2} + \sum_{\substack{i \ge 1 \\ i \ne k}}^{i \le n} \lambda i \frac{i}{j^2 - i^2} \tag{17}$$

for j odd, i and k even or for j even, i and k odd; otherwise, $b_j = 0$ for k and j even or k and j odd.

To compute the Lagrangian multipliers λ_i for i odd we multiply eq (17) by $\frac{h}{j^2 - h^2}$ and sum over all even j to obtain

$$O = \sum_j b_j \frac{h}{j^2 - h^2} = \sum_j \frac{k\ h}{(j^2 - k^2)(j^2 - h^2)} + \sum_j \sum_{\substack{i \geq 1 \\ i \neq k}}^{i \leq n} \frac{i\ \lambda_i\ h}{(j^2 - i^2)(j^2 - h^2)} \qquad (18)$$

for every h odd and ≤n and for j even and m≥j≥n+ 1. That eq (18) sums to zero follows from constraint eq (16). A similar set of linear equations exists for λ_i i even.

This is a simultaneous set of linear equations in λ_i. Although the computation is somewhat tedious it is straightforward.

Each matched filter so derived serves as an independent measure of the proper sync point. just as the NRZ and SØ cases, thresholds can be set, and the resulting pulses can be used as inputs to a phase locked loop.

REFERENCES

(1) "Trigonometric Pulse Forming Networks Revisited," P. H. Halpern IEEE Trans on Circuit Theory, January 1972, pp. 81-86.

(2) "Band Pass Filters with Linear Phase," R. M. Lerner, Proc IEEE, March 1964, pp 249-268.

(3) "Signal Theory," L. E. Franks, Englewood Cliffs, New Jersey, Prentice-Hall, 1969.

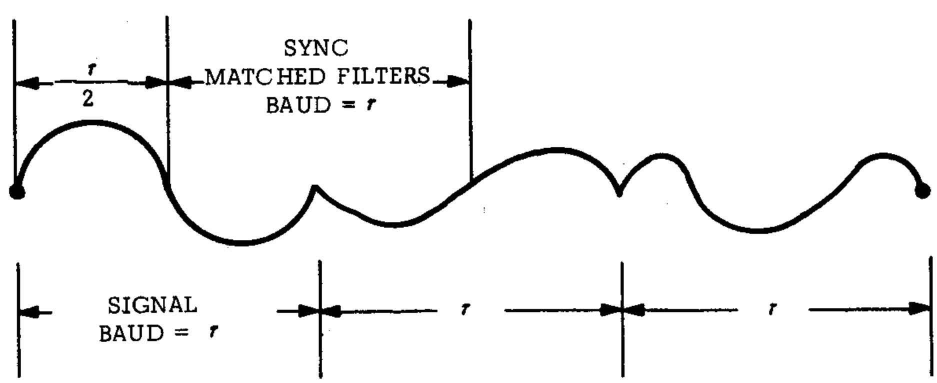

Figure 1. Signaling Interval and the Synchronizers Filter Interval

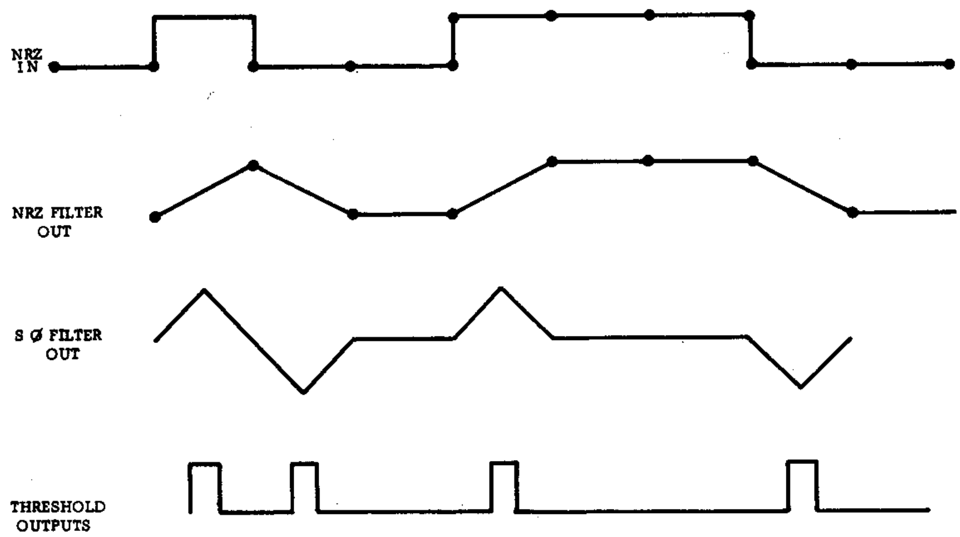

Figure 2. Ideal Filter Responses for NRZ Input

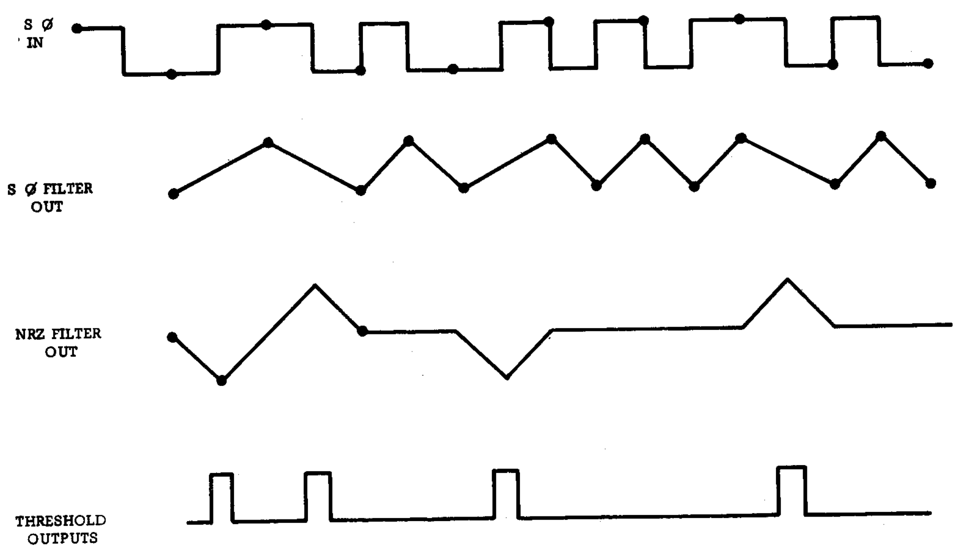

Figure 3. Ideal Filter Responses for Di-Phase Input

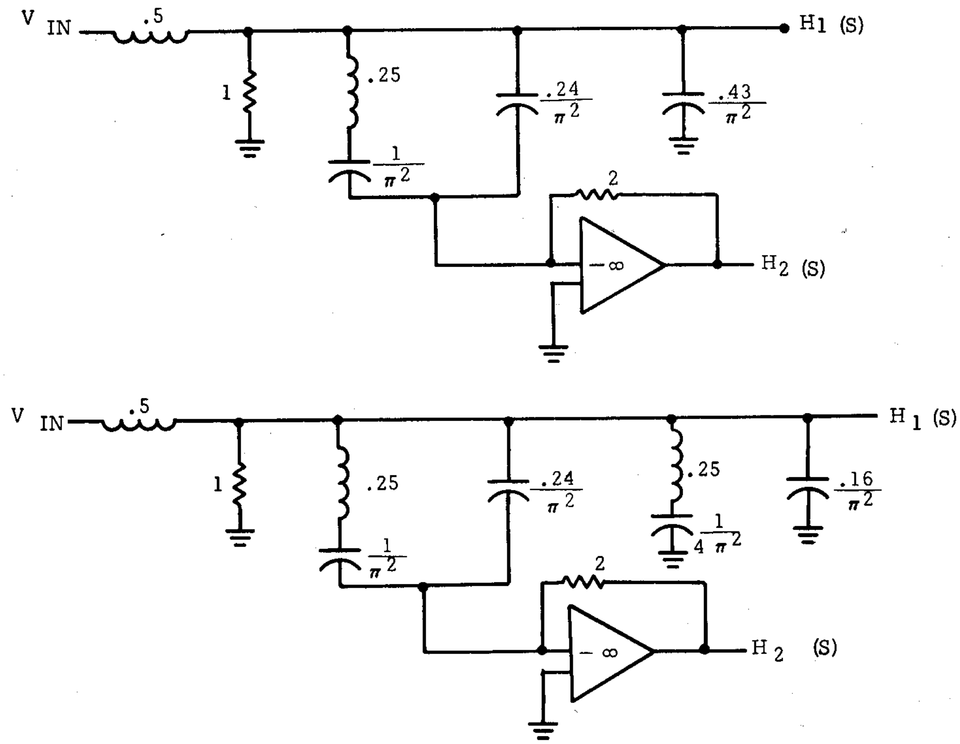

Figure 4. Normalized Realizations for Di-Phase and NRZ Matched Filters

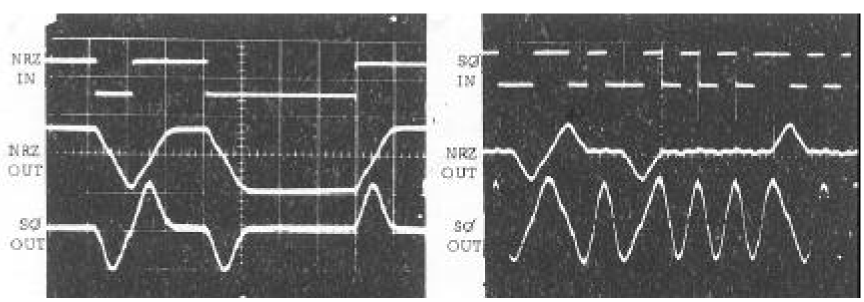

Figure 5. Responses of Two-Pole Approximations

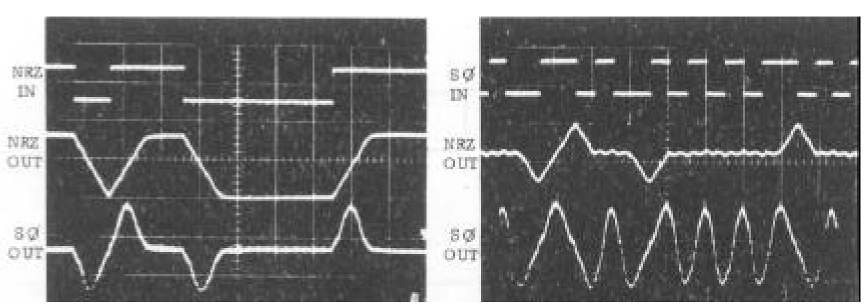

Figure 6. Responses of Three-Pole Pair Approximations