AN EXTENSION OF SOQPSK TO M-ARY SIGNALLING

Chris Bishop, PhD, PE
www . electromagnetics . biz

Mike Fahey, PhD, PE
Phase IV Systems, Inc.

ABSTRACT

Shaped Offset Quadrature Phase Shift Keying (SOQPSK) has the advantages of low sidelobes and high detection probability; however, its main lobe has a fixed width set by the number of constellation points. By slightly modifying the modulation scheme, the four constellation points of quadrature shift keying can be changed to M constellation points where M is a power of 2. After this change, the power spectral density (PSD) retains low sidelobes, and the desirable property of being able to detect the signal by integrating over two symbol periods is retained.

KEYWORDS

Power Spectral Density, Shaped Offset Quadrature Phase Shift Keying, Shaped Offset M-ary Phase Shift Keying, Markov Chain

INTRODUCTION

SOQPSK is a modulation scheme commonly used for satellite telemetry and other forms of communication [1] [2]. The original (MIL-STD) version of SOQPSK allows the phase to transition between constellation points in a linear fashion during a single bit period. Recently, more spectrally efficient versions of SOQPSK have been found [3] that dramatically lower the sidelobes of the PSD while keeping the main lobe width approximately the same. This improvement arises by allowing the phase to transition between constellation points in a non-linear fashion during multiple bit periods.

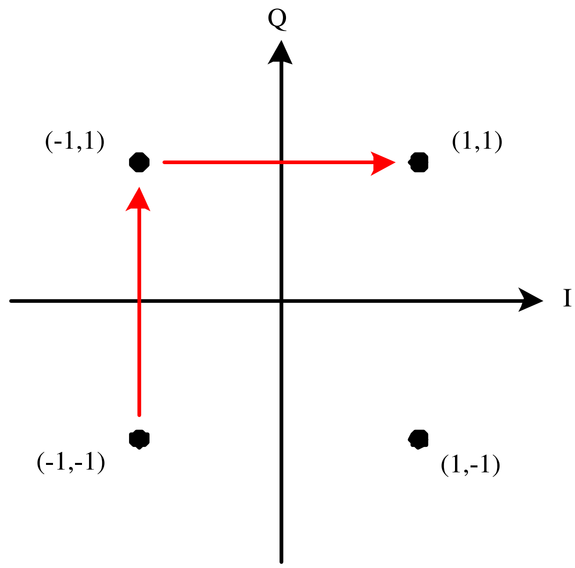

Fig. 1. The Constellation of SOQPSK

What has not been shown to date is an extension of the SOQPSK waveform to similar waveforms with more constellation points. Increasing the number of constellation points decreases the spectral width of the waveform at the expense of required power. This paper shows that such an extension is possible. The resulting waveforms share much in common with the SOQPSK waveform including the low sidelobes, the availability of variants with even lower sidelobes, and the ability to integrate for two symbol periods when detecting the transmitted signal.

THE SOMPSK MODULATION SCHEME

The chief advantage of SOQPSK concerns the ability to detect the transmitted modulation. In reference to the constellation diagram in Fig. 1, SOQPSK is constructed so that the phase remains in the left or right half-planes for two symbols in a row. Beginning in the alternate bit, it also remains in the upper or lower half-planes for two symbols in a row. Thus, the waveform might begin at the point $(-1,-1)$. It can then stay where it is or move up to the point $(-1,1)$. In either case it remains in the left half-plane for two bit periods. Assuming it moved to $(-1,1)$, it can next stay where it is or move to point $(1,1)$. In either case it remains in the upper half-plane for two bit periods.

The strength of this scheme is realized when one divides the data into I and Q channels. I corresponds to the left- and right-half planes, and Q corresponds to the upper and lower half-planes. Keeping I or Q constant for two subsequent bit periods means that the processor has twice as long to determine whether the bit is a -1 or a $+1$. The result is a very efficient detection scheme, one that is equivalent to boosting the power by about 3dB when compared to other modulation schemes.

To derive the SOMPSK waveform, we first place M equally-spaced constellation points on the unit circle. Next, we understand that every symbol will direct us to one of $\frac{M}{2}$ constellation points in

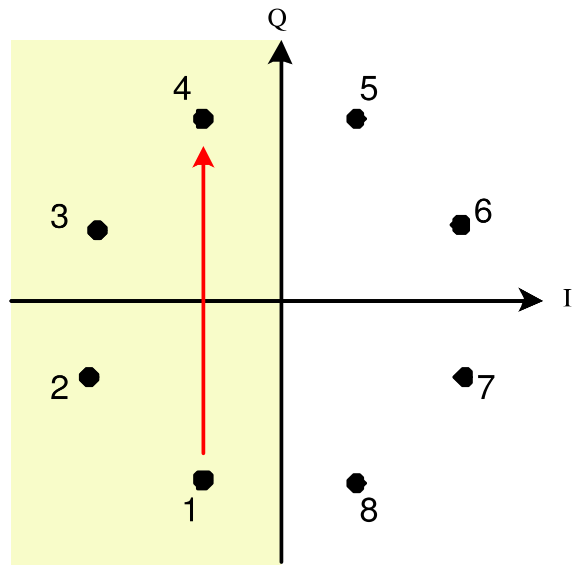

Fig. 2. The Constellation of SOMPSK with $M = 8$

such a way that we stay in the same half-plane for at least two symbols in a row.

One such scheme is illustrated by Fig. 2 for the case, $M = 8$. We assume that we began at the point marked 1. The next symbol directs us to move to any of the four points in the left-hand plane. Suppose we move to the point marked 4. The next symbol then directs us to any point in the upper half-plane. Continuing this approach, we fulfill the goal of staying in the same half-plane for two symbols in a row.

THE PSD OF SOMPSK

Using the Markov chain approach, the PSD of a modulated signal with constant bit period is given with minor modification as [4, Eq. 22]

$$G(f) = \frac{1}{t_0^2}\left|\sum_{i=1}^{N_s} p_{i,i}h_i\right|^2 \sum_{n=-\infty}^{\infty}\delta\left(f-\frac{n}{t_0}\right) + \frac{1}{t_0}(\mathbf{ph})^*\mathbf{h} + \frac{2}{t_0}\mathrm{Re}\left\{(\mathbf{ph})^*\overline{\mathbf{P}}\left(e^{-j\omega t_0}\right)\mathbf{h}\right\} \quad (1)$$

In this equation, t_0 is the bit period, $\mathbf{p}$ is a diagonal matrix whose non-zero elements are the stationary probabilities of being in the i^{th} state out of N_s possible states, $\mathbf{h}$ is a vector that contains the Fourier transform of the transmitted signal during each state, $\mathbf{h}^*$ is the conjugate transpose of $\mathbf{h}$, f is frequency, and ω is radian frequency. $\overline{\mathbf{P}}\left(e^{-j\omega t_0}\right)$ is a term that describes how the modulation scheme affects the frequency domain. This term can be written as

$$\overline{\mathbf{P}}\left(e^{-j\omega t_0}\right) = \sum_{k=1}^{\infty}(\mathbf{P}z)^k \quad (2)$$

where $\mathbf{P}$ is a matrix containing the probabilities of transition and

$$z = e^{-j\omega t_0} \quad (3)$$

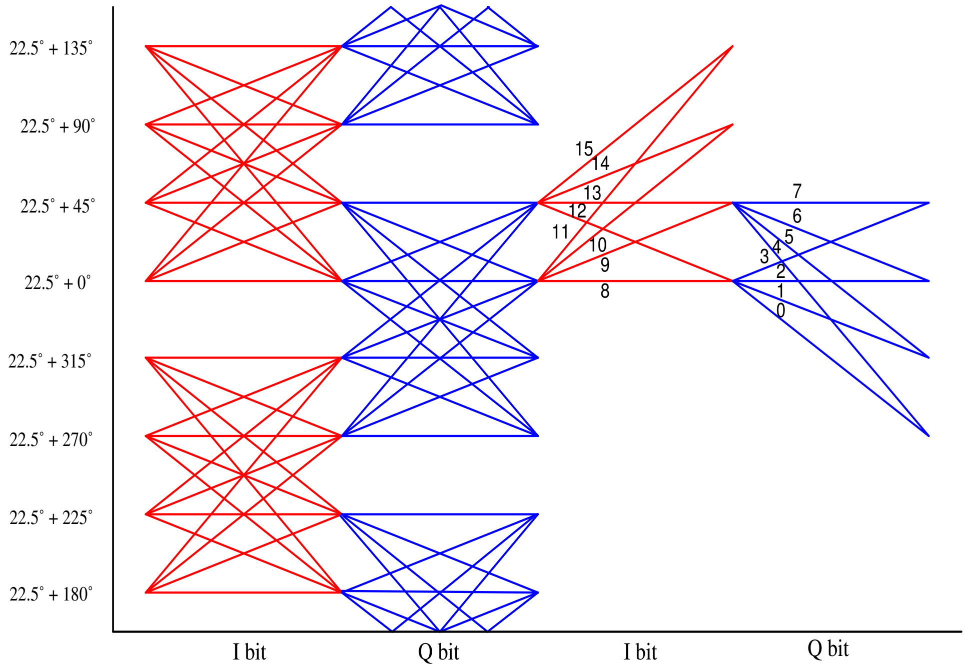

Fig. 3. The SOMPSK Trellis Diagram with $M = 8$ Showing the State Numbering

Assuming that $\omega\, t_0$ is not a multiple of 2π, (2) can be rewritten as

$$\overline{\mathbf{P}}\left(e^{-j\omega t_0}\right) = \left(\mathbf{I} - \mathbf{P}z\right)^{-1} \tag{4}$$

where $\mathbf{I}$ is the identity matrix.

(1) describes the power spectral density of an aperiodic Markov process. An aperiodic process has a transition matrix, $\mathbf{P}$, whose powers converge rather than repeat. A similar equation exists for periodic processes [5, Eq. 20]. Under certain conditions valid for all modulation schemes studied here, the equation for periodic processes reduces to that of aperiodic processes. These conditions relate to the waveforms appearing in pairs with opposite phases so that the spectral spikes and an additional term vanish.

Since we require vanishing spectral spikes, we ignore the first term of (1). The second term represents the part of the spectrum due to the transmitted signals, themselves, and the third term represents the part of the spectrum that is due in some part to the particular modulation scheme.

To number the states we refer to the phase trellis for $M = 8$ illustrated by Fig. 3. We order by grouping states that begin in the same quadrant; for example, all the states that begin between $0°$ inclusive and $90°$ non-inclusive. Next, we consider all the quadrature (Q) states first and the in-phase (I) states last. Within each of these groupings, we list the states in order from the least starting phase to the greatest starting phase. Finally, we list the states from the most negative phase trajectory to the most positive phase trajectory.

TABLE I
THE STATES OF SOQPSK ($M = 4$, $N = 1$)

State No.	$\frac{4}{\pi}\phi_s$	α
0	1	-1
1	1	0
2	1	0
3	1	1
4	3	-1
5	3	0
6	3	0
7	3	1
8	5	-1
9	5	0
10	5	0
11	5	1
12	7	-1
13	7	0
14	7	0
15	7	1

For $M = 4$ (SOQPSK) the transmitted waveform during each state has the form

$$h(\omega) = \int_0^{t_0} e^{j\,[\phi_s + \alpha(\Omega(t)+\phi_c)]} e^{-j\,\omega t}\,\mathrm{d}\,t \tag{5}$$

where t_0 is half of a symbol period. ϕ_s is the starting phase of each state; it corresponds to the locations of the constellation points. $\Omega(t)$ is a phase pulse that for MIL-STD SOQPSK is a linear ramp. ϕ_c is zero. α indicates the direction of the ramp: 1 means ramp up in phase, -1 means ramp down in phase, and 0 means hold the phase constant.

Between the constellation diagram and the phase trellis, we see that there are four states that begin at $\phi_s = \frac{\pi}{4}$. For the Q bits these correspond to $\alpha = -1$ and 0, and for the I bits they correspond to $\alpha = 0$ and 1. Thus, the order of the 16 states for MIL-STD SOQPSK is given by Table I. The table reveals that we do not really need to list the last 12 states, since they are just phase-shifted versions of the first four states.

The process just described is sufficient for half-symbol SOMPSK. By “half-symbol” we mean that the frequency pulses that define each transmitted waveform are non-zero only within a half-symbol period. (With $M = 4$ [SOQPSK], a half-symbol period is also a bit period.) Half-symbol SOQPSK, for example, corresponds to MIL-STD SOQPSK. The variants SOQPSK-A, SOQPSK-B, and others have waveforms that exist over multiple half-symbol (bit) periods. Let us define the number of half-symbols over which the waveform extends as N. To analyze SOMPSK with $N > 1$, we need to list all possible combinations of the overlapping of waveforms.

Using only the first four states of the $N = 1$ SOQPSK example just given, we can extend the

TABLE II
POSSIBLE PHASE TRAJECTORIES FOR SOMPSK WITH $M = 4$ AND $N = 2$

State No.	α_1	α_0
0	-1	-1
1	-1	0
2	0	0
3	0	1
4	0	-1
5	0	0
6	1	0
7	1	1

results to $N = 2$ SOQPSK by listing all possible combinations of phase trajectories as shown in Table II. In this case there are 32 unique states. α_1 represents the older symbol, and α_0 represents the newer symbol. Ignoring the initial phase of each state, State 0 can transition to itself or to State 1. A transition to State 1 occurs if the previous two waveforms were represented by $\alpha = -1, -1$ (meaning the system is currently in State 0) and the next waveform has $\alpha = 0$ (the α's shift to the left in the table, and the new 0 enters on the right). A similar extension for higher values of N is readily found.

When $N > 1$, α in (5) becomes a row vector that contains α_{N-1} through α_0, and $\Omega(t)$ becomes a column vector that contains the segments of the waveforms valid during the current bit periods. ϕ_c is a column vector that contains the amount accumulated by the older bits (that is, bits 1 through $N-1$). ϕ_s becomes a column vector of starting phases.

RESULTS

Using the developed equations, the PSD's of SOMPSK waveforms for various values of M and N were computed. Fig. 4 illustrates the case of $N = 1$ for various values of M. In this case the phase trajectory was assumed to be linear. Fig. 5 illustrates the PSD of 4-ary and 8-ary signalling with $N = 4$. A raised cosine phase trajectory, given by $n(t)$ and $w(t)$ below, is assumed. This is the same phase trajectory used for SOQPSK-A; however, SOQPSK-A has $N = 8$. The waveform is

$$\Omega(t) = \int_{-\infty}^{t} n(\tau) w(\tau)\, \mathrm{d}\,\tau \tag{6}$$

$$n(t) = \frac{A \cos\left(\pi\rho \frac{Bt}{2t_0}\right)}{1 - \left(2\rho \frac{Bt}{2t_0}\right)^2} \frac{\sin\left(\pi \frac{Bt}{2t_0}\right)}{\pi \frac{Bt}{2t_0}} \tag{7}$$

$$w(t) = \begin{cases} 1 & |\frac{t}{2t_0}| < T_1 \\ \frac{1}{2} + \frac{1}{2}\cos\left[\pi \frac{|\frac{t}{2t_0}| - T_1}{T_2}\right] & T_1 < |\frac{t}{2t_0}| < T_2 \\ 0 & |\frac{t}{2t_0}| > T_1 + T_2 \end{cases} \tag{8}$$

TABLE III

PARAMETERS OF SOMPSK

Parameter	$N = 4$	$N = 8$
ρ	1	1
B	1.35	1.35
T_1	0.7	1.4
T_2	0.3	0.6

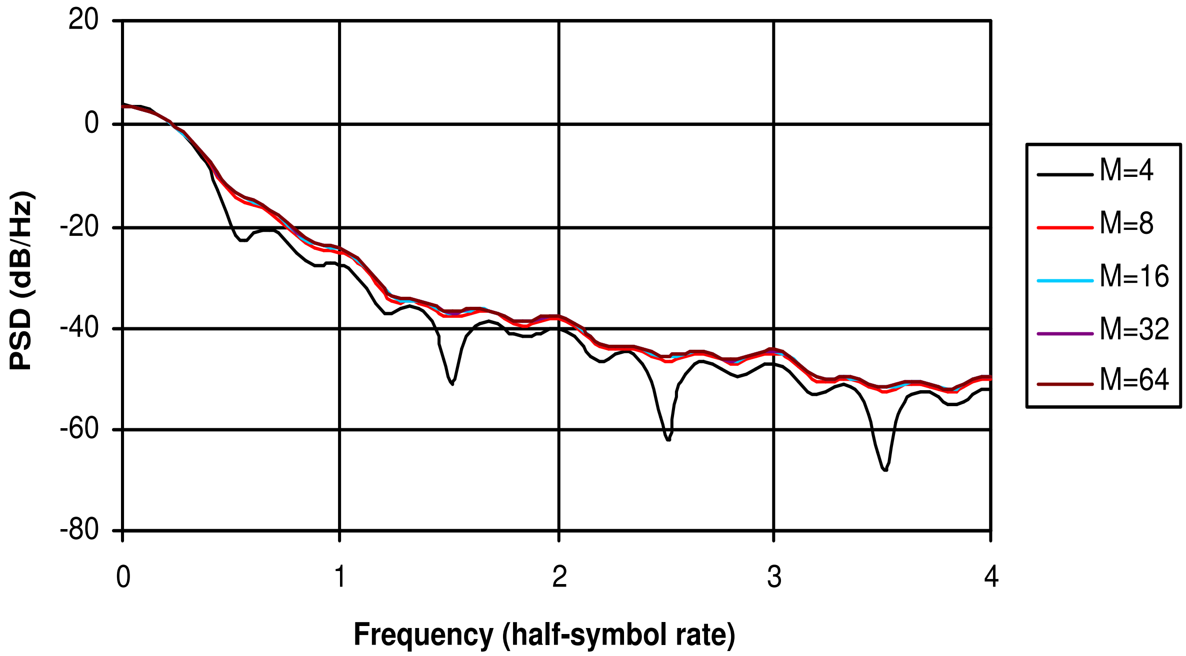

Fig. 4. The PSD of SOMPSK with Linear Phase Trajectories

(9)

The row vectors, $\overline{0}$, $\overline{1}$, $\overline{2}$, and $\overline{3}$, are filled with 0's, 1's, 2's, and 3's, respectively. The values of the parameters B, ρ, T_1, and T_2 are given by Table III. A is chosen so that the phase pulse yields $\frac{2\pi}{M}$ at $t = t_0$.

CONCLUSION

We have introduced a new set of modulation schemes that are an extension of SOQPSK. This set allows the user to decrease the spectral width of the transmitted signal at the expense of power and a slight rise in sidelobe level. We have further shown that variants exist that allow one to suppress the sidelobe levels in a manner analogous to the use of SOQPSK variants SOQPSK-A and SOQPSK-B.

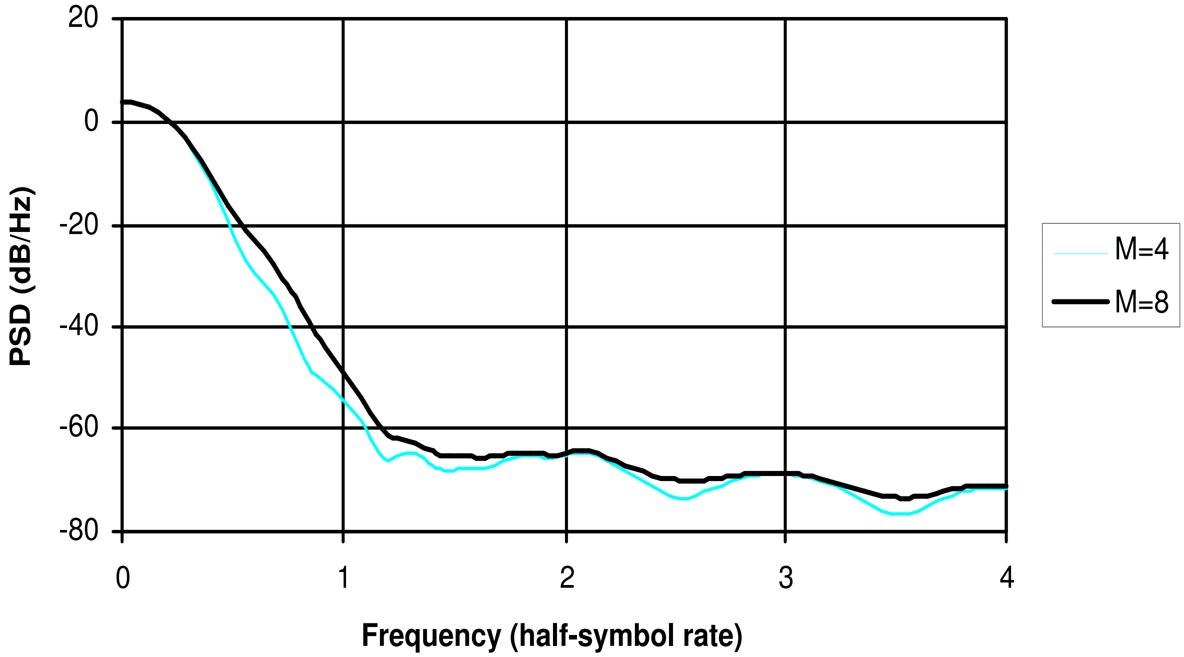

Fig. 5. The PSD of SOMPSK with $N = 4$ Raised Cosine Phase Trajectories

Because of the great numbers of states involved for modulation schemes such as SOQPSK-A and SOMPSK with $M = 32$ and higher, it was necessary to implement the computation of the PSD in a very efficient manner. Though not shown here, efficiency was gained by *directly computing* the required inverse matrix and matrix multiplications.

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