# NOVEL ANGLE OF ARRIVAL ALGORITHM FOR USE IN ACOUSTICAL POSITIONING SYSTEMS WITH NON UNIFORM RECEIVER ARRAYS

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## ABSTRACT

Traditional angle of arrival algorithms operate with uniform receiver arrays. Non-uniform arrays typically introduce significant elevation of computation complexity. This paper utilizes the double-integration method for the accurate estimation of the angle of arrival with non-uniform receiver arrays, while maintaining high computation efficiency. Because of the simplicity, the double-integration method is not significantly affected by the increase of the number of receivers or the non-uniform configuration. This approach allows us to perform high-speed high-accuracy estimation of the two-dimensional bearing angle without the constraints of structured receiver arrays, which is important to the realization of real-time tracking of mobile acoustic sources.

## KEY WORDS

Angle of Arrival (AOA), Non Uniform Arrays, Time Delay Estimation

## INTRODUCTION

In order to explore deeper and harder to reach portions of the worlds oceans it is often necessary to rely on Autonomous Underwater Vehicles(AUV) to perform various tasks that are too dangerous or outright impossible for a human to perform. The missions can vary from performing operations on deepwater well-heads to, ocean floor mapping, or mine detection and removal. Each mission type relies on the AUV accurately knowing its position relative to a known reference point, for example, in the case of the oil well the AUV must know where it is relative to the oil well. Acoustical positioning systems are the most common way for AUV's to determine their geospatial location [Mil83]. The key software component of the acoustical positioning systems is the angle of arrival (AOA) estimation technique. Traditional angle of arrival techniques fall into two categories: time delay estimation [Car81] and subspace techniques [Sch86]. Both angle of arrival techniques work best when the receiver array of the acoustical positioning system is of a uniform design. Requiring the receivers to be placed in an orderly fashion may not be feasible given the physical constraints of the vessel carrying the acoustical positioning system. Hence work has been done on expanding the existing angle of arrival techniques to work on non-uniform arrays. While these techniques can handle array geometries outside of the traditional uniform rectangular and uniform circular arrays, they often still place some constraints on the placement of the receivers or incur heavy computational costs due to the non-uniform nature of the array. This paper will show that the double integration method first developed for a particular homing and docking system can be modified to work on non-uniform arrays [Utl06]. In addition the work will show that as the number of receivers increase the double integration method incurs a much smaller, versus traditional methods, computational cost penalty.

## BACKGROUND

Acoustical positioning system relies on a transponder in a known position transmitting a known signal. When the signal from the transponder reaches the receiver array it is possible to use the signal on each receiver to estimate a direction of arrival. If the transponder is far from the receiver array, then the incident wave can be assumed to be planar. It is assumed that the receiver array lies in the two dimensional plane defined by the x and y axes. The transponder lies somewhere in three-dimensional space and is assumed to be much further away from the center of the array than the largest difference between any two receivers. The far field approximation produces the assumption that the wave incident on the receiver array is planar and thus all of the received signals can be modeled as:

$$x_i[n] = s[n - D_i] + v_i[n] \quad (1)$$

Where $x_i$ is the $i^{th}$ receiver and $D_i$ is the $i^{th}$ sample delay determined by the location of the receiver and the location of the transmitter. It is common to define the sample delay for a particular signal as referenced to the first receiver. For this argument the delay will be relative to an imaginary receiver placed at the origin. While this notation may seem a bit unnecessary it will help simplify the derivation for the random array later in the paper. Since each delay is referenced to the imaginary receiver at the origin it is now possible to calculate each sample delay by first deriving the time delay and then dividing by the sampling rate of the analog to digital converter. The time delay is the difference in distance from the origin to the transmitter and the $i^{th}$ receiver and the transmitter divided by the propagation speed. The difference in travel distance can be shown to be the inner product of vector to the receiver and transmitter vector, resulting in:

$$\tau_i = \frac{1}{c}\left(x_i \cdot x_s + y_i \cdot y_s\right) \quad (2)$$

Where c is the propagation speed of sound in the medium, for this paper water. Dividing this equation by the sample rate, $f_s$, produces the sample delay shown in equation (3).

$$D_i = \frac{1}{c \cdot f_s}\left(x_i \cdot x_s + y_i \cdot y_s\right) \tag{3}$$

Knowing all of the sample delays and receiver positions allows for the calculation of the angle of arrival. Placing the transducers can have a big effect on the type of angle of arrival estimation that can be used and corresponding computationally efficiency.

The most common and simplest to use geometry is the uniform linear array. We are most interested in 2-D angle estimation, azimuth and elevation, and as such a uniform linear array would consist of one ULA placed along the x axis and another placed along the y axis. A logical extension of the ULA is to place the receivers on a uniform rectangular spacing, this array is the uniform rectangular array, the final uniform array is the circular array, which has the M receivers evenly spaced in angle at a given radius lying in the xy plane.

All of these arrays require the receivers be placed in very particular locations and this can be in direct conflict with shape of the vessel upon which the array is being placed. Recent attempts have been made to do angle of arrival on sparse rectangular arrays where the spacing is still uniform but certain receivers have been removed and even more recently there has been much work done on interpolating truly random arrays to uniform virtual arrays that can then be used to estimate the angle of arrival [Ger10]. The double integration method presented in the next section can estimate the angle of arrival on a truly random array without the need to interpolate to uniform virtual array.

## DOUBLE INTEGRATION METHOD

The double integration method was first introduced in [Doo04, Utl06] to solve a very particular homing and docking system and as such the original formulation was for a two-element array. The algorithm will be re-derived here to take into account the additional receivers that exist in the acoustical positioning systems mentioned in the previous section. The algorithm is broken down into four key pieces: channel equalization, channel weighting, double integration, and final value to angle conversion. These four steps will be covered in this section.

The first step of the double integration method is channel normalization. Channel normalization seeks to correct amplitude differences between the various M channels. These amplitude variations can be cause by channel fading or variations in the amplifiers of each receiver but irrespective of the cause a mismatch in channel amplitudes can cause significant errors in the double integration method and hence must be corrected. To accomplish the normalization each received signal is divided by a correction term giving:

$$\tilde{x}_i[n] = \frac{s[n - D_i] + v_i[n]}{c_i} \tag{4}$$

Where the correction term is given by equation (5).

$$c_i = \sum_{n=1}^{N} x_i[n] \tag{5}$$

The next step in this version of the double integration method involves weighting each channel and then summing all the channels. In the original work of the double integration method there were only two receivers and the weights were positive and minus one but a more detailed weighting algorithm must be used in the case of M receivers. While many different types of weights could be used, this paper will use a weighting based on the location of each receiver. In order to calculate the weight each receiver will receive it is first necessary to determine the geometric center of the array. The center of the array is defined by two values shown in equations (6) and (7).

$$m_x = \frac{1}{N}\sum_{i=1}^{N} x_i \tag{6}$$

$$m_y = \frac{1}{N}\sum_{i=1}^{N} y_i \tag{7}$$

With the centroid of the receiver array calculated, the receiver positions will now be represented by their centered positions shown below in vector form:

$$\hat{r}_i = \left\langle x_i - m_x \quad y_i - m_y \quad 0 \right\rangle \tag{8}$$

With these center receiver vectors it is now possible to define the weights as:

$$w_i = \left\|\hat{r}_i\right\| \cdot e^{j*\gamma_i} \tag{9}$$

Where gamma is defined as the angle of each receiver vector, shown below as:

$$\gamma_i = \angle \hat{r}_i \tag{10}$$

With the weights defined for each channel it is necessary to multiply each corrected channel vector by its respective weight. All of the weighted channels are now summed together to produce a single vector that will be processed by the double integration method to produce an estimate of the azimuth and elevation angles. The vector to be used by the double integration method is shown below:

$$l[n] = \sum_{i=1}^{M} w_i \cdot \tilde{x}_i[n] \tag{11}$$

The vector shown above is then integrated a single time, shown next:

$$l_1[n] = \sum_{k=1}^{n} l[k] \tag{12}$$

The result from (12) is integrated a second time and this is represented in (13).

$$l_2[n] = \sum_{k=1}^{M} l_1[k] \tag{13}$$

An analysis of the double integration method shows that assuming that the weighting functions are chosen wisely the final value is equal to:

$$l_2 = \sum_{i=1}^{N} w_i \cdot \boldsymbol{\tau}_i \tag{14}$$

Now taking the final value of the double integration and plugging in the weights defined above it is possible to show that the final value is equal to:

$$l_2 = \sum_{i=1}^{N} \|\hat{r}_i\| \cdot e^{j*\gamma_i} \cdot \boldsymbol{\tau}_i \tag{15}$$

Simplifying the above equation gives:

$$\begin{aligned} l_2 = &\frac{1}{c}\sum_{i=1}^{N} \|\hat{r}_i\|^2 \cdot \cos^2(\gamma_i) \cdot x_s + \frac{1}{c}\sum_{i=1}^{N} \|\hat{r}_i\|^2 \cdot \cos(\gamma_i) \cdot \sin(\gamma_i) \cdot y_s \\ &+ \frac{j}{c}\sum_{i=1}^{N} \|\hat{r}_i\|^2 \cdot \cos(\gamma_i) \cdot \sin(\gamma_i) \cdot x_s + \frac{j}{c}\sum_{i=1}^{N} \|\hat{r}_i\|^2 \cdot \sin^2(\gamma_i) \cdot y_s \end{aligned} \tag{16}$$

Now define the following known constants:

$$A = \sum_{i=1}^{N} \|\hat{r}_i\|^2 \cdot \cos^2(\gamma_i) \tag{17}$$

$$B = \sum_{i=1}^{N} \|\hat{r}_i\|^2 \cdot \cos(\gamma_i) \cdot \sin(\gamma_i) \tag{18}$$

$$C = \sum_{i=1}^{N} \|\hat{r}_i\|^2 \cdot \sin^2(\gamma_i) \tag{19}$$

Which allows the final value to be represented by:

$$l_2 = \frac{1}{c} \cdot \left(A \cdot x_s + B \cdot y_s + j \cdot B \cdot x_s + j \cdot C \cdot y_s\right) \tag{20}$$

It is now possible to break the final value shown in equation (20) into its real and imaginary components. The real and imaginary equations, shown in equations (21 & 22), represent a two equation two unknown linear system that can be solved.

$$real(l_2) = \frac{1}{c} \cdot (A \cdot x_s + B \cdot y_s) \quad (21)$$

$$imag(l_2) = \frac{1}{c} \cdot (B \cdot x_s + C \cdot y_s) \quad (22)$$

The variables A, B and C are known as long as the geometry of the receiver is known. Thus $x_s$ and $y_s$ are the only two unknowns and there are two equations allowing for a solution of the two unknowns to be found. The equations for the two unknowns are the following:

$$x_s = \frac{imag(l_2) \cdot B - real(l_2) \cdot C}{B^2 - A \cdot C} \quad (23)$$

$$y_s = \frac{real(l_2) \cdot B + imag(l_2) \cdot A}{B^2 - A \cdot C} \quad (24)$$

With the x and y components of the source vector estimated it is a simple matter of trigonometric identities to estimate the azimuth and elevation angles of the source. The equations for transformation from vector to angle are shown below:

$$\tilde{\theta} = \arcsin\left(\frac{y_s}{f_s} \cdot c\right) \quad (25)$$

$$\tilde{\phi} = \arcsin\left(\frac{x_s}{f_s} \cdot c \cdot \cos(\tilde{\theta})\right) \quad (26)$$

## SIMULATED RESULTS

There are many different parameters that can affect the angle estimation effectiveness of the double integration method. Signal-to-noise ratio (SNR), number of receivers, array geometry, and the total span of a given array can have an effect on the overall performance of the system. This section will use computer simulations to look at the performance of the double integration method against the various system parameters defined above.

All estimators are judged based on the bias and variance of the estimated value. For these simulations, variance will be the primary metric used to gauge the effectiveness of the estimator under different system setups. The simulations will make certain basic assumptions about the virtual system. The system will be assumed to have a 2 MHz sample rate while transmitting and receiving a 50 kHz gated sine wave. This transmitted signal will propagate from a distance significantly further than the span of the arrays through water with a sound speed of 1500 meters per second. These overall system requirements will be used on a non uniform array (NUA) where the span of the random array is defined as the standard deviation of the receiver positions, assuming that their position is defined as a normally distributed random variable with zero mean. Figure 1, below, shows the placement of receivers for the 9-element non-uniform array.

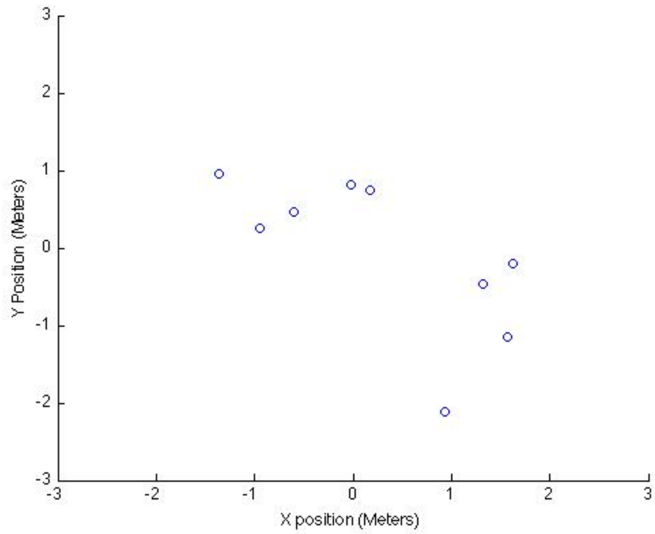


Fig. 1 – Showing example random array, with each receiver point denoted by a circle.

The first simulation, figure 2, shows the standard deviation, in degrees, versus the signal to noise ratio. The simulation assumes a 36 element array whose position is determined by a Gaussian distribution with a mean of zero and a standard deviation of 1 meter.

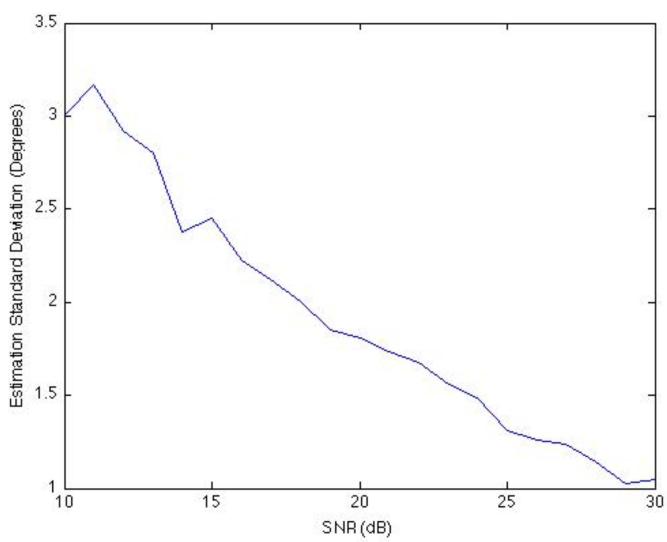


Fig. 2 – Standard deviation of angle estimation versus signal to noise ratio with a 9 element array.

The second simulation is a contour plot (Fig. 3) showing how the number of receivers and the standard deviation of the receiver locations affect the estimation accuracy. The accuracy is depicted by the contour lines in the plot, which represent standard deviation of estimated angle in degrees.

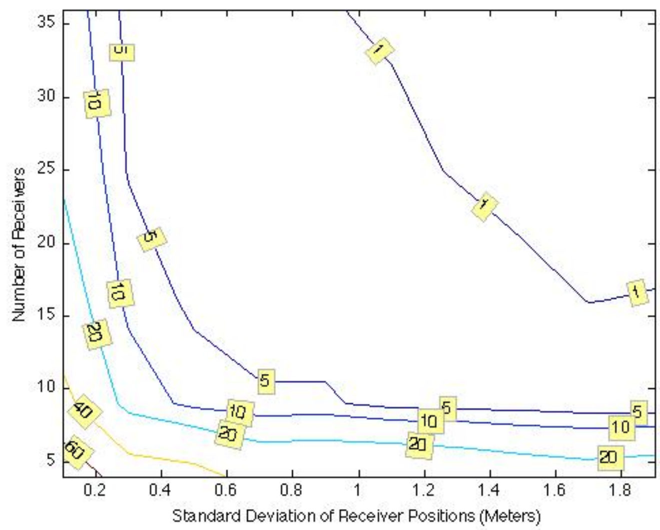


Fig. 3 – Contour plot showing the standard deviation of the angle estimation, in degrees, versus number of receivers and standard deviation of the randomly placed receivers.

Lastly, figure 4 shows the computation time required to determine the angle of 250 simulation runs. The figure clearly shows that there is only a linear increase in computational complexity as the number of receivers increases.

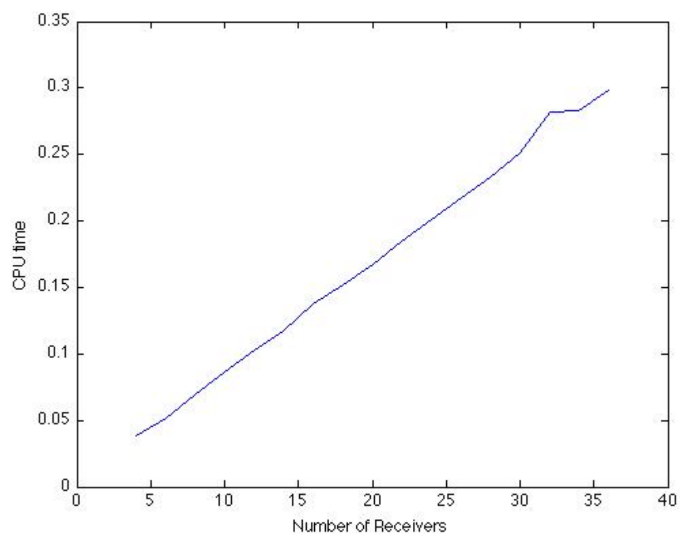


Fig. 4 – Processor time versus number of receivers, processor time is given in a machine specific time and hence the units are not stated.

The linear increase in computation complexity is a favorable alternative to even subspace methods which have a computational complexity on the order of N^3 [Wu03]. The N order of the double integration method means that the double integration method can accurately estimate the azimuth and elevation angles while suffering a smaller computation increase for larger array sizes.

## CONCLUSION

This paper has extended the double integration method from the two-element system it was originally designed for to a M element non uniform array. The results in the paper clearly show that the system can accurately estimate the direction of arrival of an active source. More importantly it shows that the double integration does not suffer from an exponential increase in computational cost as the number of elements increase. The double integration method has a clear computational advantage over existing non-uniform angle of arrival estimators and hence could be used in situations where computational resources are limited. Future work will look at a closed form solution of the estimation error so that systems can be designed with a particular performance criterion.

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