THE ROLE OF PARTIAL CORRELATION IN MULTIPLE ACCESS

D. R. ANDERSON
System Engineering Laboratory
TRW System

Summary This paper presents results on the partial correlation properties of PN and frequency-hopping sequences considered as multiple access address signals.

Introduction The basic requirement of multiple access modulation is to provide distinguishability of a desired channel from all other active channels both in the synchronization mode (if the information is coded) and in the information mode. Time-division multiple access (TDMA) and frequency division multiple access (FDMA) are well-known and widely used in current systems, particularly civilian systems. Where the user receiver signal-to-noise ratios is small, there is a requirement for high processing gain. This requires in general that the users transmit signals each of such wide bandwidth that each must share its passband with. a number of the others. However, if TDMA cannot be used to guarantee that no pair of users have significant time overlaps, crosstalk must occur. In this case, two types of address signals with the date phase-modulated on them are used:

1) Periodic high chip rate PSK modulation with a sharply peaked autocorrelation function (PN [pseudo-noise] sequences).

2) Periodic high chip rate multiple FSK modulation again with a sharply peaked autocorrelation function (frequency-hopping sequences).

It can be shown that the best families of PN sequences and the best families of frequency-hopping sequences have comparable performance with respect to correlation.

Results The role of the periodic auto- and crosscorrelation properties of pseudonoise sequences in satellite communications has been discussed by a number of authors. Aein (1), (2) Anderson and Wintz (3), Blasbalg (4), (5) and Elspas and Wolf (6) have considered this from various standpoints. Also, Anderson (7) and Gold (8) have considered the associated signal design problems. But, as pointed out by Aein (1) and by Anderson and Wintz (3), the partial autocorrelation and partial crosscorrelation (finite duration autocorrelation and crosscorrelation) properties of these signals are equally important. The purpose of this paper is to present results on the performance of these partial auto- and crosscorrelations as well as of the partial correlations associated with

partial and periodic correlation of the optimal frequency-hopping sequences corresponding to Reed-Solomon codes.

Lindholm (9) has obtained formulas for the arithmetic mean and root mean square of the partial autocorrelations of a maximal length. shift register sequence. Here these formulas will be generalized to the families of sequences considered by Anderson (7) and to those considered by Gold (8). Further, using inequalities due to Vinogradov (10), it is actually possible to find upper bounds. Specifically, if τ is the chip time and if a period of a normalized pseudonoise sequence of length N is displaced by $n\tau$, then the corresponding partial autocorrelation (finite-duration autocorrelation) $c(n\tau)$ is bounded in magnitude as follows:

$$|c(n\tau)| \leq (\log N)/\sqrt{N}$$

Similar results are obtainable for frequency-hopping sequences.

References

1) Aein, J. M., "On the Output Power Division in a Captured Hard-limiting Repeater," IEEE Trans., Vol. COM-14, No. 3, pp. 347-349; June, 1966.
2) Aein, J. M., "Normal Approximations to the Error Rate for Hard-limited Correlators," IEEE Trans., Vol. COM-15, No. 1; February, 1967.
3) Anderson, D. R. and Wintz, P. A., "Analysis of a Spread Spectrum Multiple Access System with a Hard Limiter," Proc. IEEE Prof. Group on Comm. Techniques, April, 1969.
4) Basbalg, H., "Random Access Communications Using Frequency Shifted PN (pseudonoise) Signals," IEEE Convention Record; March, 1964.
5) Blasbalg, H., "Signal-to-Noise Ratio of a Pulsed Pseudonoise System," in "Multiple Access to a Communication Satellite with a Hard-Limiting Repeater," edited by J. M. Aein and J. W. Schwartz, IDA Report R-108; April, 1965.
6) Wolf, J. K., and B. Elspas, "Mutual Interference Due to Correlated Constant-Envelope Signals," in "Multiple Access to a Communication Satellite with a Hard-Limiting Repeater," edited by J. M. Aein and J. W. Schwartz, IDA Report R-108; April 1965.
7) Anderson, D. R., "A New Class of Cyclic Codes," SIAM Journal of Applied Mathematics, Vol. 16, No. 1, 1968, pp. 181-197.
8) Gold, R., "Optimal Binary Sequences for Spread Spectrum Multiplexing," Proc. IEEE PGIT, Vol. IT-13, No. 4, pp. 619-620.
9) Lindholm, J. H., "An Analysis of the Pseudo-randomness Properties of Subsequences of Long m-Sequences," Proc. IEEE PGIT, Vol. IT-14, No. 4, July 1968, pp. 569-576.
10) Vinogradov, A. N., "Elements of Number Theory," Dover Press.