CODED SOQPSK-TG USING THE SOFT OUTPUT VITERBI ALGORITHM

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ABSTRACT

In this paper we present a system-level description of a serially concatenated convolutional coding scheme for *shaped offset quadrature phase shift keying*, telemetry group (SOQPSK-TG). Our paper describes the operation of various system modules. In addition, implementation details and references for each module in the system are provided. The modified *Soft Output Viterbi Algorithm* (SOVA) is employed for decoding *inner* and *outer* convolutional codes. The modified SOVA possess strong performance and low-complexity cost. The comparison of the modified (SOVA) and *Max-Log*-maximum *a posteriori* (MAP) decoding algorithm is presented. The SOVA after a simple modification displays the same performance as Max-Log-MAP algorithm, which is demonstrated by the simulation results. The advantage of the simple implementation of the modified SOVA makes it superior to Max-Log-MAP for our purposes.

INTRODUCTION

The SOVA is a very practical decoding method due to its simple implementation. When combined with *serially concatenated convolutional codes* it becomes even a more powerful tool. There are a number of modules and techniques that can enhance the performance of the iterative Viterbi decoder. In this paper, we present the description of the encoder and decoder models for *shaped offset quadrature phase shift keying*, telemetry group (SOQPSK-TG).

We begin by explaining the purpose of various blocks that are used in our models. Mainly we focus on the modified version of *Soft Output Viterbi Algorithm* (SOVA) which is employed in our decoder design and suggested in [1]. With minor additions, the original SOVA can display the same performance as the *Max-Log*-maximum *a posteriori* (MAP) algorithm. This obviously makes it a much more attractive choice for the iterative decoding algorithm due to its simpler implementation. The SOVA requires only one *forward traversal* (recursion) through the trellis where Max-Log-MAP algorithm, on the other hand, needs both forward and backward recursions [2].

The simulation results that we obtained, also suggest that the performance of the modified SOVA is the same as of the Max-Log-MAP algorithm.

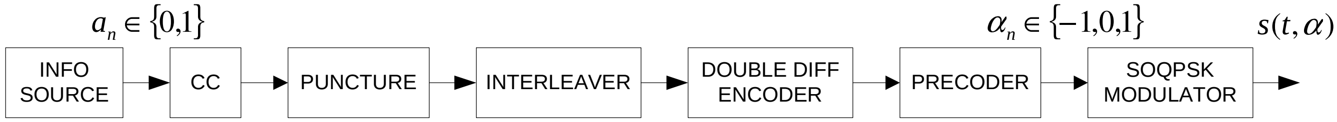

Figure 1: Block diagram of the encoder model for SOQPSK.

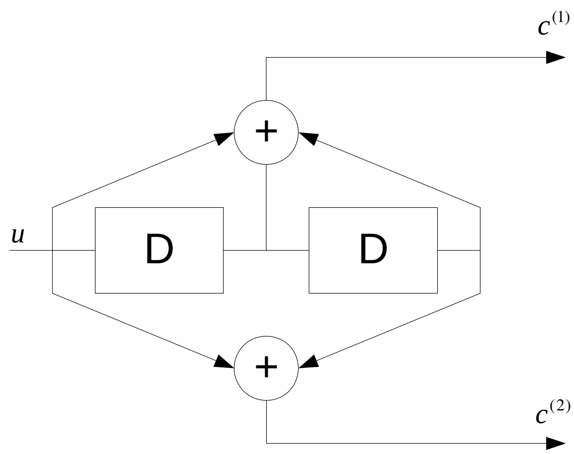

Figure 2: Block diagram of the rate-1/2 systematic convolutional encoder.

ENCODER DESCRIPTION

Figure 1 depicts the block diagram of the SOQPSK model of the encoder. First, information sequence u is encoded by using a *systematic convolutional code*. In systematic code information sequence emerges as a part of the encoded sequence [3]. In our model, we used a (5,7) encoder with a rate-1/2 code and a constraint length ν=3, illustrated in Figure 2. The transform domain generator matrix for this code is described by

$$G(D) = \left|1 + D^2 \quad 1 + D + D^2\right|.$$

The resultant convolutional code is realized with feedback registers. The convolutional encoder outputs two coded bits for every input information bit.

High rate convolutional codes are essential in systems with high data rate. In order to construct a higher code rate convolutional encoder, a *puncturing* technique is applied to the coded sequence. Punctured convolutional codes reduce the complexity of the decoding Viterbi algorithm. As explained in [3], we achieve rate-2/3 code by perforating the output of the initial rate-1/2 code, according to the *puncturing matrix*

$$P = \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}.$$

This technique allows generating a high-rate punctured convolutional code from a lower rate-1/n code, by simply deleting bits with respect to the puncturing matrix. Thus, the complexity of the resultant rate-2/3 convolutional encoder is greatly simplified.

Next, the coded sequence is passed through an *S-random interleaver*, which is denoted by Π in the system block diagram depicted in Figure 1. The interleaver rearranges the bits to minimize the effect of

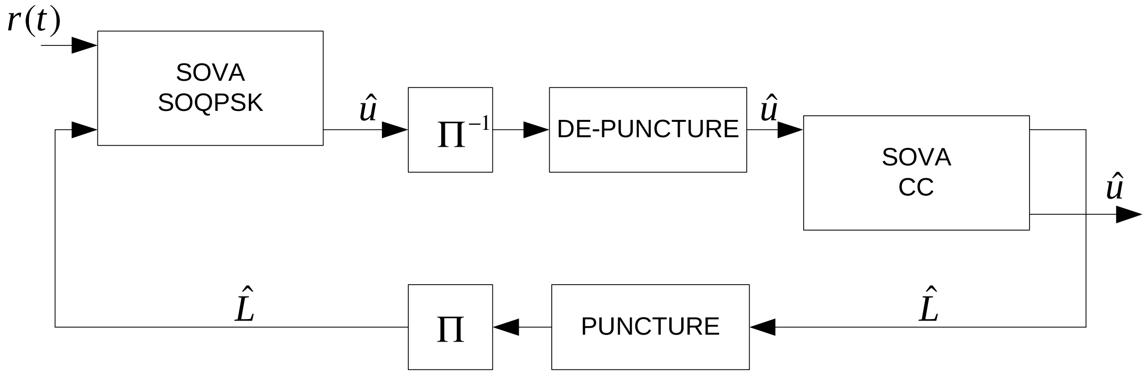

Figure 3: Block diagram of the decoder model for SOQPSK.

the burst errors on the code words. The S-random interleaver implemented in our design ensures that the distance between the nearest neighboring code words is $S = 55$.

The interleaved bits enter the *double differential encoder*. Differential encoding greatly improves the performance of serially concatenated coding systems with iterative detection (such as SOVA). Differentially encoded SOQPSK serves the purpose of the *inner code* in our coding scheme. As shown in [4], double differential encoding resolves the 4-phase ambiguity which is a common problem of carrier tracking loops. It also enhances the interleaver gains [4]. The resulting double differentially encoded sequence contains two subsequences

$$\begin{aligned} I_k &= e_k \oplus I_{k-2} \\ Q_{k+1} &= o_{k+1} \oplus Q_{k-1}. \end{aligned} \tag{1}$$

The differential encoder forms two subsequences, I_k and Q_{k+1}, that correspond to even and odd indexed bits. The last stop, before the coded sequence enters the SOQPSK modulator and leaves the encoder, is the *precoder*. In the precoder, the differentially encoded antipodal bit sequence is converted to its ternary representation. As stated in [4], the two sequences have the following relation

$$\alpha_n = \frac{1}{2}(-1)^{n+1}\hat{u}_{n-1}(\hat{u}_n - \hat{u}_{n-2}). \tag{2}$$

DECODER DESCRIPTION

The decoder receives the complex baseband SOQPSK signal in the form

$$r(t) = s(t-\tau)e^{j\phi(t)} + w(t)$$

where $w(t)$ is additive white Gaussian noise and $\phi(t)$ is a phase shift and τ is a timing offset. This paper does not consider the timing and phase synchronization which is discussed in details in [5]. The decoder model employs SOVA based iterative decoders. Figure 3 depicts the structure of the decoder. The motivation for using the modification of the soft output variation of the Viterbi Algorithm is straightforward. The original Viterbi algorithm only delivers the hard decisions (binary decisions) and does not yield any reliability information associated with the decisions. In the case with serially concatenated codes, when outer and inner codes are present, the Viterbi algorithm can be modified to produce *soft decisions* as well as to provide *reliability values* [6]. The first SOVA decoder that operates on the inner code produces soft decisions sequence or *a posteriori* information for the second SOVA decoder. The soft decisions are then de-interleaved and de-punctured. The SOVA decoder that processes the outer convolutional code, outputs the soft decisions. In addition, it feeds *a priori* reliability information to the first SOQPSK SOVA decoder. *A priori* information is punctured and permuted by the interleaver before entering the SOQPSK SOVA decoder. Thus, by using two SOVA decoders in the receiver chain to operate on both inner and outer codes, the overall SNR can be improved.

MODIFIED SOVA

When compared to the Max-Log-MAP algorithm, the SOVA has a clear advantage due to its low complexity. Max-Log-MAP algorithm is based on log-likelihood ratios. It computes the reliability values according to the probability rules. For that, it requires forward and backward recursions. On the contrary, the SOVA performs an elaborate comparison and calculates the reliability values by taking the difference between the metrics of the competing paths which requires only a forward recursion. In addition, the SOVA can be further modified, to yield the same performance as the Max-Log-MAP algorithm, while preserving its superior complexity.

Let's consider a binary trellis diagram illustrated in Figure 1 of [1]. Every node of this trellis has two entering branches and each state transition represents one information bit u_i. Since our model realizes antipodal convolutional coding, every two branches merging in one state carry distinct information bits u_i. The SOVA performs only the forward recursion through the trellis. As in the basic VA, a decoding window with a size δ is used. It is shown in [6] that while traversing the trellis, the SOVA computes and stores the cumulative metrics $\mathbf{\Gamma}(s_k)$ and $(\delta+1)$ of the most recent decisions $\hat{\boldsymbol{u}} = \{\hat{u}_{k-\delta}(s_k), \cdots, \hat{u}_k(s_k)\}$. Every decision $\hat{\boldsymbol{u}}(s_k)$ has a corresponding soft reliability value $\hat{\boldsymbol{L}}(s_k) = \{\hat{L}_{k-\delta}(s_k), \cdots, \hat{L}_k(s_k)\}$.

For each trellis node s_{k+1},the algorithm calculates the cumulative metrics $\mathbf{\Gamma}(s_k^1, s_{k+1})$ and $\mathbf{\Gamma}(s_k^2, s_{k+1})$ for the two most likely paths that originate from states s_k^1 and s_k^2 and merge in state s_{k+1}. In the same fashion as in the classical Viterbi, the path with the minimal metrics is selected according to

$$\Gamma(s_{k+1}) = \min_{i\in\{1,2\}} \{\Gamma(s_k^i, s_{k+1})\}. \tag{3}$$

The soft decision $\hat{u}_{k+1}(s_{k+1})$ is then updated in the surviving $\hat{\boldsymbol{u}}_{k+1}(s_{k+1})$ for the corresponding state s_{k+1}.

In the next step, the *reliability difference* Δ for each state s_{k+1} is obtained. This is accomplished by simply determining by what margin the accumulated metrics of the surviving path is greater than the accumulated metrics of the losing path. The reliability difference is acquired after some calculation

$$\Delta = \max_{i\in\{1,2\}} \{\Gamma(s_k^i, s_{k+1})\} - \min_{i\in\{1,2\}} \{\Gamma(s_k^i, s_{k+1})\}. \tag{4}$$

The SOVA formulated in [6] only considers the case when $\hat{u}_j^1 \neq \hat{u}_j^2$ or in other words, the two merging paths have conflicting decisions. In such case, the reliability measure is updated with

$$\hat{L}_j(s_{k+1}) = \min\{\Delta, \hat{L}_j^1\}. \tag{5}$$

The main difference between the modified SOVA and the original SOVA is the fact that modified SOVA considers the case when $\hat{u}_j^1 = \hat{u}_j^2$ for a certain $j \in \{k-\delta+1, \cdots, k\}$. In this case, we take into account a previously disregarded path-n that is merged with a losing path at decoding step $k+1$, since it now satisfies the condition $\hat{u}_j^n \neq \hat{u}_j^1$. For more in-depth explanation as well as visual demonstration of

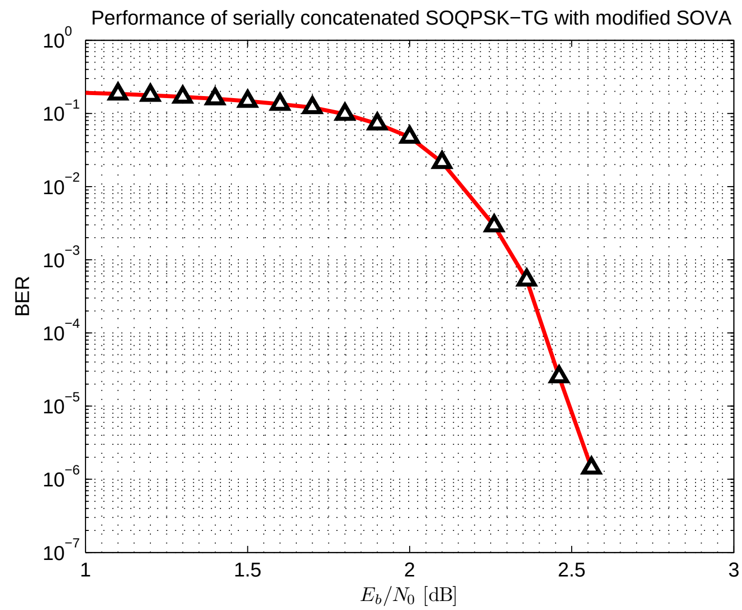

Figure 4: Performance of serially concatenated SOQPSK-TG with modified SOVA.

the trellis, please reffer to [1]. Therefore, for this particular case, the reliability measure for state s_{k+1} is updated with

$$\hat{L}_j(s_{k+1}) = \min\{\Delta + \hat{L}_j^2, \hat{L}_j^1\}. \tag{6}$$

This modification proposed in [1] can be easily implemented with the traditional SOVA with no additional cost. In addition, it has been proven in [1] that this simple modification enables SOVA to achieve the same performance as the Max-Log-MAP decoding algorithm.

CONCLUSIONS

We have provided the description of our proposed serially concatenated coding scheme. We have demonstrated various modules that are used in the encoder and decoder. We have stated the advantages of the modified SOVA over the Max-Log-MAP algorithm. Figure 4 illustrates the simulated bit error rate (BER) results for our SOVA based decode. The obtained BER curve resembles the Max-Log-MAP curve under similar conditions (curve is not shown).

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