Adaptive Space-Time Waveform Design in Ad hoc Networks using the IMMSE Algorithm *

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Abstract

An Ad hoc network with unicasting is considered, in which each node has an M element antenna array. Transmission from node $l(i)$ to i is quasi-synchronous, so that code acquisition is not required. Space-Time (S-T) waveforms are transmitted with temporal dimension N_s Nyquist samples. An adaptive, distributed S-T waveform design algorithm is developed, which maintains QoS while attempting to minimize transmit power. The resulting Iterative Minimum Mean-Square Error–Time Reversal algorithm (IMMSE-TR) sets the transmit S-T vector at node i to the conjugate time-reverse of the linear MMSE S-T detector. It is shown that IMMSE-TR corresponds to a noncooperative game which attempts to minimize transmit power while paying an interference tax. Simulation results are presented demonstrating high power efficiencies for heavily-loaded systems.

Keywords

Beamforming, Ad hoc networks, adaptive sequence construction, game theory.

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Introduction

We consider an Ad hoc network where each node has an array of M elements and transmits an N_s Nyquist sample long waveform. Each node i has a single link (unicasting) destination node $l(i) \neq i$, with $i, l(i) \in \{1, 2, \ldots, N\}$. The optimization problem is to adaptively construct the S-T transmit waveform $\tilde{\mathbf{g}}_i \in \mathbb{C}^{MN_s}$ to maintain a constant SNR γ_0 (QoS) while minimizing the sum transmit power $P_{sum} = \sum_{i=1}^{N} ||\tilde{\mathbf{g}}_i||^2$. The problem is an extension of optimal transmit/receive beamforming [1][2][3] which has been addressed using iterative MMSE techniques. Iterative methods have also been employed for temporal-only signal adaptation [4][5][6] and for S-T waveform design in uplink CDMA [7].

Synchronization (code acquisition) is a challenging problem in an Ad hoc network. In contrast to cellular topologies, power control is infeasible and hence the near-far effect greatly hinders the code acquisition process. Here, we consider a quasi-synchronous transmission scheme, in which node $l(i)$ purposefully advances its symbol transmission by $r_{i,l(i)}/c$, where $r_{i,j}$ is the distance between nodes i, j. QS operation eliminates the need for code acquisition [8][9], and can be implemented using network-wide timing/positioning available from a combination of atomic clocks, GPS and cooperative radiolocation [10].

The IMMSE-TR algorithm is summarized as follows: Node i uses the LMS/RLS algorithm to construct its linear MMSE S-T detector $\mathbf{w}_i \in \mathbb{C}^{MN_s}$ using a training sequence (e.g. embedded in a RTS/CTS handshake). The normalized transmit S-T vector $\mathbf{g}_i$ is then set to the conjugate time-reverse of $\mathbf{w}_i$, denoted by $\mathbf{w}_i^{r,*}$ as defined in the sequel. The process is iterated between nodes $i, l(i)$ until convergence. It is shown that IMMSE-TR then corresponds to the power algorithm with $\mathbf{g}_i$ the maximizing eigenvector of an SNR-related objective matrix. IMMSE-TR equivalently corresponds to a noncooperative game that maximizes normalized SNR while minimizing an approximate measure of interference between nodes.

Space-Time Signals and Channels

A generic direct-sequence type spread-spectrum system is considered, in which the symbol duration T is much longer than the multipath spread, and the system bandwidth is $W = PG/T$, where $PG > 1$ is the processing gain. The Nyquist sampling interval is then $T_s = T/(2PG)$. Further define $N_s = T/T_s$ as the number of samples/symbol. The transmit S-T signal from node i, $\tilde{\mathbf{g}}_i \in \mathbb{C}^{MN_s}$, is then defined by

$$\tilde{\mathbf{g}}_i = [\mathsf{g}_i(N_s - 1)^T, \mathsf{g}_i(N_s - 2)^T, \ldots, \mathsf{g}_i(0)^T]^T \qquad (1)$$

where $\mathsf{g}_i(n) \in \mathbb{C}^M$ is the vector of inputs to the transmit array at Nyquist sample n. Symbols $b_i(m)$ are transmitted at a rate $1/T$.

The S-T channel is represented by a Nyquist sampled response $\mathsf{H}_{i,j}(p) \in \mathbb{C}^{M \times M}$, for $p = 0, 1, \ldots, 2N_s - 1$. Let $\mathsf{H}_{i,j}^{\nu}$ represent the array response at node i due to transmission from j on the ν-th multipath, where $r_{i,j}^1 = r_{i,j}$ is the length of the direct path. Then $\mathsf{H}_{i,j}(p)$

is given by the interpolation formula, where transmit node j applies a timing advance of $r_{j,l(j)}/c$ sec. for QS operation.

$$\mathsf{H}_{i,j}(p) = \sum_{\nu=1}^{N_p} \mathsf{H}_{i,j}^{\nu} \mathrm{sinc}\left(\left(pT_s - \frac{r_{i,j}^{\nu}}{c} + \frac{r_{j,l(j)}}{c}\right)/T_s\right). \tag{2}$$

It is assumed that the multipath spread $T_m = (r_{i,j}^{N_p} - r_{i,j})/c$ satisfies $T_m = LT_s << N_sT_s$ as in a typical spread-spectrum system. Hence, the channel $\mathsf{H}_{i,l(i)}(p)$ is approximately limited to samples $p = \{0, 1, \ldots, L-1\}$. However, reception at i from nodes $l \neq l(i)$ is asynchronous, and the effective channel thus has support $p = \{0, 1, \ldots, 2N_s - 1\}$.

Given the channel model in (2) and the short multipath spread assumption, the array output vector $\mathsf{r}_i(n) \in \mathbb{C}^M$ at node i at time $mN_s + k$, for $k = 0, 1, \ldots, N_s - 1$ is

$$\mathsf{r}_i(mN_s + k) = \sum_{p=0}^{L-1} \mathsf{H}_{i,l(i)}(p)\mathsf{g}_{l(i)}((k-p))b_{l(i)}(m) + \tag{3}$$
$$\sum_{l \neq i, l(i)} \sum_{q=0}^{2} \sum_{p=0}^{2N_s-1} \mathbf{H}_{i,l}(p)\mathsf{g}_l((k-p) + qN_s)b_l(m-q) + \mathsf{n}_i(mN_s + k),$$

where $\mathsf{n}_i(k)$ is circular white Gaussian noise with covariance matrix $\mathbf{I}$.

The S-T received signal is then $\mathbf{r}_i(m) \in \mathbb{C}^{MN_s}$, defined by $\mathbf{r}_i(m) = [\mathsf{r}_i((m+1)N_s - 1)^T, \ldots \mathsf{r}_i(mN_s)^T]^T$. This S-T vector can be written in vector-matrix form as

$$\mathbf{r}_i(m) = \mathbf{H}_{i,l(i)}^0 \tilde{\mathbf{g}}_{l(i)} b_{l(i)}(m) + \sum_{l \neq i, l(i)} \sum_{q=0}^{2} \mathbf{H}_{i,l}^q \tilde{\mathbf{g}}_l b_l(m-q) + \mathbf{n}_i(m). \tag{4}$$

The matrices $\mathbf{H}_{i,j}^q \in \mathbb{C}^{MN_s \times MN_s}$ are block-Toeplitz, with subblock n, m (Matlab notation) given by

$$\mathbf{H}_{i,j}^q((n-1)M + 1 : nM, (m-1)M + 1 : mM) = \mathsf{H}_{i,j}(m - n + qN_s).$$

for $n, m = 1, \ldots, N_s$.

IMMSE-TR Algorithm

The IMMSE-TR algorithm attempts to minimize transmit power while maintaining QoS (SNR). In terms of the unit-norm S-T linear detector $\mathbf{w}_{l(i)} \in \mathbb{C}^{MN_s}$, the SNR at node $l(i)$ is

$$\Gamma_{l(i)} = \frac{|\mathbf{w}_{l(i)}^H \mathbf{H}_{l(i),i} \tilde{\mathbf{g}}_i|^2}{\mathbf{w}_{l(i)}^H \mathbf{R}_{l(i)}(\tilde{\mathbf{g}}_{-l(i)}) \mathbf{w}_{l(i)}}, \tag{5}$$

where the multiaccess interference (MAI) plus noise covariance matrix is $\mathbf{R}_i(\tilde{\mathbf{g}}_{-i}) \in \mathbb{C}^{MN_s \times MN_s}$, defined by

$$\mathbf{R}_i(\tilde{\mathbf{g}}_{-i}) = \sum_{l \neq i, l(i)} \sum_{q=0}^{2} \mathbf{H}_{i,l}^q \tilde{\mathbf{g}}_l \tilde{\mathbf{g}}_l^H (\mathbf{H}_{i,l}^q)^H + \mathbf{I}. \tag{6}$$

The game theoretic notation $\tilde{\mathbf{g}}_{-i}$ indicates dependence on all transmit vectors $\tilde{\mathbf{g}}_l$ for $l \neq i$.

The optimum linear S-T receiver is MVDR, given in unnormalized form by $\tilde{\mathbf{w}}_{l(i)} = \mathbf{R}_{l(i)}^{-1} \mathbf{H}_{l(i),i} \tilde{\mathbf{g}}_i$. Using this solution in $\Gamma_{l(i)}$ yields the following optimization problem.

$$\begin{aligned} \text{Minimize} \quad & \sum_{i=1}^{N} ||\tilde{\mathbf{g}}_i||^2 \\ \text{Subject to} \quad & \Gamma_{l(i)}(\tilde{\mathbf{g}}_i, \tilde{\mathbf{g}}_{-i}) = \tilde{\mathbf{g}}_i^H \mathbf{H}_{l(i),i}^H \mathbf{R}_{l(i)}^{-1}(\tilde{\mathbf{g}}_{-i}) \mathbf{H}_{l(i),i} \tilde{\mathbf{g}}_i \geq \gamma_0, \end{aligned} \tag{7}$$

where γ_0 is the target SNR.

The optimization problem (7) is non-convex, and a closed-form solution for the power minimizing $\tilde{\mathbf{g}}_i$ does not exist [3]. However, it is also shown in [3] that the IMMSE algorithm for beamforming can satisfy the necessary (though not sufficient) conditions for optimality. The IMMSE beamforming method is extended to S-T waveform design in Table 1. IMMSE-TR hinges on the following definition of time-reverse matrices and vectors.

Definition 1 *Time-Reversal: Let* $\mathbf{x} \in \mathbb{C}^{MN}$ *be a vector with* N *temporal subvectors* $\mathbf{x}(n) \in \mathbb{C}^M$*, hence* $\mathbf{x} = [\mathbf{x}(N)^T \mathbf{x}(N)^T \ldots \mathbf{x}(1)^T]^T$*. The time-reverse is then* $\mathbf{x}^r = [\mathbf{x}(1)^T \mathbf{x}(2)^T \ldots \mathbf{x}(N)^T]^T$ *with subvectors* $\mathbf{x}^r(n) = \mathbf{x}(N-n+1)$ *for* $n = 1, \ldots, N$*.*

Let $\mathbf{A} \in \mathbb{C}^{MN \times MN}$ *be a matrix with temporal subblocks of dimension* $M \times M$*, such that (Matlab notation)* $\mathbf{A}(n,m) \equiv \mathbf{A}((n-1)M : nM, (m-1)M : mM)$*. The time-reverse matrix* $\mathbf{A}^r$ *is defined by the subblocks* $\mathbf{A}^r(n,m) = \mathbf{A}(N-n+1, N-m+1)$ *for* $n, m = 1, \ldots, N$*.*

The following properties of time-reversed systems are then readily derived.

Proposition 1 *Time-Reversed System Relations: Let* $\mathbf{y} = \mathbf{A}\mathbf{x}$*, where* $\mathbf{y}, \mathbf{x} \in \mathcal{C}^{MN,MN}$ *are composed of temporal subvectors* $\mathbf{y}(n), \mathbf{x}(n) \in \mathbb{C}^M$*, and* $\mathbf{A}$ *is divided into temporal subblocks* $\mathbf{A}(n,m) \in \mathbb{C}^{M \times M}$*. Then*

$$\mathbf{y}^r = (\mathbf{A}\mathbf{x})^r = \mathbf{A}^r \mathbf{x}^r. \tag{8}$$

Let $\mathbf{A}, \mathbf{B} \in \mathbb{C}^{MN \times MN}$ *be S-T matrices with temporal subblocks* $\mathbf{A}(n,m), \mathbf{B}(n,m) \in \mathbb{C}^{M \times M}$*. The time-reverse of the product satisfies* $(\mathbf{AB})^r = \mathbf{A}^r \mathbf{B}^r$*.*

Let $\mathbf{A} \in \mathbb{C}^{MN \times MN}$ *with temporal subblocks* $\mathbf{A}(n,m) \in \mathbb{C}^{M \times M}$ *be invertible. Then* $(\mathbf{A}^{-1})^r = (\mathbf{A}^r)^{-1}$*.*

The next proposition defines space-time channel reciprocity.

Proposition 2 *Space-Time Channel Reciprocity: Let* $\mathbf{H}_{i,l(i)} \in \mathbb{C}^{N_s M \times N_s M}$ *represent the space-time channel from transmit array* $l(i)$ *to receive array* i*. Nyquist sample* n *of the S-T channel is denoted by* $\mathsf{H}_{i,l(i)}(n) \in \mathbb{C}^{M \times M}$ *for* $n = 0, \ldots, N_s - 1$*. The temporal subblocks of size* $M \times M$ *are then given by* $\mathbf{H}_{i,l(i)}(n,m) = \mathsf{H}_{i,l(i)}(m-n) \in \mathbb{C}^{M \times M}$ *for* $n, m = 1, \ldots, N_s$*. Assume spatial channel reciprocity, so that* $\mathsf{H}_{i,l(i)}(p)^T = \mathsf{H}_{l(i),i}(p)$*. Then* $\mathbf{H}_{i,l(i)}^T = \mathbf{H}_{l(i),i}^r$*. That*

For $n = 1, 2, \ldots$
    For each node i
        $m \leftarrow 1$
        While $||\mathbf{g}_i(m+1) - \mathbf{g}_i(m)|| > \epsilon$
            Update normalized S-T MVDR detector at i
            $\mathbf{w}'_i(m+1) = \mathbf{R}_i(\tilde{\mathbf{g}}^n_{-i})^{-1}\mathbf{H}_{i,l(i)}\mathbf{g}_{l(i)}(m)$
            $\mathbf{w}_i(m+1) \leftarrow \mathbf{w}'_i(m+1)/||\mathbf{w}'_i(m+1)||$
            $\mathbf{g}_i(m+1) \leftarrow \mathbf{w}^{r,*}_i(m+1)$
            Transmit packet to node $l(i)$ –
            Update node $l(i)$ S-T detector.
            $\mathbf{w}'_{l(i)}(m+1) = \mathbf{R}_{l(i)}(\tilde{\mathbf{g}}^n_{-i})^{-1}\mathbf{H}_{l(i),i}\mathbf{g}_i(m+1)$
            $\mathbf{w}_{l(i)}(m+1) \leftarrow \mathbf{w}'_{l(i)}(m+1)/||\mathbf{w}'_{l(i)}(m+1)||$
            $\mathbf{g}_{l(i)}(m+1) \leftarrow \mathbf{w}^{r,*}_{l(i)}(m+1)$
            $m \leftarrow m+1$
        End while
        $m_i \leftarrow m$
        Update SVD approximation $\mathbf{g}^n_i = \mathbf{s}_i \otimes \mathbf{a}_i \approx \mathbf{g}_i(m_i)$
        Transmit estimated SNR $\Gamma_{l(i)}(\sqrt{P^n_i}\mathbf{g}^n_i, \tilde{\mathbf{g}}^n_{-i})$ from $l(i) \rightarrow i$
        Update power $P^n_i \leftarrow P^n_i\gamma_0/\Gamma_{l(i)}$
    Next i
Next n

Table 1: IMMSE-TR adaptive space-time waveform algorithm.

is, the transpose of the S-T block Toeplitz channel response at node i due to $l(i)$ is the time-reverse of the response at $l(i)$ to i. Note that this space-time reciprocity does not *hold for channels $\mathbf{H}_{i,j}$, for $j \neq l(i)$, due to the QS time-advance $\mathbf{r}_{j,l(j)}/c$.*

The basic operation of IMMSE-TR is as follows: On each iteration n, nodes $i = 1, 2, \ldots, N$ are updated sequentially. Each nodal update i is composed of IMMSE subiterations m. Specifically, at subiteration $m+1$, node i computes its MMSE/MVDR S-T detector $\mathbf{w}_i(m+1)$ based on a training sequence transmitted by node $l(i)$. In practice, the MMSE S-T detector is updated using the LMS or RLS algorithm, however in the simulations in the sequel, it is assumed that the unnormalized $\mathbf{w}'_i(m+1)$ takes on its optimum value $\mathbf{w}'_i(m+1) = \mathbf{R}^{-1}_i(\tilde{\mathbf{g}}^n_{-i})\mathbf{H}_{i,l(i)}\tilde{\mathbf{g}}_{l(i)}(m)$. The transmit beamformer is then set to the conjugate time-reverse, $\mathbf{g}_i(m+1) = \mathbf{w}_i(m+1)^{r,*}$. Node i then transmits a training sequence back to $l(i)$, which computes its S-T detector $\mathbf{w}_{l(i)}(m+1)$, and resets its transmit vector to $\mathbf{g}_{l(i)}(m+1) = \mathbf{w}_{l(i)}(m+1)^{r,*}$. The result of the subiterations is characterized by the following proposition.

Proposition 3 *The S-T transmit vector $\mathbf{g}_i(m)$ converges to the maximum eigenvector of the objective matrix*

$$\mathbf{G}_i = (\mathbf{R}^{-1}_i)^{r,*}\mathbf{H}^H_{l(i),i}\mathbf{R}^{-1}_{l(i)}\mathbf{H}_{l(i),i}, \tag{9}$$

Equivalently, $\lim_{m\to\infty} \mathbf{g}_i(m)$ *solves*

$$\mathbf{g}_i = \mathit{arg}\max_{\mathbf{g}} \frac{\mathbf{g}^H \mathbf{H}_{l(i),i}^H \mathbf{R}_{l(i)}(\tilde{\mathbf{g}}_{-i})^{-1} \mathbf{H}_{l(i),i}\mathbf{g}}{\mathbf{g}^H \mathbf{R}_i(\tilde{\mathbf{g}}_{-i})^{r,*}\mathbf{g}}. \tag{10}$$

Proof: From Table 1, $\mathbf{g}_i(m+1) = \mathbf{w}_i^{r,*}(m+1)$. Assuming exact convergence of the LMS/RLS MMSE algorithm, and using the rules for time-reversal in Definition 1 we then have $\mathbf{g}_i(m+1) = (\mathbf{R}_i^{-1})^{r,*}\mathbf{H}_{i,l(i)}^{r,*}\mathbf{g}_{l(i)}^{r,*}(m)$. However, from S-T channel reciprocity in Proposition 2, $\mathbf{H}_{i,l(i)}^{r,*} = \mathbf{H}_{l(i),i}^H$. Furthermore, $\mathbf{g}_{l(i)}^{r,*}(m) = \mathbf{w}_{l(i)}(m)$. Substituting for $\mathbf{w}_{l(i)}(m) = \mathbf{R}_{l(i)}^{-1}\mathbf{H}_{l(i),i}\mathbf{g}_i(m)$ yields the final update for the transmit vector.

$$\mathbf{g}_i(m+1) = \frac{1}{c}(\mathbf{R}_i^{-1})^{r,*}\mathbf{H}_{l(i),i}^H \mathbf{R}_{l(i)}^{-1}\mathbf{H}_{l(i),i}\mathbf{g}_i(m), \tag{11}$$

where c normalizes $||\mathbf{g}_i(m+1)|| = 1$. Following [3], eq. (11) corresponds to the power algorithm, leading as $m \to \infty$ to the solution in eqs. (9),(10).

The noncooperative game theory interpretation of IMMSE-TR is similar to [3]. The utility function corresponding to Table 1 is

$$\begin{aligned} &u_i(\tilde{\mathbf{g}}_i, \tilde{\mathbf{g}}_{-i}^n) = \\ &\quad \nu(\gamma_0 - \Gamma_{l(i)}(\tilde{\mathbf{g}}_i, \tilde{\mathbf{g}}_{-i})) + \ln\left(\tilde{\mathbf{g}}_i^H \mathbf{H}_{l(i),i}^H \mathbf{R}_{l(i)}(\tilde{\mathbf{g}}_{-i}^n)^{-1}\mathbf{H}_{l(i),i)}\tilde{\mathbf{g}}_i\right) - \ln\left(\tilde{\mathbf{g}}_i^H \mathbf{R}_i(\tilde{\mathbf{g}}_{-i}^n)^{r,*}\tilde{\mathbf{g}}_i\right), \end{aligned} \tag{12}$$

where $\nu(x)$ is an arbitrary concave, continuous function with maximum at $x = 0$. The IMMSE-TR algorithm at iteration n, i then corresponds to the noncooperative game $\tilde{\mathbf{g}}_i^n = \arg\max_{\tilde{\mathbf{g}}_i} u_i(\tilde{\mathbf{g}}_i, \tilde{\mathbf{g}}_{-i}^n)$. The utility $u_i()$ increases with normalized SNR (first ln term) and decreases with increasing interference to other nodes (second ln term – an interference tax.) The utility is maximized when the power $P_i = ||\tilde{\mathbf{g}}_i||^2$ is set so that the SNR constraint is met with equality ($\Gamma_{l(i)} = \gamma_0$). The interference tax can be rewritten using the time-reversal definitions 1 and replacing $\mathbf{w}_l^{r,*}$ by $\mathbf{g}_l$ as

$$\begin{aligned} &\mathbf{g}_i^H \mathbf{R}_i^{r,*}\mathbf{g}_i = \\ &\quad \mathbf{g}_i^H \sum_{l\neq i,l(i)} \mathbf{H}_{i,l}^{r,*}\mathbf{g}_l^{r,*}(\mathbf{g}_l^H)^{r,*}(\mathbf{H}_{i,l}^H)^{r,*}\mathbf{g}_i + 1 = \sum_{l\neq i,l(i)} |\mathbf{g}_i^H \mathbf{H}_{i,l}^{r,*}\mathbf{w}_l|^2 + 1. \end{aligned} \tag{13}$$

If S-T channel reciprocity held for all nodes i, j, then $\mathbf{H}_{i,l}^{r,*} = \mathbf{H}_{l,i}^H$, and the interference terms in (13) would indeed correspond to the interference at node l caused by transmitter i, $|\mathbf{w}_l^H \mathbf{H}_{l,i}\mathbf{g}_i|^2$, as in [3]. Unfortunately, S-T reciprocity only holds for node pairs $i, l(i)$ (Definition 2), hence eq. (13) is not part of a Total Interference Function [3].

Simulation results demonstrated slow convergence when $\mathbf{g}_i^n$ was set to the unconstrained IMMSE-TR solution (10). Furthermore, allowing $\mathbf{g}_i^n$ to be an arbitrary MN_S length complex vector increases transmitter complexity. The IMMSE-TR algorithm in Table 1 enforces a separable S-T solution $\mathbf{g}_i = \mathbf{s}_i \otimes \mathbf{a}_i$, where $\otimes$ is the Kronecker product. The temporal signature is $\mathbf{s}_i \in \mathbb{C}^{N_s}$, with spatial signature $\mathbf{a}_i \in \mathbb{C}^M$. The separable S-T waveform is then the solution to

$$\mathbf{s}_i, \mathbf{a}_i = \arg\min_{\mathbf{s}\in\mathbb{C}^{N_s}, \mathbf{a}\in\mathbb{C}^M} ||\mathbf{g}_i - \mathbf{s}_i \otimes \mathbf{a}_i||^2. \tag{14}$$

It is well known that the minimum norm solution (14) can be found using the SVD of the matrix (Matlab notation)

$$\mathbf{G}_i = [\mathbf{g}_i(1:M)\mathbf{g}_i(M+1:2M)\ldots\mathbf{g}_i((N_s-1)M+1:N_sM)]^T.$$

The vectors $\mathbf{s}_i, \mathbf{a}_i$ are then the left and right singular vectors of $\mathbf{G}_i$ corresponding to the largest singular value.

Results and Conclusions

Simulation results for IMMSE-TR are shown in Figures 1, 2 and 3 for $N = 10$ nodes with $M = 4$ elements and a processing gain of $PG = 7$, or $N_s = 14$ Nyquist samples per symbol. Three ray channels $N_p = 3$ were chosen at all nodes with a $\pi/2$ rad. angular spread. The temporal multipath duration was $2T_s$ sec. The power efficiency $0 \leq \eta \leq 1$ in Fig. 1 is the ratio $\eta^n = P_i^{su}/P_i^n$, where P_i^{su} is the power required to maintain SNR γ_0 in the absence of MAI. That is, P_i^{su} is

$$P_i^{su} = \frac{\gamma_0}{\lambda_{max}\left(\mathbf{H}_{l(i),i}^H\mathbf{H}_{l(i),i}\right)}, \tag{15}$$

where $\lambda_{max}(\mathbf{A})$ is the largest eigenvalue of matrix $\mathbf{A}$. Note that $\lambda_{max}\left(\mathbf{H}_{l(i),i}^H\mathbf{H}_{l(i),i}\right)$ is the maximum normalized SNR on link $i \rightarrow l(i)$ in the absence of MAI. Hence $\eta = 1$ when MAI is absent, since $\mathbf{g}_i^n$ is the maximum eigenvector of $\mathbf{H}_{l(i),i}^H\mathbf{H}_{l(i),i}$ from (9). The power efficiencies are then seen in Fig. 1 to satisfy $\eta \geq .8$ at all nodes.

The final temporal signatures $\mathbf{s}_i$ and beamformers $\mathbf{a}_i$ are shown in Figs. 2, 3 for this scenario. The temporal signatures are OFDM-like, but not pure sinusoids. As shown in [11], the optimal S-T waveforms in the absence of MAI are indeed separable, with temporal signatures given by sinusoids. However, the SNR-maximizing temporal waveforms for the Ad hoc network are not sinusoidal, even with the inclusion of a cyclic prefix, since the effective whitened channel matrix $\mathbf{R}_{l(i)}^{-1/2}\mathbf{H}_{l(i),i}$ is no longer block-Toeplitz. Nevertheless, the sinusoidal nature of the $\mathbf{s}_i$ suggests that a more structured solution for the S-T waveforms may exist.

The same scenario as in Figs. $1-3$ was considered with the processing gain increased to $PG = 15$. The resulting power efficiency is plotted in Fig. 4, which shows a slight increase in η over the $PG = 7$ case, due to the reduced temporal correlation achievable.

To summarize, a new iterative MMSE space-time waveform design algorithm was developed, which was shown to correspond to a noncooperative game. Quasi-synchronous transmission on the desired link was employed to minimize the code acquisition problem. The resulting S-T waveforms $\mathbf{g}_i$ maximize a similar utility function to that in the pure beamforming case [3], with utility increasing with normalized SNR, and decreasing with (approximate) interference to other nodes. The QS channel model alters the direct relationship between the interference tax and Total Interference Function in pure beamforming [3]. As a result, the optimum solution for the $\mathbf{g}_i$ via the Lagrangian does not have the generic IMMSE form, in contrast to beamforming-only [3]. Nevertheless, simulation results showed excellent power efficiencies in a scenario with more nodes than beamforming elements, due to the additional degrees of freedom afforded by the temporal signature.

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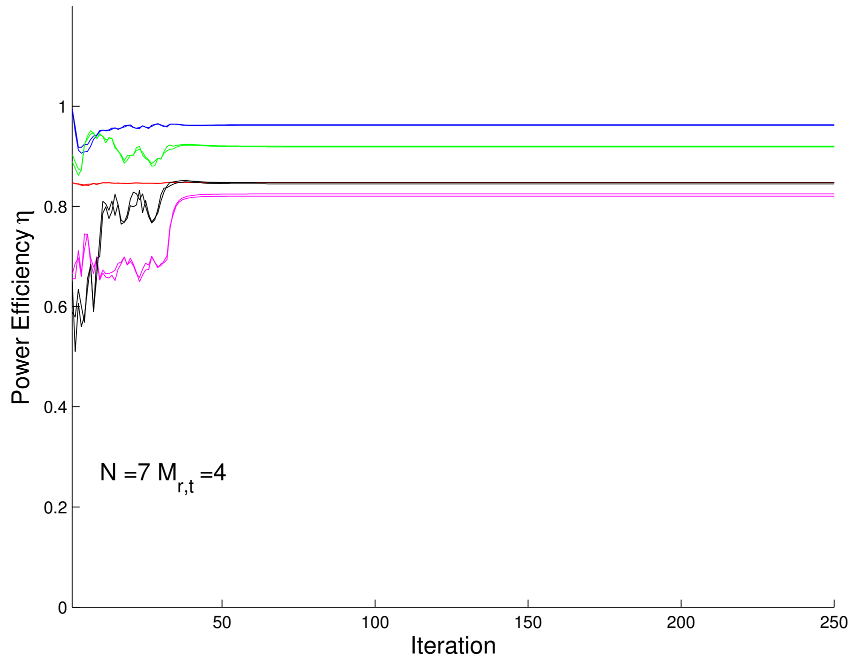

Figure 1: Power efficiency η for $M = 4, N_s = 14,\ N = 10$ nodes.

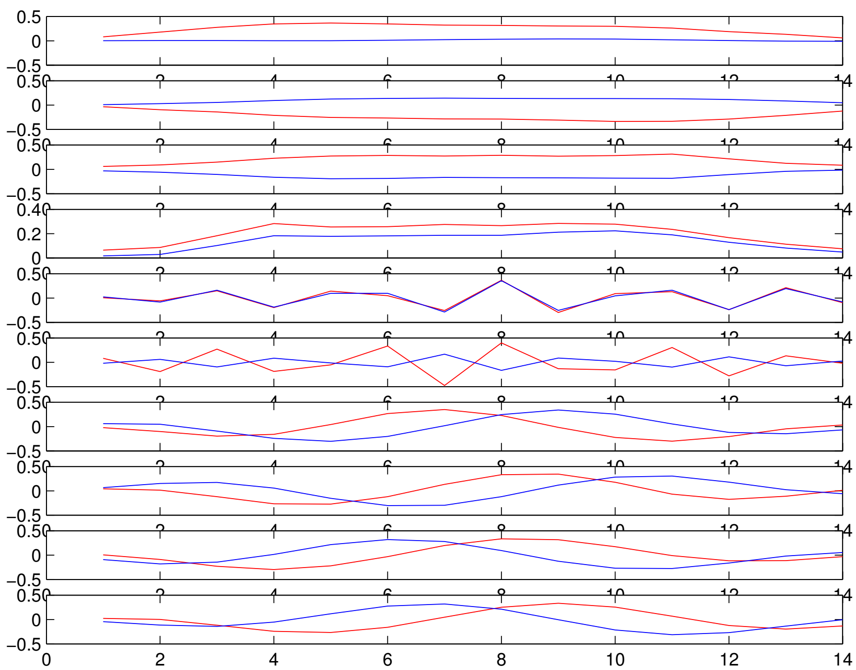

Figure 2: Temporal signatures for $M = 4, N_s = 14$.

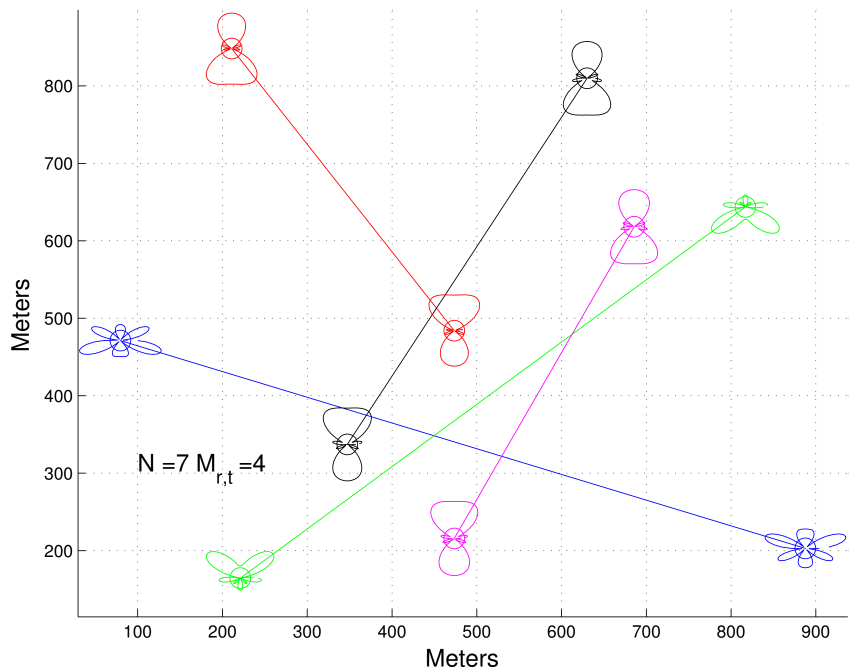

Figure 3: Spatial signatures for $M = 4, N_s = 14$.

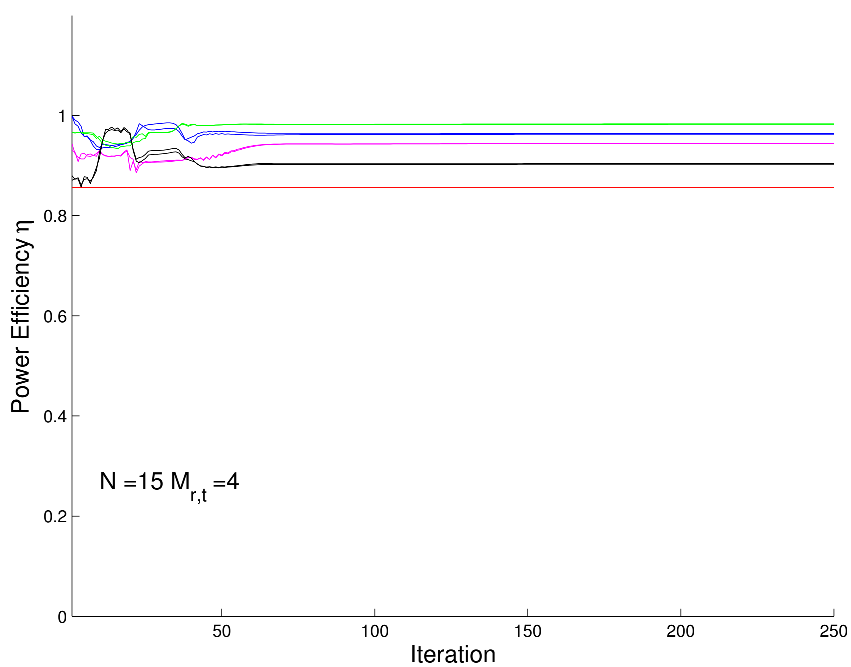

Figure 4: Power efficiency η for $M = 4, N_s = 30$, $N = 10$ nodes.