ANALYSIS OF CYCLOSTATIONARY AND SPECTRAL CORRELATION OF FEHER-KEYING (FK) SIGNALS

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ABSTRACT

Feher Keying (FK) signals are clock shaped baseband waveforms with the potential to attain very high spectral efficiencies. Two FK signals which have different level rectangular waveforms (named as FK-1) or sinusoidal waveforms (named as FK-2) for two binary symbols are considered in this paper. These signals have periodic components in the time domain. Therefore they have cyclostationary properties. This means that spectral correlation exists in the frequency domain. For each type of waveforms, spectral correlation has been investigated. FK signals can be expressed mathematically into two parts in the frequency domain – discrete part and continuous part. The discrete part has one or more discrete impulse(s) in their spectra and the continuous part has periodically the same shape of harmonics in their spectra. The correlations of their spectra have been obtained mathematically and by simulation. It is shown that FK signals have high correlation related to the symbol rate.

Finally, some suggestions how these properties can be used to improve their performance by devising better demodulators are discussed. These properties can be used for interference rejection at the receiver, which results in low bit error rate performance.

KEY WORDS

Feher Keying (FK), autocorrelation, power spectral density (PSD), cyclostationarity, bit error rate (BER)

I. INTRODUCTION

Well tailored waveforms are essential for efficient digital communications. Several parameters can be considered to evaluate efficient communications – spectral efficiency, power efficiency, symbol error rate, system complexity for implementation, etc.. Among these waveforms are Feher Keying (FK) waveforms [1]. In these signals, clock is shaped according to the incoming data. Therefore they are called clock shaped. So far, two waveforms are suggested: rectangular shaped is named as FK-1 and sinusoidal shaped is named as FK-2 with different levels according to data values. These have the potential to attain very high spectral efficiency [2],[3]. In this paper, these two binary waveforms – FK-1 and FK-2 – are considered to analyze their properties. The waveforms are shown in Fig. 1..

Data communication signals have periodic characteristics because information data are transmitted periodically while synchronized with clock signals. Therefore the transmitted signals have cyclostationary properties. This means that in the frequency domain, the spectrum has one or more impulse(s) at the clock frequency (or symbol rate) and is correlated to the symbol rate. In this paper, for FK clocked waveforms, how these cyclostationary properties work and can be interpreted and how these properties can be utilized to have better performance are described.

FK-2 signal has one impulse at the symbol rate frequency and the same shape harmonics in its spectrum while FK-1 has periodic impulses at the multiples of symbol rate frequency. The autocorrelations of these spectra have been investigated.

For the conclusion, some suggestions how we can utilize these correlations are proposed.

II. MATHEMATICAL REPRESENTATION AND NUMERICAL EVALUATION OF GENERIC DIGITAL COMMUNICATION SIGNALS

A. Mathematical Representation of Baseband Waveform of Digital Communications

<u>Mathematical representation of binary baseband waveforms in the time domain</u>

Let $g_0(t)$ and $g_1(t)$ be the waveforms for state 0 and state 1 respectively in a binary system which has the probability of state 0, ρ. Then most baseband waveforms for specific data string is expressed as:

$$\mathrm{s}(t) = \sum_{n=-\infty}^{\infty} a_n \prod (t - nT_S) \qquad \textbf{(1)}$$

where a_n is

$$a_n = \begin{cases} g_0(t - nT_S) & \text{for binary 0} \\ g_1(t - nT_S) & \text{for binary 1} \end{cases}$$

and

$$\prod(t) = \mathrm{rect}\left(\frac{t}{T_S}\right)$$

and T_S is the symbol duration.

Power spectral density for binary waveforms

Power spectral density for binary signals can be derived as [4]

$$P_S(f) = 2 f_S \rho (1-\rho) \,|\, G_0(f) - G_1(f) \,|^2 + f_S^2 \, [\rho \, G_0(0) + (1-\rho) \, G_1(0)]^2 \, \delta(f)$$

$$+2 f_S^2 \sum_{m=1}^{\infty} \,|\, \rho \, G_0(mf_S) + (1-\rho) \, G_1(mf_S) \,|^2 \, \delta(f - mf_S) \quad \textbf{(2)}$$

where f_s is the symbol rate and $G_0(f)$ and $G_1(f)$ are the Fourier transformations of $g_0(t)$ and $g_1(t)$, respectively. As described in [4], this equation is divided into three parts – DC component, continuous component, and harmonics.

B. Cyclostationarity of Binary Baseband Waveforms

Wide-sense cyclostationarity can be determined by two properties in the time domain – mean and autocorrelation. These have to be satisfied with [5],[6]:

mean: $$\mathrm{E}[\mathrm{x}(t)] = \mathrm{E}[\mathrm{x}(t - mT_S)] \quad \textbf{(3)}$$

and

autocorrelation: $$\mathcal{R}_X(t + \tau) = \mathrm{E}[\mathrm{x}(t)\, \mathrm{x}^*(t - \tau)] = \mathrm{E}[\mathrm{x}(t + mT_0)\, \mathrm{x}^*(t + mT_0 - \tau)] \quad \textbf{(4)}$$

for any integer m.

Means of the binary baseband waveform expressed in (1) at times t and $(t - mT_S)$ are:

$$\mathrm{E}[s(t)] = \mathrm{E}\Big[\sum_{n=-\infty}^{\infty} a_n \prod(t - nT_S)\Big] \quad \textbf{(5)}$$

and

$$\mathrm{E}[s(t - mT_S)] = \mathrm{E}\Big[\sum_{n=-\infty}^{\infty} a_n \prod(t + (m-n)T_S)\Big]. \quad \textbf{(6)}$$

Then

$$\mathrm{E}[s(t)] = \mathrm{E}[s(t - mT_S)] \quad \textbf{(7)}$$

is satisfied. Its autocorrelations are

$$\mathrm{E}[\mathrm{s}(t)\, \mathrm{s}^*(t - \tau)] = \mathrm{E}\Big[\sum_i \sum_j a_i a_j \prod(t) \prod(t - \tau)\Big] \quad \textbf{(8)}$$

and

$$E[s(t+mT_S)\, s^*(t + mT_S - \tau)] = E[\sum_i \sum_j a_i a_j \prod(t + iT_S) \prod(t + jT_S - \tau)]. \quad \textbf{(9)}$$

Then

$$E[s(t)\, s^*(t - \tau)] = E[s(t+mT_S)\, s^*(t + mT_S - \tau)] \quad \textbf{(10)}$$

is satisfied. Therefore with equations (7) and (10) it can be shown that binary baseband signals expressed in (1) are cyclostationary.

III. MATHEMATICAL REPRESENTATION OF FK SIGNALS

FK signals can be expressed mathematically in the time domain and in the frequency domain as following. With these expressions, cyclostationarity can be tested for FK signals.

A. Representation in the Time Domain and Frequency Domain

In the time domain, FK signals can be represented by

$$s_{FK}(t) = \sum_{n=-\infty}^{\infty} a_n \prod(t - nT_S)$$

where

$$a_n = \begin{cases} A_0\, p_{FK}(t - nT_S) & \text{for binary 0} \\ A_1\, p_{FK}(t - nT_S) & \text{for binary 1} \end{cases}$$

where A_0 and A_1 are constant and

$$p_{FK}(t) = \begin{cases} \prod 2(t + \frac{T_S}{4}) - \prod 2(t - \frac{T_s}{4}) & \text{for FK-1} \\ \sin\frac{2\pi}{T_S}(t + \frac{T_S}{2}) \prod(\frac{t}{T_s}) & \text{for FK-2.} \end{cases} \quad \textbf{(11)}$$

In the frequency domain, their Fourier transformations and power spectral densities can be taken. If Fourier transformed signals have spectral correlations, then power spectral densities also have these properties. The power spectral densities (from eq. (2) above) of FK signals are expressed as:

$$P_{FK}(f) = 2\, f_S\, \rho\, (1-\rho)\, |\, G_0(f) - G_1(f)\, |^2 + f_S^2\, [\rho\, G_0(0) + (1-\rho)\, G_1(0)]^2\, \delta(f)$$

$$+2\, f_S^2 \sum_{m=1}^{\infty} |\, \rho\, G_0(mf_S) + (1-\rho)\, G_1(mf_S)\, |^2\, \delta(f - mf_S)$$

where $G_0(f) = A_0\, \mathcal{F}\, [p_{FK}(t)]$ and $G_1(f) = A_1\, \mathcal{F}\, [p_{FK}(t)]$ **(12)**

B. Cyclostationarity

The FK signals presented in this paper have a periodic component in the power spectrum density appearing as discrete line(s) which contributes to the cyclostationarity of the signal. In this paper, we investigate the cyclostationary properties of two FK signals. The generic details of this feature have been analyzed previously by Gardner [6], [7] and collaborators [8], [9], [10]. From these studies, a range of possible applications can be devised in which the signal can be detected in the presence of noise or other interfering signals [6]. Gardner in his patent [11] engineered a frequency-shift (FRESH) filter to shift a MSK type signal to another frequency where noise can be rejected. In the review of literature, other authors cite this feature to enhanceperformance of different types of communications systems. Carroll [12] states that cyclostationarity may be used to recover a signal modulated onto the periodic parts of a chaotic signal. Adlard [13] uses this feature in the structure of a frequency-shift filter (FRESH) to achieve interference rejection.

In the time domain, for FK signals the mean and autocorrelation are calculated as follows:

$$\mathrm{E}[s_{FK}(t)] = \frac{1}{2}\,[\, A_0 + A_1\,] \sum_{n=-\infty}^{\infty} P_{FK}(t - nT_S) \prod (t - nT_S) \quad \textbf{(13)}$$

and

$$\mathcal{R}_{SFK}(t,\tau) = \mathrm{E}[s_{FK}(t)\, s_{FK}^*(t-\tau)]$$

$$= \frac{1}{4}\,[\, A_0^2 + A_1^2 + 2\, A_0\, A_1]\; \mathrm{E}[\sum_i \sum_j P_{FK}(t - iT_S) \prod (t - iT_S) P_{FK}(t - jT_S - \tau) \prod (t - jT_S - \tau)\,].$$

(14)

Then the following can be easily proved:

$$\mathrm{E}[s_{FK}(t)] = \mathrm{E}[s_{FK}(t + nT_S)]$$

and

$$\mathcal{R}_{SFK}(t,\tau) = \mathcal{R}_{SFK}(t + nT_S, \tau). \quad \textbf{(15)}$$

With the above derivations, it can be shown that FK signals have cyclostationary properties.

C. Spectral Correlation

Most digital communication signals are cyclostationary. Therefore they have spectral correlations. In the frequency domain, autocorrelation can be derived between itself and its frequency shifted version with the frequency shift, Δf:

For the Fourier transform of $s_{FK}(t)$, $S_{FK}(f)$, the autocorrelation is expressed as

$$\mathcal{R}_{SFK}(f,\Delta f) = \lim_{F \to \infty} \frac{1}{2F}\int_{f-F}^{f+F} S_{FK}(f)S_{FK}(f-\Delta f)df \qquad \textbf{(16)}$$

For the power spectral density of $s_{FK}(t)$, $P_{FK}(f)$, the autocorrelation is as follows:

$$\mathcal{R}_{PFK}(f,\Delta f) = \lim_{F \to \infty} \frac{1}{2F}\int_{f-F}^{f+F} P_{FK}(f)P_{FK}(f-\Delta f)df \qquad \textbf{(17)}$$

In the frequency domain, spectral correlation exists for both Fourier transformed version of the signal and its power spectral density.

With thw above equations, FK signals are simulated in the following section.

IV. SIMULATION RESULTS AND INTERPRETATION

Simulation has been done by using System View from Elanix. Power spectral densities are shown in Fig. 2. for two FK signals. In this figure, it is shown that there is one spike (or impulse) at the symbol rate frequency for FK-2 and for at each multiple of the symbol rate frequency for FK-1. The two power spectral densities have discrete and continuous components, and the discrete line or lines come from the periodic component introduced in the formation of the FK signal. This means that clock information can be extracted to be used for symbol synchronization. Since in the construction of these signals, there is this periodic component, these signals have cyclostionary properties. In Fig. 3., it can be shown that autocorrelation is periodically repeated with the period of symbol duration in the time domain. This means that FK signals have cyclostationarity.

Fig. 4. shows the spectral correlations of the signals. These curves were calculated and plotted by applying the autocorrelation function to the power spectral densities by the simulation program. In FK-1, at the multiples of two times the symbol rate frequency, correlations have spikes while in FK-2, at the multiples of the symbol rate frequency correlations have them. This means that at the multiples of symbol rate frequency, considerable correlation exists. By using this fact, we may extract the information from various frequency bands.

Waveforms containing information have correlations in the frequency domain while noise or interference does not have correlation. Since there is a considerable amount of correlation between a waveform and its shifted version in the frequency domain, a waveform and a correlated frequency shifted version of the same signal can be linearly combined to improve the SNR. This feature can be exploited to detect weak signal or signal in the presence of noise. Using this fact, interference can be reduced at the receiver with frequency shifted filtering [13].

V. CONCLUSION

In this paper, cyclostationarity is derived mathematically and is tested for two types of FK signals by computer simulation. The simulation indicates that both have considerable spectral correlations at the multiples of symbol rate frequency. For FK-1, the correlation is locally maximized at the multiple of $2f_{s,}$ for FK-1 and at the multiple of $f_{s,}$ for FK-2.

Using these properties of cyclostationarity, some of the following may be accomplished:

- synchronization information can be extracted using spectral correction
- signal identification and detection and
- interference rejection as described before.

The above properties can be applied for other digital communication waveforms in a similar way. Therefore we can have clues by which more efficient waveforms or detection methods at the receiver may be devised.

This paper also introduced ideas for receiver optimization in addition to the work already published regarding transmitter design for more general class of clock modulated signals [1-3],[15-17]. However, any idea, which has the possibility of far exceeding the performance of conventional systems is an excellent choice for further study [14].

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SIGNAL PARAMETERS
Clock : pulse train, f = 10 Hz
PN Generator: register lengh=10, register tap= 3, $L = 2^{10} - 1 = 1023$
To make FK#1: 2 square waves of amplitude 1 and 3 V respectively, f = 10 Hz
To make FK #2: 2 sinusoid waves of amplitude 1 and 3 V respectively, f= 10 Hz

SYSTEM VIEW COMPUTER SIMULATION PARAMETERS
Run time =109.2 sc
Sample rate= 300 Hz
No. of samples= 32768
Frequency resolution = Sample rate/ No. of samples = $9.1 \; 10^{-3}$ Hz
Before applying the spectral analysis to the signals, we use the Bartley window.
Autocorrelation and crosscorrelation were performed with zero padding.

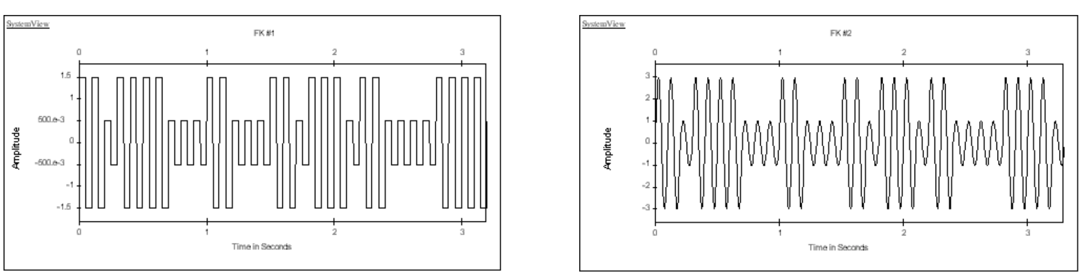
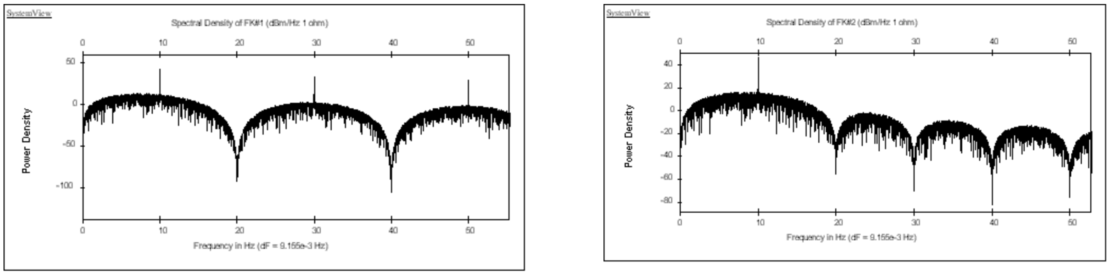


Fig. 1. FK signal waveforms in the time domain of FK-1 and of FK-2.



a.) b.)

Fig. 2. Power spectral densities of FK signals for (a) FK-1 and (b) FK-2.

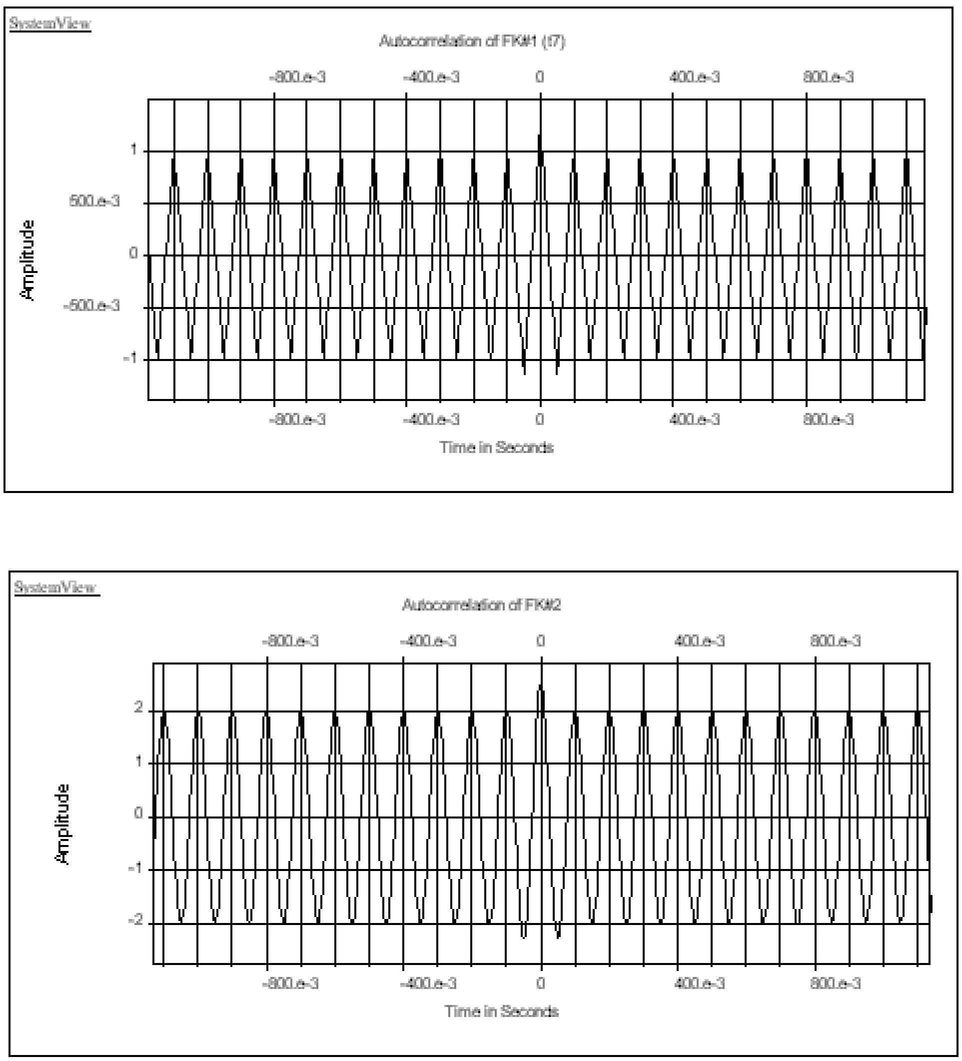


Fig. 3. Autocorrelations of FK signals in the time domain for FK-1 and FK-2 shown only for 1 sec of the data.

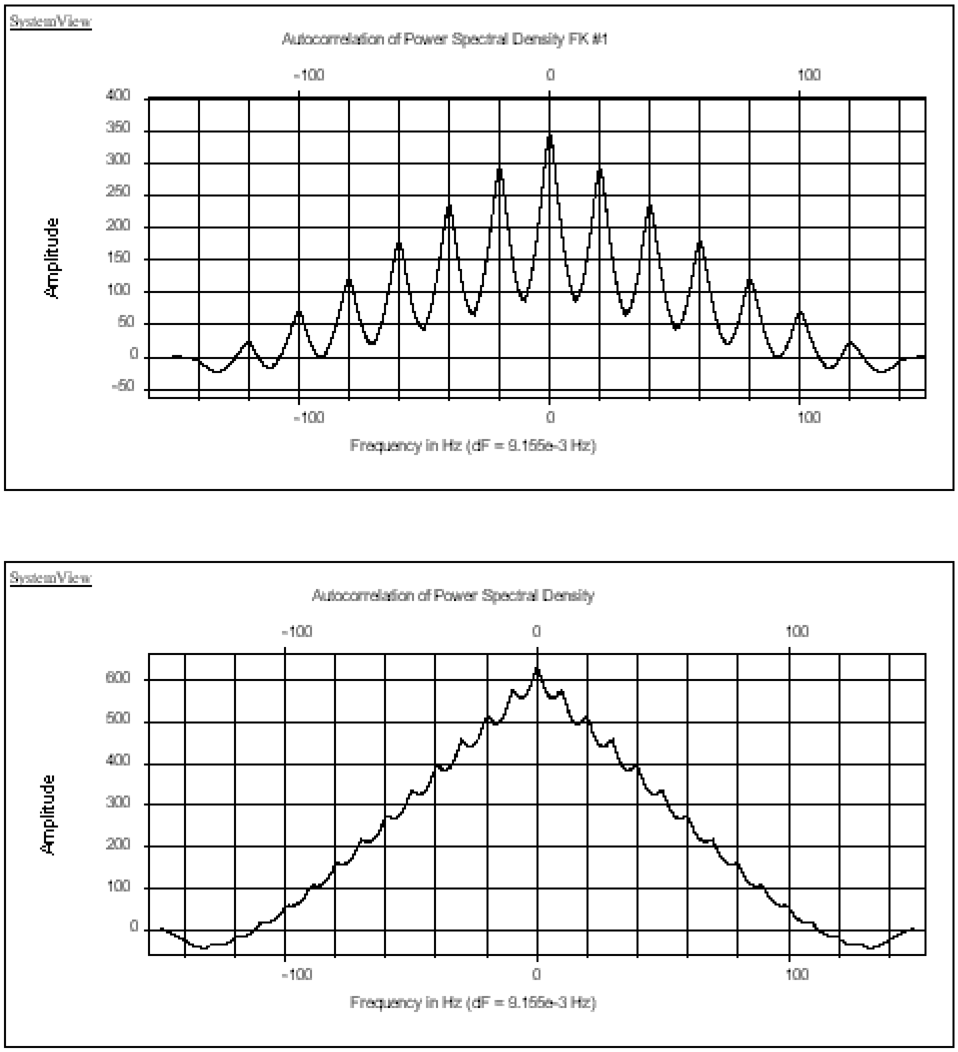


Fig. 4. Autocorrelations of FK signals in the frequency domain for FK-1 and FK-2.