COMPARISON OF DIGITAL MODULATION METHODS

DAVID J. GOODMAN
Bell Laboratories
Murray Hill, N. J.

Summary Important characteristics of a digital modulation technique are economy, efficiency, and quantizing distortion. As in most engineering situations, the choice of a specific digital modulation method for a practical application represents a compromise among cost and performance objectives. The purpose of this paper is to identify important design variables of digital modulators and to discuss the manner in which they influence cost and performance. In particular, we derive formulas that show the variation of quantizing noise with sampling rate and quantizer resolution in basic PCM and DPCM modulators. These formulas imply a conflict between economy and efficiency and we describe three current approaches to the resolution of this conflict. The first is efficiency oriented, the second economy oriented, and the third relies on digital signal processing to convert between efficient and economical modulation formats.

Economy and Efficiency Viewing digital modulation as the representation of a continuous waveform by a binary pulse train, we consider efficiency to be measured by the number of pulses per second. We measure economy by the cost of the hardware that performs the analog-to-binary and binary-to-analog conversions. Although these are commonly accepted measures, we are obliged to observe that they take a narrow view of efficiency and economy. Placed in the context of an application , an apparently inefficient or expensive digital modulator may provide opportunities for substantial efficiencies or cost savings in other parts of a system.

Cost and Design Parameters Fundamental to digital modulation are time sampling and amplitude quantization and in generalizing about cost, we observe that it is often economical to increase circuit operating speed if the increase admits a reduction in quantizer size. This observation follows from the fact that there is a wide range of operating speeds over which hardware costs are constant, while, on the other hand, cost is generally quite sensitive to analog accuracy requirements.

Performance and Design Parameters In addition to cost, we consider the manner in which efficiency and distortion depend on sampling rate and quantizer resolution. In particular, we refer to Figure 1, a block diagram that is sufficiently general to encompas many modulation methods. We shall consider the simplest methods, uniform pulse code modulation (PCM) and differential PCM (DPCM) and derive formulas for quantizing noise power as a function of sampling rate and the number of quantizer bits. In PCM, there is no feedback loop in the modulator (A is an open circuit) and demodulation consists of conversion of the binary pulse stream to a quantized sequence of amplitude modulated pulses (PAM) followed by low-pass filtering. (B is a closed circuit.) In DPCM, A and B are identical integrators and y is a quantized approximation to the input, x.

In the M-bit uniform quantizer, the spacing between adjacent decision levels and the spacing between adjacent output levels is a constant, d, and N_Q, the mean-square difference between quantizer input and output, is proportional to d^2 when the input is within the dynamic range of the quantizer. Often, it is quite accurate to assume that the quantizing error is equally likely to be at any point in the range (-½d, ½d), in which case,

$$N_Q = \mathrm{Av}(z-q)^2 = d^2/12. \quad (1)$$

To maintain a prescribed dynamic range as M changes, d must vary inversely with the number of quantization intervals, $d \sim 2^M-1)^{-1}$ so that

$$N_Q \sim (2^M-1)^{-2}, \quad (2)$$

in which the constant of proportionality depends on overload immunity requirements and on the characteristics of the input signal.

To analyze the effect of sampling rate on distortion, we consider PCM and DPCM separately. In PCM, the overload characteristics and, therefore, d and N_Q are independent of f_S, and only in the low pass filter of the demodulator does the sampling rate influence quantizing noise. The cutoff frequency of the filter is W Hz, the bandwidth of x(t) [which we assume to be bandlimited to $W \leq ½f_S$ to prevent aliasing], while the frequency components of $(z-q)^2$ are distributed uniformly over the range (0, $½f_S$) as indicated in Figure 2. It follows that the fraction of N_Q in the passband of x(t) is $2W/f_S$ so that the overall (filtered) quantizing noise power is

$$N_F = \mathrm{Av}(x-\hat{x})^2 = 2WN_Q/f_S$$
$$= c_1(2^M-1)^{-2} f_S^{-1} \quad (\mathrm{PCM}). \quad (3)$$

The overload mechanism of DPCM differs from that of PCM. When A in Figure 1 is an inegrator, z = x - y is related to the change in x over the f_S^{-1} sec sampling interval. The quantizer overloads when the rate of change of x exceeds $½(2^M-1)\, d\, f_S$, the greatest possible rate of change of y. Hence with M fixed, d proportional to f_S^{-1} maintains a fixed overload immunity. It follows that $N_Q \sim d^2 \sim f_S^{-1}$, or combining the influence of f_S and M,

$$N_Q \sim (2^M-1)^{-2}\ f_S^{-2} \qquad \text{(DPCM)}. \tag{4}$$

To account for the effect of the low pass filter, we note that for $M \geq 2$ bit DPCM, $(z\text{-}q)^2$ is uniformly distributed in frequency as in Figure 2. However, one-bit DPCM - delta modulation (ΔM) - is exceptional. Here, the power at frequencies near $\frac{1}{2}f_S$ is significantly greater than at lower frequencies and the proportion of N_Q passed by the filter is approximately W/f_S rather than $2W/f_S$. See Figure 3. Hence

$$N_F = c_2(2^M-1)^{-2}\ f_S^{-3}; \qquad M \geq 2 \qquad \text{(DPCM)} \tag{5}$$

$$= \tfrac{1}{2}c_2 f_S^{-3} \qquad ; \qquad M = 1 \qquad (\Delta M). \tag{6}$$

Although they omit many details, Equations 3, 5 and 6 exhibit important properties of a large class of digital modulation methods: noise varying exponentially with number of quantizer bits; noise varying algebraically with sampling rate. To interpret the effects of f_S and M on efficiency, we recall that our measure of efficiency is the bit rate

$$f_B = Mf_S. \tag{7}$$

Considering Equations 3, 5, 6 and 7 together, we conclude that over a broad range of quality objectives it is efficient to have f_S low and to control N_F by the value of M. Figures 4 and 5 consolidate these equations. To illustrate our point we consider reducing N_F by a factor of 4 in a DPCM modulator. Assuming the original quantizer size to be 5 bits, we can make the quality improvement by increasing M to 6, thereby increasing f_B by the factor 6/5 which is a 20% increase. On the other hand, to bring about the same improvement with M constant, it is necessary that f_S, and therefore f_B, be increased by the factor $\sqrt[3]{4}$ which is a 59% increase. (PCM is far more extreme; a fourfold increase in f_S and f_B is required.)

Contrasted with this relationship of efficiency to f_S and M is our earlier comment that over a wide range of conditions cost is relatively independent of speed, but is quite sensitive to quantizer size. Hence, the need to compromise between cost and efficiency in simple PCM and DPCM, which we summarize as follows:

	Economical	Efficient
Sampling	High speed	Low speed
Quantization	Few bits	Many bits

Tradeoffs Between Cost and Efficiency   Existing and proposed modulation techniques comprise a wise variety of responses to the conflict between cost and efficiency. At one extreme is the modulation scheme used for digital transmission over telephone trunks. In this application, efficiency is essential because the overall system cost is controlled by the time-multiplexed transmission line and PCM, using the lowest sampling rate consistent with constraints on aliasing distortion, is the common modulation scheme for trunk transmission. To enhance the efficiency relative to the uniform quantization we have described, non-uniform quantization is introduced. This permits distortion requirements to be met over a large class of input signals with fewer bits per sample than would be possible with uniform quantization. In many existing systems non-uniform quantization is achieved by non-linear processing (companding) of the unquantized signal samples at the modulator. The quantizer is uniform[1] In more modern systems a non-uniform quantizer, one with unequally spaced output levels and decision levels, is implemented directly.

Although this modulation scheme is basically more costly (for the sake of greater efficiency) than the uniform PCM scheme, considerable cost savings can be realized through time sharing when many signals are to be modulated at the same point. Thus the Bell System Dl digital channel bank[1] is shared among 24 channels and the D2 bank[2] among 96 channels, and as a result, the cost per channel of the most expensive elements of the modulator is a fraction of what it would be if each signal were coded individually.

Although time-shared, efficient modulators are appropriate to telephone trunk transmission, there are many newer applications that call for cheaper, though perhaps less efficient methods. In many of these applications, opportunities for time sharing of coding equipment are limited or absent and the cost of analog-to-digital conversion has an important influence on total system cost. The emphasis on economy, rather than efficiency, in such applications has led in recent years, to renewed interest in ΔM, in which the quantizer is binary, responding only to the polarity of a signal, rather than to the precise level.

The original single-integration and double integration ΔM schemes[3] are extremely inefficient[*] and considerable effort has been devoted to the discovery of more efficient ΔM techniques which sacrifice some of the economy of the simplest approaches but retain the cost advantages inherent in two-level quantization. The variable-step-size approach to AM has brought the greatest success to this effort, resulting in code formats with efficiencies rivalling those of non-uniform PCM. The cost of variable-step-size ΔM is considerably higher than that of the simpler methods, but when signals are coded individually, it is lower than that of PCM. In applications with time-sharing opportunities, it is impossible to generalize about the relative efficiency and economy of the two approaches. It is clear that both will receive extensive application.

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[*] For example, a proposal to use ΔM as the code format of a digital switching system[4] calls for voice channels encoded at the rate of 1,500 kbits/sec in contrast to the 64 kbits/sec required with non-uniform PCM.

In the most promising variable-step-size schemes, the step size (the gain of the modulator feedback loop and the gain of the demodulator) depends on the binary ΔM signal. When the ΔM signal indicates that the rate of change of the input is low, the step size is also low so that the approximation signal changes in small increments to provide close tracking of the input. On the other hand, when the rate of change of the input is high, overload is prevented by a step size that is high enough to allow the approximation signal to follow the input.

Among the many variable-step-size schemes, a principal distinguishing characteristic is the speed with which the step size changes. For example, the modulator of the subscriber loop multiplex system[5] developed at Bell Laboratories, allows the step size to change by a factor of two at 125 µsec intervals. By contrast, the step size of the digitally controlled ΔM techniques[6], invented at Philips and incorporated in a 60 voice channel multiplex[7] system by TRT in France, varies with a time constant of a few milliseconds.

We have thus far given examples of two approaches to the problem of reconciling the goals of economy and efficiency. Our first example was efficiency-oriented and achieved economies through time-sharing. The second began with the most economical modulator and modified it at some cost to enhance efficiency. A third approach, illustrated in Figure 6, retains the simplest fixed-step-size M modulator and introduces digital signal processing hardware to convert the high speed ΔM signal to a more efficient format. This approach is based on the observation that standardized digital circuit elements are economical and are becoming more so and therefore the combination of a simple inefficient modulator and a digital code converter may, in some cases, be more economical than an efficient modulator. Applications of this approach have been studied with conversion from fixed-step-size ΔM to PCM[8.9], DPCM,[10] and variable step size ΔM[11].

Conclusions The widespread current interest in digital modulation is stimulated by technological advances which provide new opportunities for converting between analog and digital signal formats and by the proliferation of new digital communication, computer and control applications which use digitally encoded analog signals. The diversity of current approaches to digital modulation has created for the system designer the flexibility to choose a modulation method that meets his specific cost and performance requirements.

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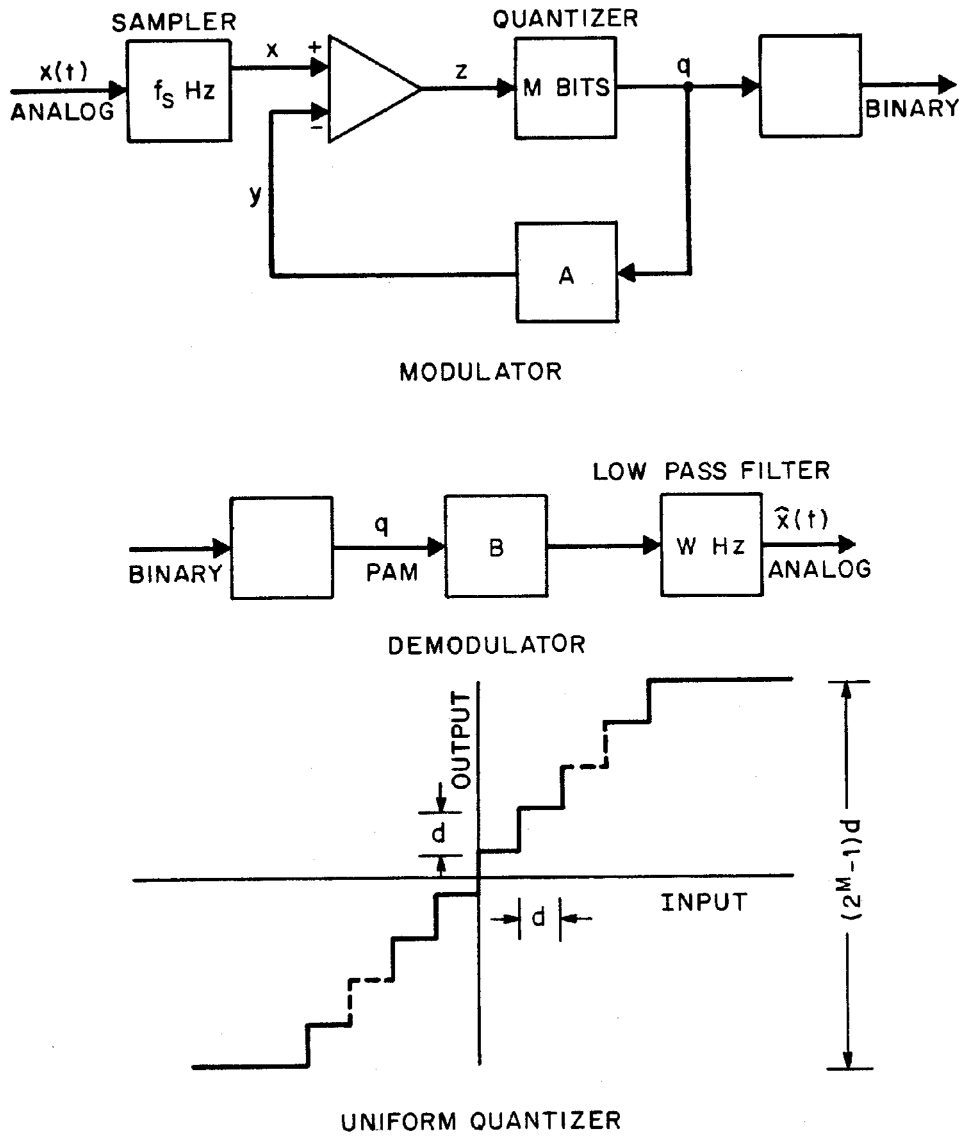

Fig. 1 - Digital modulation system.

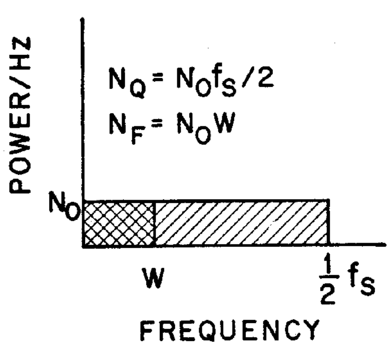

Fig. 2 - Effect of filtering on quantizing noise.

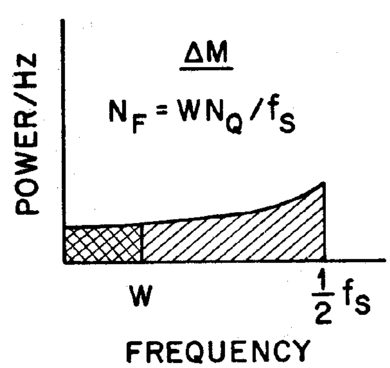

Fig. 3 - Filtering effect in ΔM.

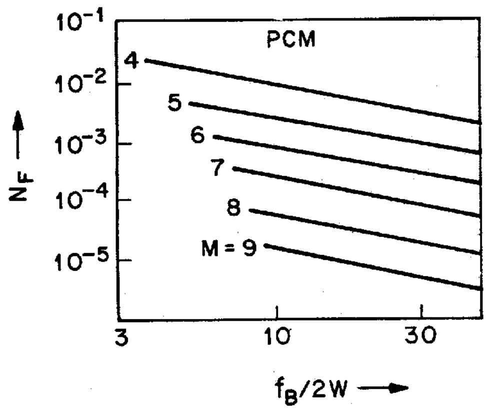

Fig. 4 - Quantizing noise of PCM.

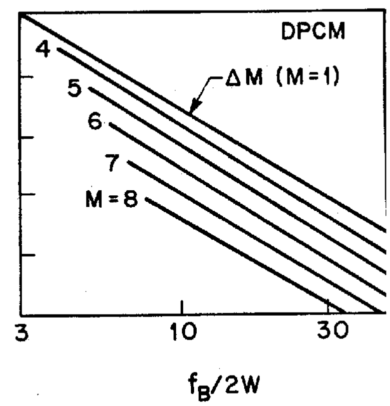

Fig. 5 - Quantizing noise of DPCM.

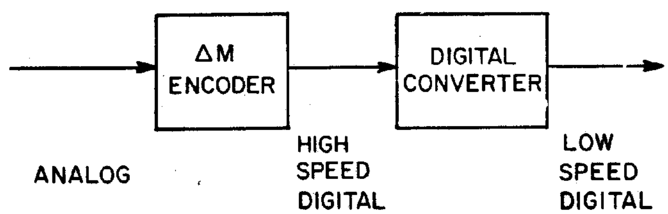

Fig. 6 - Use of code conversion in modulation.