DISTRIBUTED TERRESTRIAL RADIOLOCATION USING THE RLS ALGORITHM

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ABSTRACT

This paper presents the development of two distributed terrestrial radiolocation algorithms that use range estimates derived from DS-CDMA waveforms. The first algorithm, which is RLS-based, is derived as the solution of an approximate least-squares positioning problem. This algorithm has the advantage of reduced computational complexity, compared with the EKF-based algorithm that is presented. It is shown via simulations that both positioning algorithms perform well, with the performance of the EKF-based algorithm being superior.

KEY WORDS

Radiolocation, RLS, EKF, GSIC, DS-CDMA

INTRODUCTION

A terrestrial radiolocation system has previously been proposed using DS-CDMA waveforms and a hand-shaking protocol. In such a system, a master node which wishes to learn its position transmits a request-to-send (RTS) packet, and reference nodes with known (or estimated) positions immediately and simultaneously transmit acknowledge (ACK) packets based on CDMA waveforms [1],[2],[3]. A multi-user synchronization algorithm based on Generalized Successive Interference Cancellation (GSIC) [1] then estimates the times-of-arrivals, or equivalently round-trip travel times (RTTs) and hence relative ranges. Other synchronization approaches for estimating ranges in CDMA systems include subspace-based algorithms [4],[5],[6], expectation-maximization [7], and maximum-likelihood methods [8]. Alternatively, relative ranges can be estimated using field strength measurements [9]. Finally, the range estimates and known or estimated reference node positions are used to estimate the position on the master node.

In positioning systems such as the one described above the most substantial errors occur when no line-of-sight (LOS) path exists between the transmitter and the reference node: this is the non-line-of-sight (NLOS) scenario. Several authors have proposed methods for dealing with the NLOS problem. In [10] it is shown that NLOS measurements can be distinguished from LOS measurements by using a time history of the range measurements and that the NLOS measurements can be corrected if the statistics of the ranging sensor are known. Alternatively, in [11] a residual weighting algorithm that does not require knowledge of the sensor statistics is proposed for reducing the positioning error caused by NLOS measurements, provided that a sufficient number of range measurements are available. The NLOS effect can also be combated by sharing relative range measurements between nodes and computing the estimated positions either at a central processor or in manner that distributes the processing between the nodes.

In this paper, distributed positioning algorithms based on an approximate recursive least-squares (RLS) algorithm and an adaptation of the extended Kalman filter (EKF) algorithm are presented. For the RLS-based algorithm (Section II) it is shown that the use of differenced squared range measurements results in a linearized measurement model, and hence provides an alternative to the use of extended Kalman filter algorithms, as in [12], [13]. For completeness and to facilitate the comparison of the RLS-based algorithm with the EKF-based algorithm, a distributed EKF-based positioning algorithm is derived in Section III. In Section IV the performance of the RLS- and EKF-based algorithms is evaluated using simulated range estimates, and in Section V the performance of both algorithms in a complete simulated terrestrial radiolocation system is presented.

RLS-BASED POSITIONING ALGORITHM

The RLS-based positioning algorithm is derived as follows. Assume that for packet index n, all nodes $k = 1, 2, \ldots, K$ sequentially execute the handshaking ranging protocol. Then let $\mathbf{x}_k$ represent the x-y position of node k (assumed time-invariant over the observation duration of less than one second.) The GSIC algorithm at node k yields a range estimate from nodes $i = 1, 2, \ldots, k\text{-}1, k\text{+}1, \ldots, K$ for packet n given by

$$\rho_{i,k}(n) = \|\mathbf{x}_i - \mathbf{x}_k\| + v_k(n), \tag{1}$$

where $v_k(n)$ is the estimation error. Consider the differenced squared ranges

$$\rho_{i,j,k}(n) = \frac{1}{2}\left(\rho_{i,k}^2(n) - \rho_{j,k}^2(n)\right) \approx \frac{1}{2}\left(\|\mathbf{x}_i\|^2 - \|\mathbf{x}_j\|^2\right) + \left(\mathbf{x}_j - \mathbf{x}_i\right)^T \mathbf{x}_k + v_{i,j,k}(n). \tag{2}$$

For simplicity, $v_{i,j,k}(n)$ is modeled as zero-mean Gaussian, which ignores the effect of squaring and position-dependent terms on the original noise $v_k(n)$. Then the following transformed measurements $z_{i,j,k}(n)$ are linear in $\mathbf{x}_k$, when $\mathbf{x}_i$, and $\mathbf{x}_j$ are known.

$$z_{i,j,k}(n) = \rho_{i,j,k}(n) - \frac{1}{2}\left(\|\mathbf{x}_i\|^2 - \|\mathbf{x}_j\|^2\right) = \left(\mathbf{x}_j - \mathbf{x}_i\right)^T \mathbf{x}_k + v_{i,j,k}(n). \tag{3}$$

An exact least-squares cost function for nodes $k = 1, 2, \ldots, K$ over packet indices $l = 1, 2, \ldots, n$ is given by

$$J(n) = \sum_{k=1}^{K}\sum_{l=1}^{n}\|\mathbf{z}_k(l) - \mathbf{A}_k\mathbf{x}_k\|^2, \tag{4}$$

where $\mathbf{z}_k(l) \in \Re^{(K-1)(K-2)/2}$ is

$$\mathbf{z}_k(l) = \begin{bmatrix} z_{1,2,k} \\ \vdots \\ z_{1,k-1,k} \\ z_{1,k+1,k} \\ \vdots \\ z_{K-1,K,k} \end{bmatrix}. \tag{5}$$

The resulting matrix $\mathbf{A}_k \in \Re^{(K-1)(K-2)/2x2}$ (for 2-D positioning) is

$$\mathbf{A}_k = \begin{bmatrix} (\mathbf{x}_2 - \mathbf{x}_1)^T \\ \vdots \\ (\mathbf{x}_{k-1} - \mathbf{x}_1)^T \\ (\mathbf{x}_{k+1} - \mathbf{x}_1)^T \\ \vdots \\ (\mathbf{x}_K - \mathbf{x}_{K-1})^T \end{bmatrix}. \tag{6}$$

Note that both $\mathbf{z}_k(l)$ and $\mathbf{A}_k$ are functions of the positions $\{\mathbf{x}_l\}$, and hence the cost function (4) is non-quadratic.

The non-quadratic cost function $J(n)$ can be forced into quadratic form by approximating $\mathbf{z}_k(l)$ and $\mathbf{A}_k$ using previous estimates of positions $\mathbf{x}_i(l\text{-}1)$ and $\mathbf{x}_j(l\text{-}1)$ for nodes $i, j \neq k$. The cost function is then clearly separable in the nodes k, yielding

$$J_k(n) = \sum_{l=1}^{n} \left\| \tilde{\mathbf{z}}_k(l) - \mathbf{A}_k(l)\mathbf{x}_k \right\|^2, \tag{7}$$

where

$$\tilde{\mathbf{z}}_k(l) = \begin{bmatrix} \rho_{1,2,k}(l) - \frac{1}{2}\left(\left\|\hat{\mathbf{x}}_1(l-1)\right\|^2 - \left\|\hat{\mathbf{x}}_2(l-1)\right\|^2\right) \\ \vdots \\ \rho_{1,k-1,k}(l) - \frac{1}{2}\left(\left\|\hat{\mathbf{x}}_1(l-1)\right\|^2 - \left\|\hat{\mathbf{x}}_{k-1}(l-1)\right\|^2\right) \\ \rho_{1,k+1,k}(l) - \frac{1}{2}\left(\left\|\hat{\mathbf{x}}_1(l-1)\right\|^2 - \left\|\hat{\mathbf{x}}_{k+1}(l-1)\right\|^2\right) \\ \vdots \\ \rho_{K-1,K,k}(l) - \frac{1}{2}\left(\left\|\hat{\mathbf{x}}_{K-1}(l-1)\right\|^2 - \left\|\hat{\mathbf{x}}_K(l-1)\right\|^2\right) \end{bmatrix}. \tag{8}$$

The estimated measurement matrix is then

$$\mathbf{A}_k(l) = \begin{bmatrix} \left(\hat{\mathbf{x}}_2(l-1) - \hat{\mathbf{x}}_1(l-1)\right)^T \\ \vdots \\ \left(\hat{\mathbf{x}}_{k-1}(l-1) - \hat{\mathbf{x}}_1(l-1)\right)^T \\ \left(\hat{\mathbf{x}}_{k+1}(l-1) - \hat{\mathbf{x}}_1(l-1)\right)^T \\ \vdots \\ \left(\hat{\mathbf{x}}_K(l-1) - \hat{\mathbf{x}}_{K-1}(l-1)\right)^T \end{bmatrix}. \tag{9}$$

The positioning algorithm is then described by K separable RLS updates. However, it should be emphasized that the estimated measurements $\tilde{\mathbf{z}}_k(l)$ and measurement matrix $\mathbf{A}_k(l)$ depend on *previous* RLS-derived position estimates for users $i, j \neq k$. The RLS updates are computed as follows:

$$\mathbf{P}_k(n)^{-1} = \lambda\, \mathbf{P}_k(n-1)^{-1} + \mathbf{A}_k(n)^T \mathbf{A}_k(n) \tag{10}$$

$$\hat{\mathbf{x}}_k(n) = \hat{\mathbf{x}}_k(n-1) + \mathbf{P}_k(n)\mathbf{A}_k(n)^T \left[\tilde{\mathbf{z}}_k(n) - \mathbf{A}_k(n)\hat{\mathbf{x}}_k(n-1)\right]. \tag{11}$$

With $\hat{\mathbf{x}}_k(0)$ chosen based on any prior estimate of the position of node k that may be available (or, otherwise, chosen arbitrarily), the diagonal pseudo-covariance matrix $\mathbf{P}_k(0)$ can be initialized as follows:

$$\mathbf{P}_k(0) = \mathrm{Cov}\{\hat{\mathbf{x}}_k(0)\}. \tag{12}$$

EKF-BASED POSITIONING ALGORITHM

The EKF-based positioning algorithm is derived as follows. Again let $\mathbf{x}_k$ represent the x-y position of node k (assumed time-invariant over the observation duration of less than one second.) With K the total number of nodes, the vector $\mathbf{x}$ of unknown states consists of the $\mathbf{x}_k$ stacked into a column vector as follows:

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_K \end{bmatrix} \in \Re^{2K}. \tag{13}$$

Again assume that for packet index n, all nodes $k = 1, 2, ..., K$ sequentially execute the handshaking ranging protocol. However, note the following difference between the RLS- and EKF-based positioning algorithms. In the RLS-based algorithm node k updates only its own position estimate, and for packet n, node k delays its position estimate update until it has received the range estimates obtained at all other nodes. In contrast, for the EKF-based positioning algorithm, during packet epoch n, node k executes the handshaking ranging protocol, obtains its GSIC-derived ranges, and updates the global state estimate vector $\mathbf{x}$ containing the estimates of the positions of all the nodes. After user k has updated $\mathbf{x}$, user k+1 executes the handshaking ranging protocol, obtains its GSIC-derived range estimates, and updates **x**, etc. Thus, in effect, a singe range sensor and state estimator moves from node to node and uses the global state estimates computed at the previous node to compute the new global state estimates. This arrangement is, in fact, the optimal sensor and state estimator configuration, as proven analytically and demonstrated via information flow graphs in [14].

Hence for packet n at node k, denote the GSIC-derived estimate of the range between nodes k and j as the measurement

$$z_{k,j} = \left\| \mathbf{x}_j - \mathbf{x}_k \right\| + v_{k,j}\,. \tag{14}$$

Then the measurement equation is given by

$$\mathbf{z}(nK + (k-1)) = \begin{bmatrix} z_{k,1} \\ \vdots \\ z_{k,k-1} \\ z_{k,k+1} \\ \vdots \\ z_{k,K} \end{bmatrix} = \mathbf{h}_k(\mathbf{x}) + \mathbf{v}_k(n) = \begin{bmatrix} \left\|\mathbf{x}_1 - \mathbf{x}_k\right\| \\ \vdots \\ \left\|\mathbf{x}_{k-1} - \mathbf{x}_k\right\| \\ \left\|\mathbf{x}_{k+1} - \mathbf{x}_k\right\| \\ \vdots \\ \left\|\mathbf{x}_K - \mathbf{x}_k\right\| \end{bmatrix} + \begin{bmatrix} v_{k,1} \\ \vdots \\ v_{k,k-1} \\ v_{k,k+1} \\ \vdots \\ v_{k,K} \end{bmatrix} \in \Re^{(K-1)}, \tag{15}$$

The $v_{k,j}$ are assumed to be i.i.d. Gaussian with zero mean and variance σ_v^2, so that $\mathbf{v}_k(n)$ has a diagonal covariance matrix $\mathbf{R}_k$ with nonzero entries σ_v^2.

The measurements $\mathbf{z}(nK + (k - 1))$ are clearly nonlinear in $\mathbf{x}$ because of the dependence of $\mathbf{h}_k(\mathbf{x})$ on $\|\mathbf{x}_j - \mathbf{x}_k\|$. However, the measurement equation can be linearized by expanding $\mathbf{h}_k(\mathbf{x})$ in a Taylor series about $\mathbf{x}$. Define $\mathbf{H}_k^{T}(n)$ as

$$\mathbf{H}_k^{\mathbf{T}}(n) = \frac{\partial \mathbf{h}_k(\mathbf{x})}{\partial \mathbf{x}}\Big|_{\mathbf{x}=\hat{\mathbf{x}}(nK+(k-1)|nK+(k-2))} \in \Re^{(K-1)x(2K)} \tag{16}$$

Then $\mathbf{H}_k^{T}(n)$ is composed of 1-by-2 rows $H_{k,p,q}^{T}(n)$ of the form

$$H_{k,p,q}^{T}(n) = \begin{cases} \dfrac{(\mathbf{x}_p - \mathbf{x}_k)^T}{\left\|\mathbf{x}_p - \mathbf{x}_k\right\|},\ p = q \\ -\dfrac{(\mathbf{x}_p - \mathbf{x}_k)^T}{\left\|\mathbf{x}_p - \mathbf{x}_k\right\|},\ q = k \\ 0,\ \text{otherwise} \end{cases} \tag{17}$$

where $p = 1, 2, \ldots, k\text{-}1, k\text{+}1, \ldots, K$ indexes the rows in $\mathbf{H}_k^{T}(n)$ and $q = 1, 2, \ldots, K$ indexes the $(K\text{-}1)$-by-2 column blocks in $\mathbf{H}_k^{T}(n)$. The linearized measurement equation is then

$$\begin{aligned} \mathbf{z}(nK + (k-1)) &= \mathbf{h}_k(\hat{\mathbf{x}}(nK + (k-1) \mid nK + (k-2))) \\ &\quad + \mathbf{H}_k^{\mathbf{T}}(n)\left[\mathbf{x} - \hat{\mathbf{x}}(nK + (k-1) \mid nK + (k-2))\right] + \mathbf{v}_k(n). \end{aligned} \tag{18}$$

Finally, the EKF updates $\hat{\mathbf{x}}(nK+(k-1)\,|\,nK+(k-1))$ may be computed using the following relations (in reverse order):

$$\begin{aligned}\hat{\mathbf{x}}(nK+(k-1)\,|\,nK+(k-1)) &= \hat{\mathbf{x}}(nK+(k-1)\,|\,nK+(k-2)) \\ &\quad + \mathbf{L}(nK+(k-1))\big[\mathbf{z}(nK+(k-1)) - \mathbf{h}_k(\hat{\mathbf{x}}(nK+(k-1)\,|\,nK+(k-2)))\big]\end{aligned} \tag{19}$$

$$\hat{\mathbf{x}}(nK+(k-1)\,|\,nK+(k-2)) = \hat{\mathbf{x}}(nK+(k-2)\,|\,nK+(k-2)) \tag{20}$$

$$\begin{aligned}\mathbf{\Sigma}(nK+(k-1)\,|\,nK+(k-1)) &= \\ &\mathbf{\Sigma}(nK+(k-1)\,|\,nK+(k-2)) \\ &-\mathbf{\Sigma}(nK+(k-1)\,|\,nK+(k-2))\mathbf{H}_k(n) \\ &\times\big[\mathbf{H}_k^T(n)\mathbf{\Sigma}(nK+(k-1)\,|\,nK+(k-2))\mathbf{H}_k(n)+\mathbf{R}_k(n)\big]^{-1} \\ &\times\mathbf{H}_k{}^T(n)\mathbf{\Sigma}(nK+(k-1)\,|\,nK+(k-2))\end{aligned} \tag{21}$$

$$\mathbf{L}(nK+(k-1)) = \mathbf{\Sigma}(nK+(k-1)\,|\,nK+(k-2))\mathbf{H}_k(n)\mathbf{\Omega}^{-1}(nK+(k-1)) \tag{22}$$

$$\mathbf{\Omega}(nK+(k-1)) = \mathbf{H}_k{}^{\mathbf{T}}(n)\mathbf{\Sigma}(nK+(k-1)\,|\,nK+(k-2))\mathbf{H}_k(n)+\mathbf{R}_k(n) \tag{23}$$

$$\mathbf{\Sigma}(nK+(k-1)\,|\,nK+(k-2)) = \mathbf{\Sigma}(nK+(k-2)\,|\,nK+(k-2)), \tag{24}$$

where $\mathbf{\Sigma}(nK + (k - 1) \,|\, nK + (k - 2))$ is the covariance matrix for the state estimates $\hat{\mathbf{x}}(nK+(k-1)\,|\,nK+(k-2))$ computed at node k during packet epoch n using measurements collected during packet epoch n at nodes 1, 2, …, k-1 and for packets 1, 2, …, n-1 at all nodes. The initializations $\hat{\mathbf{x}}(0\,|\,0)$ can be chosen based on any available prior estimates of the node positions, and $\mathbf{\Sigma}(0 \,|\, 0)$ can be chosen based on the estimated uncertainty in the prior estimates:

$$\mathbf{\Sigma}(0\,|\,0) = \mathrm{Cov}\{\hat{\mathbf{x}}(0\,|\,0)\}. \tag{25}$$

SIMULATION RESULTS

The performance of the RLS- and EKF-based distributed positioning algorithms was first evaluated using simulated range measurements equal to the true ranges plus additive white Gaussian noise of variance σ_v^2 (a variable parameter). In these simulations the prior position estimates were set to the true node positions plus additive white Gaussian noise of variance σ_x^2, and the initial covariances were initialized accordingly. K = 6 nodes were simulated; three of the nodes were GPS-assisted ($\sigma_x^2 = 100$ meters^2), while three of the nodes were unassisted ($\sigma_x^2 = 10{,}000$ meters^2). Figures 1 and 2 present the average position estimation error convergence results obtained over 5000 independent runs of the RLS- and EKF-based algorithms for $\sigma_v^2 = 900$ meters^2 and $\sigma_v^2 = 10{,}000$ meters^2, respectively. (Note: a range estimation variance of $\sigma_v^2 = 900$ meters^2 could correspond to a range error of 30 meters, or one CDMA chip, for a chip duration of 100nS.) From Figure 1 it is clear that both the RLS- and EKF-based algorithms function well under realistic conditions. The EKF-based algorithm converges more quickly to the final position estimates, but the average error of the final estimates is nearly identical for both algorithms. From Figure 2 it is clear that the EKF-based algorithm performs much better than the RLS-based algorithm when the measurement noise is very high. Under these conditions, the EKF-derived position estimates still converge well, but the RLS-derived estimates converge more slowly and never achieve the level of error minimization that is obtained with the EKF-based algorithm.

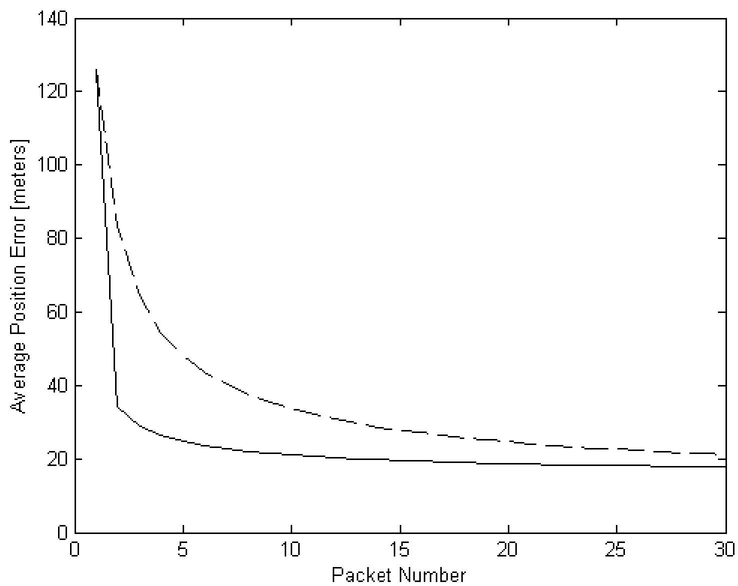

Figure 1: Plot of average position estimation error versus iteration number for the RLS-based (dashed) and EKF-based (solid) positioning algorithms using simulated range measurements with a variance of $\sigma_v^2 = 900$ meters^2.

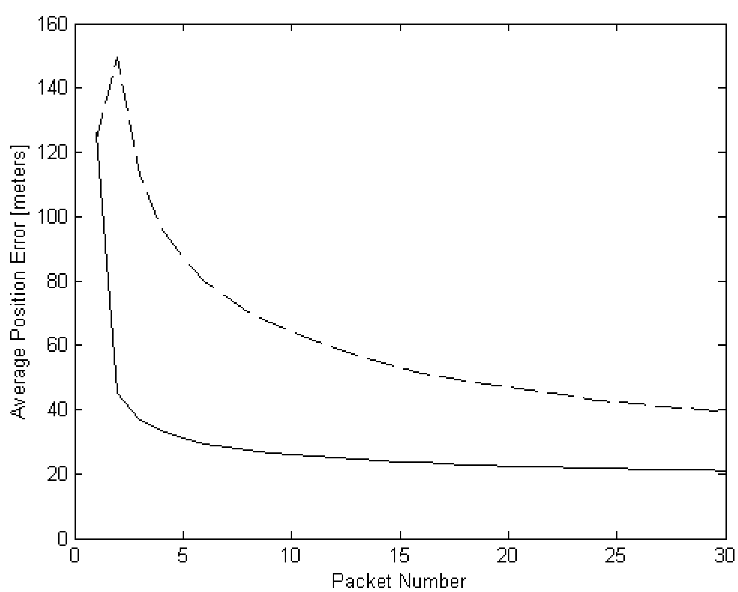

Figure 2: Plot of average position estimation error versus iteration number for the RLS-based (dashed) and EKF-based (solid) positioning algorithms using simulated range measurements with a variance of $\sigma_v^2 = 10,000$ meters^2.

POSITIONING RESULTS FROM COMPLETE RADIOLOCATION SIMULATION

In a full-scale terrestrial radiolocation simulation, multipath channel estimates for an urban environment were obtained using a 2-D ray tracing simulation (as in [15]), and estimated ranges (or, equivalently, TOAs) were acquired using the GSIC algorithm [1]. Finally, the RLS- and EKF-based distributed positioning algorithms were applied to obtain position estimates for each node. In these simulations the total number of nodes was $K = 6$. Three simulated reference nodes where equipped with GPS positioning devices, and three simulated user nodes did not have GPS positioning information. It was assumed that the position estimation error for the GPS-assisted reference nodes was Gaussian with a variance of 100 meters^2 (corresponding to 95% certainty within 20 meters). For the unassisted users the initial uncertainty in the position estimates was modeled as Gaussian with a variance of 10,000 meters^2. The range estimates used by the RLS- and EKF-based distributed positioning algorithms corresponded to the application of GSIC to 28 sets of ACK packets at each of the six nodes (i.e. each node acted as the master node and received ACK packets from each of the other nodes 28 different times). In order to rigorously test the convergence properties of the positioning algorithms, the range estimates corresponding to the 28 sets of ACK packets were used in a randomized order, and different prior position estimate initializations were used in 5000 recorded tests of each positioning algorithm. Figure 3 traces the performance of the RLS-based positioning algorithm for one realization of packet ordering and prior position estimate initialization. With the GPS-assisted reference nodes labeled as diamonds and the unassisted user nodes labeled as circles, the position estimates are shown to converge quickly and accurately to the true positions. In Figure 4 the performance of the algorithms over 5000 different realizations of range estimate orderings and prior position estimate initializations is demonstrated. It is shown that the average position estimation error for the unassisted users decreases rapidly and consistently to an average error of only 20 meters. Thus the feasibility of the complete CDMA-based terrestrial radiolocation system using GSIC-derived ranges and the RLS-based or EKF-based positioning algorithms that are presented in this paper is verified.

CONCLUSION

Two effective radiolocation algorithms have been presented in this paper. Both algorithms are distributed, though the communication requirements and processing configurations differ for the two algorithms. The RLS-based algorithm is attractive because of its low computational complexity. On the other hand, the EKF-based algorithm tends to converge in fewer iterations, and when the range estimates are very noisy the latter converges to a lower average position estimation error. Finally, it has been shown via a complete radiolocation simulation that the RLS- and EKF-based algorithms perform well in a realistic setting.

ACKNOWLEDGMENTS

This work was supported in part by DARPA under grant No. MDA972-00-1-0036, and by a grant from the International Foundation for Telemetering.

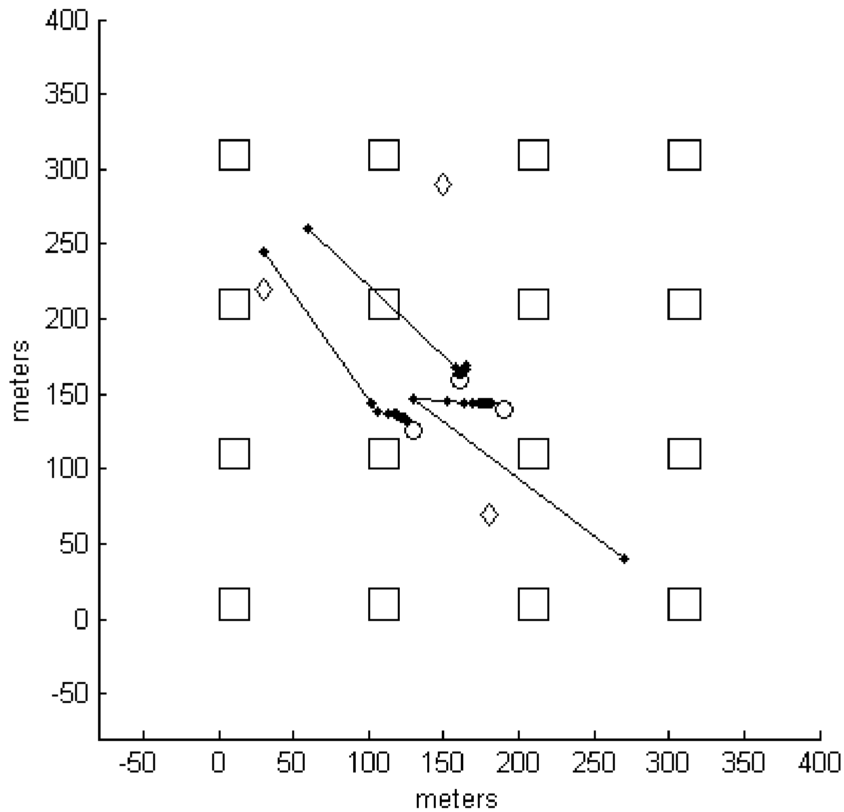

Figure 3: Simulated urban geometry and traces of the RLS-based node position estimates.

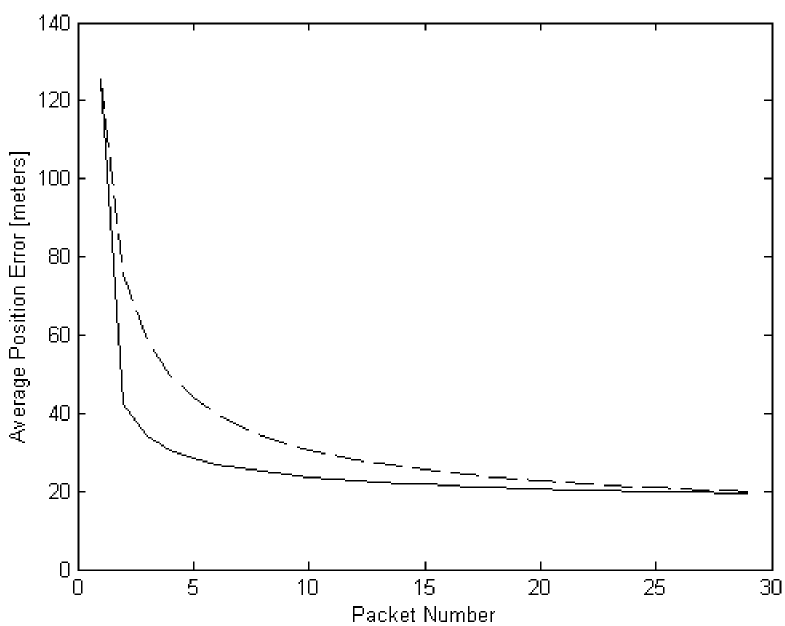

Figure 4: Plot of average position estimation error versus GSIC packet number for the RLS-based (dashed) and EKF-based (solid) positioning algorithms.

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