THE INFLUENCE OF CARRIER FREQUENCY ON SNR FOR FM SYSTEMS[1]

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Summary. The influence of carrier frequency on broadband signal-to-noise ratio is derived for a frequency modulated tape recording system. Optimum signal-to-noise performance occurs at the value of carrier frequency where the carrier-to-noise ratio is falling at 6 dB/octave. Signal-to-noise ratio is relatively insensitive to changes in carrier frequency about the optimum value.

Introduction. It is well known that fm systems can give significant improvements in signal-to-noise performance over am systems. However, in order to take full advantage of the improvements possible for a given system, the designer must known the system parameters and their effect on signal-to-noise performance. Then he can alter the system design in order to obtain the optimum performance. This paper presents briefly the derivation of signal-to-noise ratio for fm systems. The derivation is adapted to express more accurately the conditions found in magnetic recorders and to show how a particular parameter, carrier frequency, might be chosen for optimum signal-to-noise.

Analysis. Figure 1 shows a block diagram of the limiter and discriminator portion of an fm system. For high signal-to-noise ratios as found in tape recorders it is quite accurate to calculate the noise output in the presence of an unmodulated carrier and then to calculate the signal output without the noise present. The noise voltage with a spectral density, N_i, will produce noise fm on the carrier which will be demodulated and appear as a base-band noise spectral density, N_o, as shown in Figure 2. The noise at frequency f contained in a small bandwidth df can be expressed as an equivalent sinusoidal voltage

$$e_n = A_n(f) \cos[2\pi ft + \Phi_n(f)] \tag{1}$$

where $A_n(f) = \sqrt{[2N_i(f)df]}$,

and $\Phi_n(f)$ is a random phase angle. For a carrier voltage

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$$e_c = A_c \cos(2\pi f_c t), \quad (2)$$

noise voltage (1) will produce amplitude and angle modulation. After limiting, only the angle modulation will remain, giving for the voltage in ut to the discriminator

$$e_L = A_L \cos\{2\pi f_c t + \frac{A_n(f)}{A_c} \sin[2\pi(f-f_c)t + \Phi_n(f)]\} \quad (3)$$

The base-band or frequency-demodulated noise component at the discriminator output caused by noise at frequency f is equal to

$$e_d = \frac{2\pi K_d(f-f_c)A_n(f)}{A_c} \cos[2\pi(f-f_c)t + \Phi_n(f)] \quad (4)$$

with mean-squared value

$$\overline{e_d^2} = \frac{(2\pi K_d)^2(f-f_c)^2 A_n^2(f)}{2A_c^2} \quad (5)$$

K_d is the frequency discriminator gain transfer constant.

In order to add the noise produced at different frequencies, we must sum the mean-squared noise voltages. For a low-pass filter with bandwidth f_m following the discriminator, noise at the limiter input from a frequency band f_c-f_m to f_c+f_m will appear at the filter output. The total mean-squared noise voltage at the output will be given by

$$\overline{E_{on}^2} = \frac{4\pi^2 K_d^2}{A_c^2} \int_{f_c-f_m}^{f_c+f_m} (f-f_c)^2 N_i(f)\, df \quad (6)$$

This expression may be written in terms of the base-band frequency, $f_b = |f-f_c|$. By a suitable change of variables, the output noise can be more conveniently expressed as

$$\overline{E_{on}^2} = \frac{4\pi^2 K_d^2}{A_c^2} \int_0^{f_m} f_b^2 [N_i(f_c+f_b) + N_i(f_c-f_b)]df_b \quad (7)$$

$$= \int_0^{f_m} N_o(f_b)\, df_b$$

where, $N_o(f_b)$, the base-band noise spectral density is given by

$$N_o(f_b) = \frac{4\pi^2 K_d^2}{A_c^2} f_b^2 \left[N_i(f_c+f_b) + N_i(f_c-f_b)\right] \tag{8}$$

Equation (8) is used to obtain the curve of Figure 2(b) from the curve of Figure 2(a).

The output noise can be calculated from equations (7) and (8) whenever the limiter input noise spectral density, N_i. is known. In magnetic recording systems N_i is usually not constant with frequency, as shown in Figure 2(a). However, it often can be sufficiently represented over the frequency band by a sloping straight line approximation. Thus, let N_i vary linearly with frequency and be equal to $N_c[1 + s(f-f_c)/fc]$, where s is the slope of the noise density with frequency, and N_c is the value at the carrier frequency, f_c. The output mean-squared voltage becomes independent of slope, s, and is given by

$$\overline{E_{on}^2} = \frac{4\pi^2 K_d^2}{A_c^2} \int_0^{f_m} f_b^2 \, (2N_c) df_b$$

$$= \frac{8\pi^2 K_d^2}{A_c^2} \cdot \frac{N_c f_m^3}{3} \tag{9}$$

Next, the mean-squared signal voltage from the discriminator is known to be

$$\overline{E_s^2} = \frac{4\pi^2 K_d^2 (\Delta f)^2}{2} \tag{10}$$

where Δf is the peak frequency derivation. Signal-to-noise ratio is then given by

$$\left(\frac{S}{N}\right) = \frac{\dfrac{4\pi^2 K_d^2 (\Delta f)^2}{2}}{\dfrac{8\pi^2 K_d^2 N_c f_m^3}{3A_c^2}} \tag{11}$$

$$= 3\left(\frac{\Delta f}{f_m}\right)^2 \cdot \frac{\dfrac{A_c^2}{2}}{2N_c f_m}$$

$$= \frac{3}{f_m^2} \left(\frac{\Delta f}{f_c}\right)^2 \cdot f_c^2 \cdot \left(\frac{C}{N}\right)_i$$

$(C/N)_i$ is the carrier-to-noise ratio at the input to the limiter for a spectral density, N_i. that varies linearly with frequency over the band from f_c-f_m to f_c +f_m . Equation (11) completes the derivation of signal-to-noise ratio.

Design Considerations. When choosing the carrier frequency of an fm system for magnetic tape recorders it is desirable to examine signal-to-noise ratio performance for a fixed ratio of peak frequency deviation to carrier frequency, $\Delta f/f_c$. Holding $\Delta f/f_c$ constant provides constant protection against flutter. Circuits and tape signal transmission capability usually allow Δf to increase as f_c increases. We see that signal-to-noise ratio becomes

$$\left(\frac{S}{N}\right)_o = (\text{constant}) \cdot f_c^2 \left(\frac{C}{N}\right)_i \qquad (12)$$

allowing the possibility of choosing f_c, the carrier frequency, for optimum signal-to-noise.

As an example, let the variation of $(C/N)_i$ for a hypothetical tape recorder be sinusoidal with frequency, as shown in Figure 3. Figure 3 also shows a plot of the corresponding $(S/N)_o$ vs. f_c. We note that there is an optimum $(S/N)_o$, for an f_c which is 1.45 times the frequency at which $(C/N)_i$ is maximum. Placing the carrier frequency at this value improves the signal-to-noise ratio by 2 dB or 60% over that obtained by optimizing the carrier-to-noise ratio at the input to the limiter. Given the relationships derived above and a theoretical or experimental carrier-to-noise ratio, the designer can then optimize signal-to-noise ratio by choice of carrier frequency, using analytical or graphical methods to determine the optimum f_c and the sensitivity of $(S/N)_o$ to changes in f_c.

An application of this method to experimental data from a tape recorder is shown in Figure 4, here plotted on semi-logarithmic coordinates. The choice of the carrier frequency, based on output signal-to-noise only, is seen not to be critical since the ratio, $(S/N)_o$, is within 1 dB of maximum over a carrier frequency range from 700 kHz to 1.25 MHz.

If one does not need the entire curve of $(S/N)_o$ but only the value of the carrier frequency which gives the maximum value of signal-to-noise ratio, a simple graphical procedure can be used as shown in Figure 4. It is only necessary to plot on semi-logarithmic scales the carrier-to-noise ratio, $(C/N)_i$ in dB, vs. carrier frequency. The best carrier frequency occurs where the slope of the curve is -6 dB/octave.

This relation follows from equation (12) by the method of taking its derivative and setting it to zero.

$$\frac{d(S/N)_o}{df_c} = f_c^2 \frac{d(C/N)_i}{df_c} + 2f_c(C/N)_i = 0$$

Thus,

$$\frac{d(C/N)_i}{(C/N)_i} = -2 \frac{df_c}{f_c} \tag{13}$$

or in terms of logarithms

$$\frac{d[10 \log_{10} (C/N)_i]}{d[20 \log_{10} f_c]} = -1 \tag{14}$$

(14) shows that the optimum occurs for a slope of -1. When the power ratio $(C/N)_i$ in dB is plotted vs. f_c on semi-logarithmic coordinates, a slope of -1 corresponds to -6 dB/octave.

Conclusion. An expression has been derived which shows the influence of carrier frequency on signal-to-noise ratio for fm systems. Design considerations show that there is an optimum choice for carrier frequency, but that the optimum is a fairly broad onei

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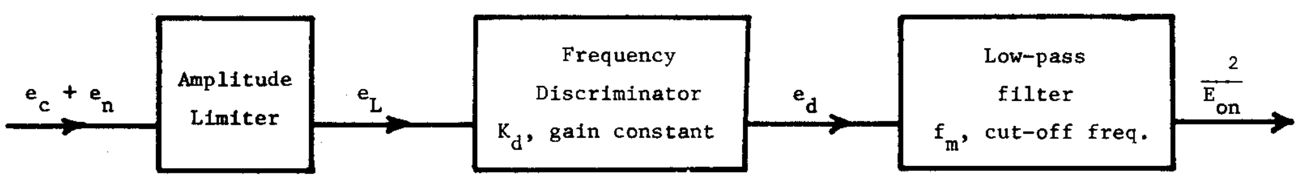

Fig. 1 - fm System Block Diagram

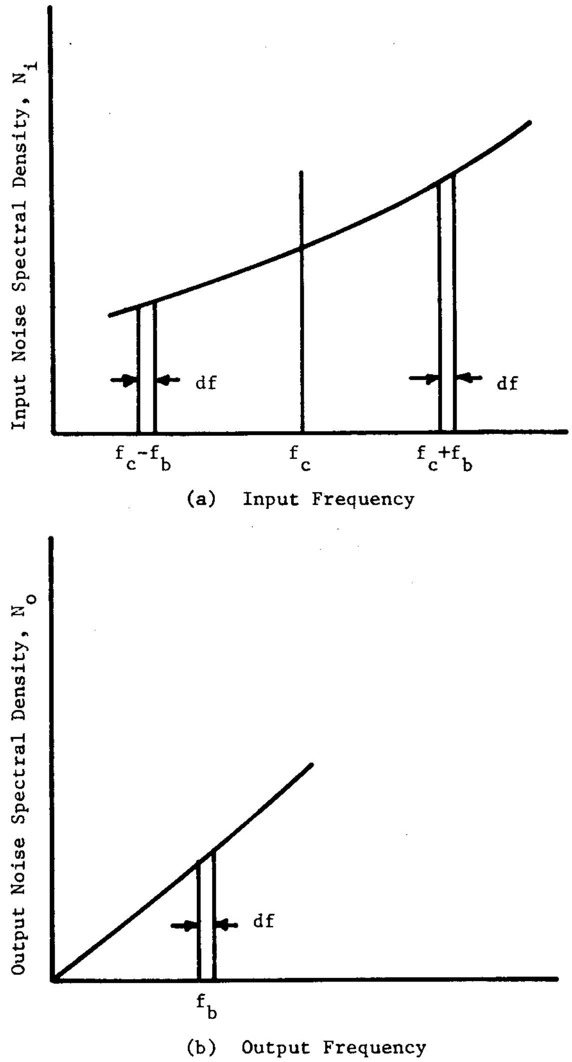

Fig. 2 - Noise Spectral Densities at Input and Output of fm System

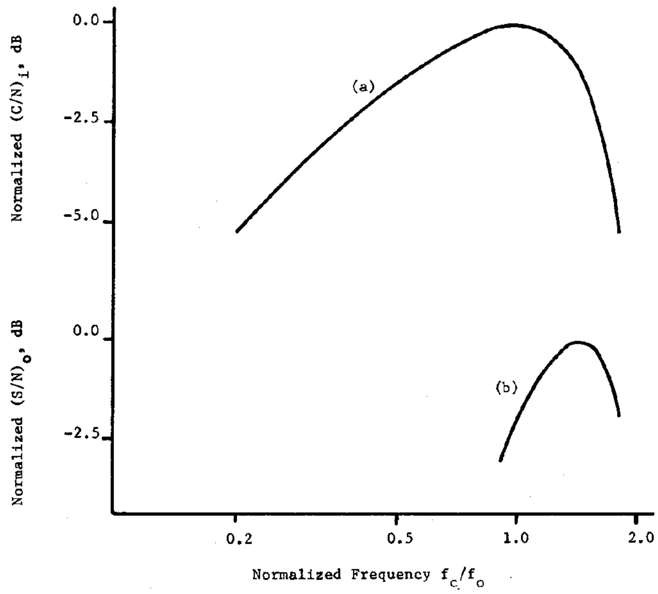

Fig. 3 - Plots of (a) Input Carrier-to-Noise Ratio and (b) Output Signal-to-Noise Ratio vs. Normalized Carrier Frequency. Input Carrier-to-Noise Ratio is Sinusoidal with Frequency.

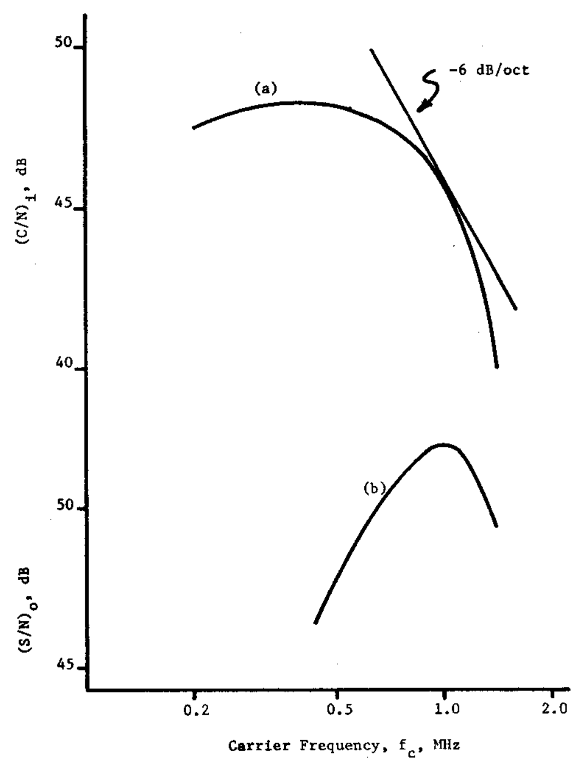

Fig. 4 - Plot (a) is the Carrier-to-Noise Ratio measured for a tape recorder running at 120 ips with a bandwidth of 256 KHz. Plot (b) is the output Signal-to-Noise Ratio calculated from (a) assuming a frequency deviation of 30%.