PERFORMANCE ANALYSIS OF REENTRY TDOA POSITIONING SYSTEM

Xie Nan   Zhang Futang

Institute of Electronic Engineering, China Academy of Engineering Physics
P.O.BOX 919-517, Mianyang, Sichuan, P.R.China
Zip Code: 621900 Tel: +86 816 2487524 Fax: +86 816 2487594
Email: xienanmail@hotmail.com

ABSTRACT

In reentry measurement, we need to measure the track of reentry vehicle. For the speed of target is very high, it is difficult for traditional positioning system to capture and track the target. TDOA (time difference of arrival) positioning system is used in reentry measurement. The position of target is calculated by using time difference of arrival. This paper introduces the principle of reentry TDOA positioning system. The key technology is synchronization of ground stations' clocks. The implementation of clock synchronization using low-cost commercial GPS receiver is presented. The sources of errors in the existing telemetry system, corresponding precision and experimental results are presented. Some methods, which are used to improve the precision, are proposed at the end of this paper.

KEY WORDS

Reentry measurement, TDOA positioning, Telemetry, GPS

INTRODUCTION

In the reentry measurement, one of our missions is to measure the track and the point of fall of the reentry vehicle. It can be named reentry-positioning technology. In general, radar can be used to locate the track and the point of fall of aircraft. It locates the position of aircraft by measuring the azimuth, the elevation and the distance of target. The optical measurement is also used sometimes. The precision and accuracy of these methods is satisfying. In the reentry-positioning measurement, the reentry vehicle is different from any other aircrafts. During reentry, the speed of target is very high and the altitude is low. Above-mentioned systems can't capture the target before it fell to the ground. These systems are not suitable for the reentry vehicle track measurement.

Another system, which is called 3R-positioning system, can be used in the track measurement of reentry vehicle. It needs at least three ground stations A, B, C and locates the position of vehicle by measuring the distance between the vehicle and each of the respective stations. The stations sent query signals to reentry vehicle and received the responses. The time of propagation from each of the respective stations to the vehicle, i.e., the time to travel the distances, r_A, r_B and r_C, are computed. The distance r is given by $r = \tau \cdot c$, where c is the speed of light. Then the position can be located. This system works in active mode. The result of test is not satisfying. So we need a useful system to realize the positioning of reentry vehicle.

In this paper, the reentry TDOA positioning system is introduced. It needs at least four ground stations. In this system, the ground stations don't send any signals; they receive the common signal, for example the frame synchronization signal, from the reentry vehicle and record the time of arrival. Because the time of the signal was sent is unknown, one more station is needed in reentry TDOA system than in 3R system. Three time differences are given by $\Delta t_i = t_i - t_{i+1}, (i = 1, 2, 3)$, where t_i is the time of arrival. The coordinates of four ground stations are known, and then the position of the reentry vehicle can be computed. The detail principle of reentry TDOA positioning system is presented in this paper. In the measurement of time difference, the time synchronization of ground stations is the key technology. Microwave, Loran-C and flight clock were used in time transfer, but they were complicated and expensive. They can only serve some limited areas on the globe.

GPS is a satellite-based positioning system available 24h everywhere on the globe with a high accuracy. GPS time transfer provides precise time that satisfies the reentry TDOA positioning system requirement. The time of ground stations in reentry TDOA system are synchronized by one GPS satellite. In simultaneous common-view mode, some common error can be cancelled. The accuracy is about 15ns. The principles, errors and implementation of GPS time transfer used in reentry TDOA positioning system will be introduced. Several sources of positioning error are discussed. The approximate precision is given. Some experimental results are presented. In order to improve the precision and the accuracy, some methods are also proposed.

PRINCIPLE OF REENTRY TDOA POSITIONING SYSTEM

In reentry TDOA positioning system, the ground stations receive the common telemetry signal sent from reentry vehicle, and record the time of arrival respectively, then

$$
\begin{aligned}
t_1 &= T_0 + \frac{r_1}{c} \\
t_2 &= T_0 + \frac{r_2}{c} \\
&\vdots \\
t_N &= T_0 + \frac{r_N}{c}
\end{aligned} \tag{1}
$$

where $t_i (i = 1,2,\cdots,N)$ is the time of arrival, T_0 is the time of signal sent, $r_i (i = 1,2,\cdots,N)$ is the distance between ground stations and reentry vehicle respectively, c is the velocity of light in meters per second.

Provided that the coordinates of ground station i is $\mathbf{X}_i = [x_i, y_i, z_i]^{\mathrm{T}}$, the coordinates of target is $\mathbf{X}_T = [x_T, y_T, z_T]^{\mathrm{T}}$, then

$$
r_i^2 = (x_T - x_i)^2 + (y_T - y_i)^2 + (z_T - z_i)^2 \tag{2}
$$

For the T_0 is unknown, then

$$
\begin{aligned}
\Delta t_1 &= t_1 - t_2 = (r_1 - r_2)/c \\
\Delta t_2 &= t_2 - t_3 = (r_2 - r_3)/c \\
&\vdots \qquad\quad \vdots \\
\Delta t_{N-1} &= t_{(N-1)} - t_N = (r_{N-1} - r_N)/c
\end{aligned} \tag{3}
$$

In this equations set, only the coordinates of target are unknown. If there are four ground stations, three Δt_i can be calculated. The equation roots of formula (3) are the coordinates of target. This is the basic idea of reentry TDOA positioning system.

GPS COMMON VIEW

The Global Positioning System (GPS) includes 24 satellites, distributed in six orbital planes equally spaced in angle. Satellite altitude is approximately 20,183 kilometers above the earth's surface. Each satellite carries an operating atomic clock and emits timed signals that include a code telling its location. In reentry TDOA system, the accuracy of the time measurement should be for example, of the order of 1ns for 30cm accuracy in range error. For the target position is calculated by using the time difference of arrival, the accuracy of the ground stations synchronization is needed to be of the order of nanoseconds. Since the GPS satellites are at about

4.2 earth radii. For example, there are two ground stations (A, B) on the globe then,

$$\begin{aligned} \rho_A(t) &= [(X - X_A)^2 + (Y - Y_A)^2 + (Z - Z_A)^2]^{1/2} + c\Delta t_A - c\Delta t + \Delta\rho_{An} + \Delta\rho_{Ap} \\ \rho_B(t) &= [(X - X_B)^2 + (Y - Y_B)^2 + (Z - Z_B)^2]^{1/2} + c\Delta t_B - c\Delta t + \Delta\rho_{Bn} + \Delta\rho_{Bp} \end{aligned} \quad (4)$$

where $\rho_A(t)$, $\rho_B(t)$ are pseudo-ranges; (X, Y, Z) is the coordinates of satellite; (X_A, Y_A, Z_A), (X_B, Y_B, Z_B) are the coordinates of ground stations; c is the velocity of light; Δt_A, Δt_B are local clock errors; Δt is satellite clock error; $\Delta\rho_{An}$, $\Delta\rho_{Bn}$ are ionosphere delay errors; $\Delta\rho_{Ap}$, $\Delta\rho_{Bp}$ are troposphere delay errors. If the distance between the ground stations is less than 1000km, the angle ∠(Station A – Satellite – Station B) will be less than 10°. In simultaneous common view of a single GPS satellite, can take advantage of common mode cancellation of GPS ephemeris errors and satellite clock error. The ionosphere delay errors and the troposphere delay errors can be reduced greatly.

From formula (4) then,

$$\Delta t_{AB} = \Delta t_B - \Delta t_A = \frac{1}{c}[\rho_B(t) - \rho_A(t) - \Delta R] \quad (5)$$

Where

$$\Delta R = [(X - X_B)^2 + (Y - Y_B)^2 + (Z - Z_B)^2]^{1/2} - [(X - X_A)^2 + (Y - Y_A)^2 + (Z - Z_A)^2]^{1/2} \quad (6)$$

The accuracy of synchronization of ground stations will be improved greatly. At the same time it has been found that inaccurate station coordinates (reaching sometimes several tens of meters) are the cause of large errors in GPS common view time transfer. Now the ground stations coordinates can be located by differential GPS.

A single GPS satellite synchronizes the clocks of ground stations in reentry TDOA system. According to the orbit of GPS satellite, we can force the GPS receivers of ground stations to track a same satellite before reentry measurement. During measurement, the GPS receivers can track the same satellite with the highest elevation. Then GPS common view is realized.

SYSTEM ERROR ANALYSIS

There are three major error sources in reentry TDOA positioning system. They are the time measurement, the ground station coordinates and PDOP (Position Dilution of Precision). We try here to estimate, for each of them, their level at present and in the near future.

A. Time Measurement

The time measurement error is resulted from signal propagation delay, ground stations synchronization, station clock quantification, telemetry receiver and Doppler shift.

1) Signal propagation delay ε_c

After the reentry vehicle going out of the blackout area, the altitude of it is very low. The signal propagation delay is the effect exerted on the telemetry signal as it traverses the troposphere. At the same time, for the difference of the environment around each ground stations, the multipath reflection is one of the error sources. For the distances between the ground stations are not very long, the signal propagation delay ε_c is estimated about 5ns.

2) Ground stations synchronization ε_s

If the ground stations coordinates are located by the differential GPS locating technology, the ground stations synchronization error ε_s may be of the order of 10ns to 20ns.

3) Ground station clock quantification ε_q

For the accuracy of time measurement is needed to be of the order of nanoseconds, and the accuracy of GPS time binary data is about 1ms, a time interval counter is needed in the ground station. Considering the equipment complexity and working toward minimum parts cost, the clock frequency of the time interval counter is 200MHz. The ground station clock quantification error ε_q is 5ns.

4) Telemetry receiver ε_d

The error ε_d resulted from the telemetry receiver is affected by the selection of the telemetry system. In the PCM system, ε_d is about one fiftieth of the chip width. If the bit rate is 2Mbps, ε_d is about 10ns. In the PPK (Pulse Position Keying) system, ε_d is affected by the signal pulse jitter ε_t, the pulse delay resulted by the signal level change Δt_{ds}, the pulse delay resulted by the difference of the decision level $\Delta t'_{ds}$ and the RF pulse jitter σ_{df}.

$$\varepsilon_t = t_r \big/ \sqrt{S/N} \tag{7}$$

Where t_r is the rise time of the pulse signal, $\sqrt{S/N}$ is the square root of the receiver's Signal-to-Noise. For the reliability of the transferred data, the S/N is about 16dB to 17dB. Then

$$\varepsilon_t = \frac{1}{7} t_r \tag{8}$$

In general, the relationship between the system bandwidth and the rise time is $t_r = 0.5/B$.

$$\Delta t_{ds} = \frac{r t_r (k_1 - 1)}{k_1} \tag{9}$$

Where r is the relative decision level, k_1 is the ratio of the actual signal level to the ideal one. $k_1 = 2$, $\Delta t_{ds} = 0.5 r t_r$. $r = 0.6$, $\Delta t_{ds} = 0.3 t_r$. It means that if the actual signal level is twice larger than the ideal one, Δt_{ds} is three times larger than ε_t. For this reason, the system gain should be well controlled to keep the received signal level invariable.

$$\Delta t_{ds}^{'} = r t_r (k_2 - 1) \tag{10}$$

Where k_2 is the ratio of the changed decision level to the original one. If the decision level changed 10%, then $k_2 = 1.1$, $\Delta t_{ds}^{'} \approx 0.42 \varepsilon_t$. The error ε_d resulted from the telemetry receiver in one ground station is

$$\varepsilon_d = \sqrt{\varepsilon_t^2 + \Delta t_{ds}^2 + \Delta t_{ds}^{'2} + \sigma_{df}^2} \tag{11}$$

In reentry TDOA system, the ground stations receive the same telemetry signal and calculate the target’s position by using the time difference of arrival. The RF pulse jitter contributes nothing. The ε_t, Δt_{ds} and $\Delta t_{ds}^{'}$ of each ground station are independent, then in determining the time difference these errors will be enlarged. The error ε_d is

$$\varepsilon_d = \sqrt{2\varepsilon_t^2 + 2\Delta t_{ds}^2 + 2\Delta t_{ds}^{'2}} \tag{12}$$

If the bandwidth of PPK system is 9MHz, $r = 0.6, k_1 = 2, k_2 = 1.1$, S/N is about 16dB to 17dB, $t_r \approx 56ns$, $\varepsilon_t \approx 8ns$, $\Delta t_{ds} \approx 16.8ns$, $\Delta t_{ds}^{'} \approx 3.36ns$, $\varepsilon_d \approx 26.7ns$。

5) Doppler shift ε_f

For the speed of the reentry vehicle is very high, the time measurement will be affected by the Doppler shift. The error resulted from the Doppler shift can’t be estimated accurately. It is about of the order of 15ns to 100ns.

The time measurement error in reentry TDOA positioning system is

$$\varepsilon = \sqrt{2\varepsilon_c^2 + 2\varepsilon_q^2 + \varepsilon_s^2 + \varepsilon_d^2 + \varepsilon_f^2} \tag{13}$$

When $\varepsilon_c = 5ns$, $\varepsilon_q = 5ns$, $\varepsilon_s = 20ns$, $\varepsilon_f = 15ns$, in PPK system, $\varepsilon_d = 27ns$, then $\varepsilon = 38ns$, the range error is about 11m. In PCM system, $\varepsilon_d = 10ns$, then $\varepsilon = 29ns$, the range error is about 9m.

B. Ground station coordinates

The ground station coordinates have effect on the ground stations synchronization and the target position calculation. As above-mentioned, they are the cause of large errors in GPS common-view time transfer. The coordinates should be known accurately in a global terrestrial reference frame before the time measurement. The target position will be calculated based on the ground stations coordinates and expressed in the same global frame. Now using the differential GPS locating technology, the accuracy of the ground station coordinates are satisfying.

C. PDOP

PDOP is a measure of the geometrical “strength of figure” of ground stations. The volume of the shape described by the unit-vectors from the target to the ground stations is inversely proportional to PDOP. In reentry TDOA positioning system, the time measurement error is multiplied by PDOP to get the positioning accuracy. As an example, let’s say the measurement accuracy is 30ns. If PDOP is 1, then the locating accuracy is 9 meters. What happens when the PDOP is 5? The best positioning accuracy is 45 meters. For best results, always do mission planning to avoid high PDOP.

REALIZATION OF THE LOCAL CLOCK

With the reentry TDOA positioning method mentioned above, all stations are synchronized at nanosecond level. A time measurement at nanosecond precision is needed for better positing result. In most case, GPS time-receiver can provide a second pulse signal with high precision and the time code of the pulse’s ascending edge, but unfortunately, the time code is accurate only at second or millisecond level, and it only outputs once per second, it can’t be used to measure the time difference in position system. A local nanosecond clock synchronized by GPS is expected.

The function diagram of the local clock is shown in Figure 1. A local frequency divider link is synchronized by the 1PPS signal from the GPS receiver, it generates a time pulse at nanosecond precision; the local clock responses to the remote signal from target, and outputs the unite-code time for later use after it completes the arrived time signal processing. In most case, the 1PPS pulse come from the GPS receiver is longer than 1ms, a reshaping circuit is used to transform the 1PPS pulse into a quite narrow synchronization signal. Take into accounting of the precision request of reentry position system, the complicacy and the cost etc., the frequency of the local clock is chosen at 200MHz. It can provide by a high stability basic frequency source with phase-locking and frequency doubling. With these parameters, the time resolution of the whole system is 5ns.

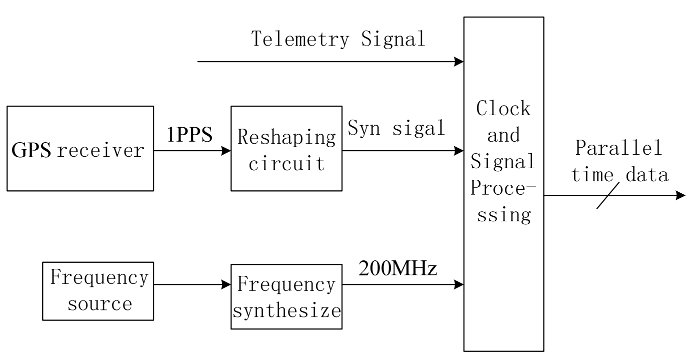

Figure 1. principle diagram of the local clock

The implementation of the local clock is consisted of the designing of analog circuit and the digital circuit. The 200MHz clock pulse (CP) is generated with analog method, and the digital clock and the time signal processing circuit implemented with digital method.

EXPERIMENTAL RESULTS

To experimentally determine the time measurement performance of reentry TDOA positioning system, three stations A, B, C work simultaneously. One transmitter in station A sends telemetry signal. The hardware for B, C stations include antenna, telemetry receiver, GPS receiver, local clock. They receive the telemetry signal and record the time of arrival. Before experiment, r_{AB} and r_{AC} are computed by differential GPS, they are 1189.855m and 2495.746m respectively. The range difference is -1305.891m. Then the time difference Δt_0 is -4353ns. The experimental results are compared with Δt_0 to give the evaluation of the time measurement performance. Figure 2. is the experimental results curve. Several error values are greater than the others. It may be resulted by interference and can be omitted in data processing. The average is -4338ns, which is 15ns greater than Δt_0. This is resulted by the difference of device delay of B, C stations, which can be compensated by calibration measurement. The square root is 36.1ns, which is the reflection of the time measurement performance. The experimental results show that if ground stations are synchronized by GPS common view, the range error of reentry TDOA positioning system may be lower than 15m (1σ), which is satisfying.

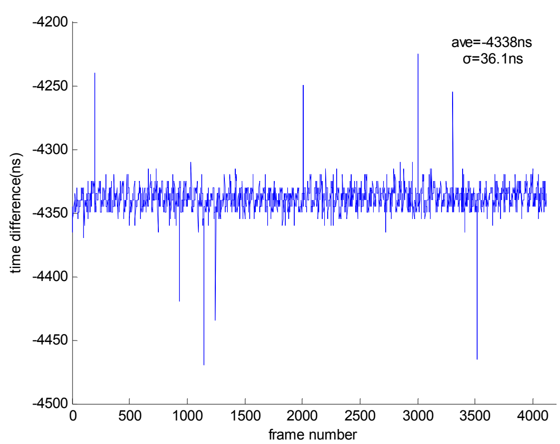

Figure 2. experimental time difference curve

POSSIBLE IMPROVEMENTS OF POSITIONING ACCURACY

In the existing telemetry system, considering the parts cost and the compatibility with the telemetry system, the most feasible methods to improve positioning accuracy are GPS differential positioning technology and arranging the ground stations to get low PDOP. Using GPS carrier phase differential positioning technology, the ground stations coordinates will be determined accurately; the measurement error and the calculation error will be reduced greatly.

As an example, table 1 shows the DOP of a 4-stations system and a rearranged 4-stations system. HDOP and VDOP are given in this table. Where HDOP is horizontal dilution of precision, VDOP is vertical dilution of precision. The coordinates of the point of fall are (0, 0, 0). In the 4-stations system, the coordinates of ground stations are 1 (-5626, 14657, 0), 2 (-3969, 2873, 0), 3 (-3096, -3265, 0), 4 (5313, -8266, 0). VDOP in this system is about 30 to 220; the positioning accuracy is not satisfying. By rearranging the ground stations, we get a new 4-stations system. In this system, the coordinates of ground stations are 1 (5000, 0, 0), 2 (20000, 0, 0), 3 (-2500,13000, 0), 4 (-2500, -13000, 0). VDOP in this system is about 1 to 90. Especially when the height of the target is greater than 1km, the positioning accuracy is acceptable.

From table 1we know that when the height is less than 1km, VDOP increases rapidly. The reason is that the ground stations located in the same plane and the height of stations are zero. As the target falling, the volume of the shape described by the unit-vectors from the target to the ground stations decreases near to zero, and then VDOP is very large. If there is an air-station, VDOP will be less than 10 when the target below 1km. Table 1shows that by rearranging the ground stations the positioning accuracy can be improved greatly.

Table 1   DOP of two systems

target altitude (m)	4-stations system		rearranged 4-stations system	
	HDOP	VDOP	HDOP	VDOP
100	3.32	225.69	1.17	88.80
500	3.51	59.35	1.16	13.67
1000	3.71	39.95	1.16	4.66
1500	3.85	34.34	1.16	2.06
2000	3.97	32.08	1.16	1.35
2500	4.11	31.14	1.17	1.51
3000	4.31	30.86	1.17	1.90
3500	4.64	31.01	1.19	2.32
4000	5.12	31.46	1.21	2.73
4500	5.76	32.15	1.25	3.12
5000	6.55	33.05	1.33	3.49
5500	7.49	34.11	1.47	3.84

6000	8.56	35.31	1.70	4.17
6500	9.76	36.64	2.05	4.48
7000	11.08	38.08	2.52	4.78
7500	12.51	39.61	3.11	5.07
8000	14.06	41.23	3.82	5.36
8500	15.71	42.94	4.63	5.65
9000	17.48	44.71	5.53	5.94
9500	19.34	46.56	6.50	6.25
10000	21.32	48.47	7.53	6.58

CONCLUSION

The ground station synchronization is the major error source in reentry TDOA positioning system. The high precision time synchronization among the ground stations is the key technology in this system. The analysis of the principles and errors of GPS common view show that it is useful to apply GPS common view to TDOA system. Using the GPS common-view technology in the PPK telemetry system, the reentry TDOA positioning system used to measure the track and the point of fall of the reentry vehicle is feasible. In this paper, the implementation of high precision local clock and the experimental results are presented. If the accuracy of the ground stations coordinates is great, the system range error is about 10m. When the PDOP is about 2 to 8, the location error is about 20m to 80m.

REFERENCES

1. Xie Mingsun, “The technology of the reentry telemetry system”, National Defense Industry Press, 1992
2. Zhang Shouxin, “Principles and Application of GPS Satellite Measurement”, Science and Technology University of National Defense Press, 1996
3. Lewandowski, W., Thomas, C., “GPS time transfer”, Proceedings of the IEEE, VOL.79, NO.7, JULY 1991
4. Sun Zhongkang, “Active and Passive positioning technology based on one or more bases”, National Defense Industry Press, 1996
5. Xie Nan, “Study of Errors in The Reentry Low Altitude Target’s Positioning Based on 3Δt Passive Position-finding Principles”, Telecommunication Engineering, Vol. 40, NO. 5, October 2000.