Summary  In deep space communications with distant planets, the data rate as well as the operating signal-to-noise ratio may be very low. To maintain the error rate also at a very low level, it is necessary to use a sophisticated coding system (longer code) without excessive decoding complexity. The concatenated coding has been shown to meet such requirements in that the error rate decreases exponentially with the overall length of the code while the decoder complexity increases only algebraically. Three methods of concatenating an inner code with an outer code are considered. Performance comparison of the three concatenated codes is made. It is shown that the concatenated code with inner code a convolutional code and outer code a Reed-Solomon code performs the best among the three.

Introduction  It has been recently established that the use of non-coherent MFSK signals is the best approach for the deep space communications especially with the small probes. As the transmitter power of the small probes is small due to the weight limitation, very low data rate must be used in order to successfully transmit a signal over the great distance from Venus to earth. The available performance with the MFSK signals is, however, limited because of the receiver complexity for a large number of signals. By concatenating two codes, it is possible to reduce the error rate in orders of magnitude while the decoder complexity increases only algebraically. Several techniques of concatenating an inner code with an outer code have been proposed (e.g. [1], [2], [3]). The error performance, efficiency and decoding complexity of the concatenated codes have also been studied ([1] - [7]). The available results however, are very limited and uncorrelated. In this paper, performance comparison of the following three concatenated codes will be made.

Code I.  inner code bi-orthogonal code, outer code a generalized Hamming code or Reed-Solomon (R-S) code.

Code II.  inner code a convolutional code, outer code a block orthogonal code (MFSK).

Code III. inner code a convolutional code, outer code a R-S code.
Other methods of concatenation are possible. For example, two \( k = 6 \) by \( v = 2 \) convolutional codes can be concatenated to give a 9 by 4 code. Here \( k \) is the constraint length and \( 1/v \) is the rate of such a code. Each convolutional code can be decoded by using Viterbi's decoding scheme. Erickson [5] has shown, however, that the concatenation of Viterbi decoders does not appear to be useful in the present context of the planetary program. He conjectures that the most appropriate outer code, in any concatenation scheme involving a Viterbi algorithm inner decoder, is a high-rate algebraic block code.

**Code I** Consider first a generalized Hamming code as the outer code [2]. The code has \( n \) elements including \( k \) information (data) elements and \( m = n - k \) check elements. Each data element is a six-bit bi-orthogonal code word. The receiver performs both error detection and correction. The generalized Hamming code which has a Hamming distance of three is a specific case of R-S code. For the R-S code, the minimum distance, \( d \), between two code words is related to the number of check elements, \( m \) by \( d = m + 1 \). The maximum number of correctable elements for each code word, \( t \), is equal to \( m/2 \). The probability of the bit error after both detection and correction is

\[
P_c = \frac{P_A A_2 + P_B A_4}{A_2 + A_4}
\]

where \( P_A \) = probability of bit error at the detector output, \( P_B \) = probability of bit error at the corrector output, and \( A_2 \) and \( A_4 \) are the data quantities at the detector and the corrector outputs respectively [2].

Without restricting to the Hamming distance of 3, Forney [1] and Simpson (4) have considered the bi-orthogonal inner code and the R-S outer code. A typical concatenated coding system is shown in Fig. 1 where the R-S code can correct up to 2 errors. The main difference among the three reports [1], [2], [4] is in the decoding method. Forney considers both the maximum likelihood decoding and the generalized minimum distance decoding of the R-S code. Both Miller and Simpson use the algebraic decoding for the R-S code although their error probability expressions are inconsistent. The digit error given by Simpson is

\[
P_D(e) \leq \sum_{i=t+1}^{n-t-1} \frac{(t + i)}{n} \binom{n}{i} p^i (1 - p)^{n-i} + \sum_{i=n-t}^{n} \binom{n}{i} p^i (1 - p)^{n-i}
\]

where \( p \) is the probability of error of the bi-orthogonal code word. Available results are shown in Fig. 2 with the error probability plotted as a function of signal energy per bit to noise density ratio of the inner code. Curve 1 has a 2^8 - symbol R-S code and a bi-orthogonal code of rate 1/16 (Forney). Curve 2 has an (18, 12) R-S outer code (12 information elements out of a total of 18 elements) and a 6 bit bi-orthogonal inner code.
(Miller). Curve 3 has a (63, 49) R-S outer code and a 6 bit bi-orthogonal inner code (Simpson). Although the code lengths are not the same, it is clear that Curve 1 is better than Curve 2 which is better than Curve 3.

**Code II** The block diagram of the concatenated system is shown in Fig 3. Although it is possible to determine the upper bound of the error probability, the bound may be too loose to be useful. Some computer simulation results were reported by Richardson et.al. [8]. Let M be the bit-duration - IF filter bandwidth product. Fig. 4 is a plot of the error probabilities for ν = 3, k = 6 inner code and ν = 5, k = 8 inner code with M = 2 and 10 along with the performance of coherent systems without concatenation. The degradation in performance from M = 2 to M = 10 is approximately 1.1 to 1.4 dB in $E_b/N_o$, the signal energy per bit to noise density ratio. It is noted that the performance improvement over the wideband noncoherent MFSK system 191 is very significant for both M = 2 and 10.

**Code III** This concatenated code was considered by Odenwalder [3]. A block diagram of the system is shown in Fig. 5. The best computer simulation result of (1/3, 8) convolutional inner code and $2^8$ - symbol R-S outer code is shown in Fig. 6. Here ν = 3, k = 8. Also plotted in the same figure are the error probabilities of a 5-bit bi-orthogonal code, an (1/2, 8) convolutional code with the Viterbi’s maximum likelihood decoding (from Odenwalder [3], chapter 5), and a concatenated code (31, 25, 5) with (31, 25) R-S outer code and 5 bit bi-orthogonal inner code. The improvement from bi-orthogonal only to bi-orthogonal/R-S code is added to the (1/2, 8) convolutional code to give an estimate of the error probability of concatenated code with (1/2, 8) convolutional inner code and (31, 25) R-S outer code. The error probability of the Code III is the best among all concatenated codes. The required interleave-buffer, however, essentially increases the word length, or reduces the effective signal-to-noise ratio.

**Comments and Conclusions** Performance curves indicate that the concatenated codes greatly improve the performance over the unconcatenated codes. The decoding complexity increases from Code I to Code II to Code III while the performance improves in the reverse order. The decoding complexity depends mainly on the inner code used.

Based on the available low error rate at the low signal-to-noise ratio and the moderate increase in decoding complexity, we conclude that the concatenated codes, especially Code II and Code III described above meet the needs of deep space communications.
References


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Figure 1     Code I
Figure 2  Word error probability of Code I

Figure 3  Code II
Figure 4  Bit error probability of Code II

Figure 5  Code III
Figure 6    Word error probabilities of several coding techniques: (1) 5-bit bi-orthogonal code, (2) (31, 25, 5) bi-orthogonal/ R-S code, (3) (1/2, 8) convolutional code (Viterbi decoding), (4) (1/2, 8) convolutional/ (31, 25) R-S code, and (5) best concatenated code, (1/3, 8) convolutional/ R-S code.