IMPROVED ERROR PERFORMANCE IN SOQPSK MODULATION USING A TERNARY SYMBOL ENCODER

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ABSTRACT

Telemetry transmissions have been evolving over the recent years from PCM/FM to other constant envelope modulation schemes such as SOQPSK and multi-h CPM. These newer modulations schemes have better spectral efficiency but tend to have worse error performance than optimally detected PCM/FM.

We present a new ternary symbol encoder to replace the existing differential encoder and pre-coder for SOQPSK. This improves error performance while minimally affecting the spectral properties. The ”reach” of the new ternary code length is not much longer than current symbol encoder, so there will not be a significant increase in synchronization time in the receiver. We provide simulation results showing the increased performance. Along the way, we also provide a simplified view of the current SOQPSK differential encoder and pre-coder.

I. INTRODUCTION

ARTM Tier-1 (SOQPSK) and Tier-2 (multi-h CPM) constant envelope telemetry modulations were originally adopted since they had much better spectral efficiency and error performance than the ARTM Tier-0 (PCM/FM) approach [1]. However PCM/FM detectors have matured from the simple limiter-discriminator based technique to optimal sequence detection methods. With these maturations, ARTM Tier-1 and Tier-2 approaches no longer have better error performance than the Tier-0 PCM/FM approach although they still have better spectral efficiency [2]. These descriptions are illustrated in Figure 1.

In many applications, link reliability (i.e., error performance) is more important than spectral efficiency. One way to achieve this is to transmit at a higher data rate and use the extra data bandwidth for error correction. However, this results in a change to the spectral mask which requires altering the frequency allocation schemes in place. Ideally, it is best if the spectral mask is not changed, or minimally changed while still conforming to the IRIG mask, so that compatibility with existing frequency allocation schemes is maintained.

We introduce a coding scheme to replace the differential encoder and pre-coder for SOQPSK which improves the error performance while minimally affecting the spectral properties. This scheme allows a drop in replacement of the SOQPSK transmitter and receiver, without altering

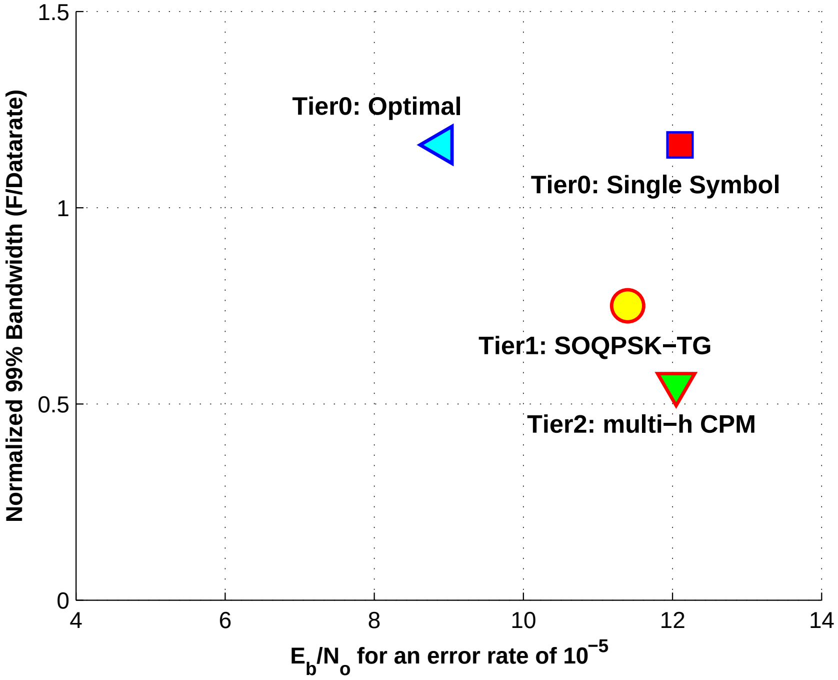

Fig. 1. Comparison of the ARTM modulation schemes [2].

a) the input data rate to the SOQPSK transmitter, and b) the spectral mask of the transmitted signal. In fact, since the improvement is in the coding scheme, in almost all cases, this can be achieved with a firmware upgrade to the transmitter and receiver.

In Section II, we go over differential encoding and the pre-coder used in SOQPSK and provide a simplified view of the SOQPSK symbol encoding process. Section III describes our ternary code and how we will replace the differential encoder and pre-coder while improving the error performance and minimally affecting the spectral efficiency. Our error performance simulation results will be presented in Section IV and compared against other modulation schemes. In Section V we look at the spectral differences between ternary coded and conventional SOQPSK. Lastly, Section VI summarizes all of our work together.

II. THE SOQPSK SYMBOL ENCODER SIMPLIFIED

A. THE TRADITIONAL VIEW OF THE SOQPSK SYMBOL ENCODER

The different components of the SOQPSK transmitter are shown in Figure 2. The SOQPSK symbol encoder consists of a differential encoder and pre-coder defined in IRIG 106-07 [1]. Dr. Rice has provided an in-depth look at how the differential encoder eliminates phase and delay-axis ambiguities associated with carrier phase synchronization [3]. The pre-coder is used to produce ternary values for carrier modulation.

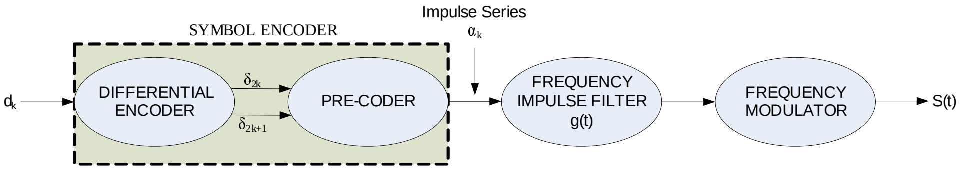

Fig. 2. SOQPSK Transmitter [1].

MAP α_k FROM δ_{2k}					MAP α_{k+1} FROM δ_{2k+1}				
δ_{2k}	δ_{2k-1}	d_{2k-2}	$\Delta\Phi$	α_k	δ_{2k+1}	δ_{2k}	d_{2k-1}	$\Delta\Phi$	α_{k+1}
0	X*	0	0	0	0	X*	0	0	0
+1	X*	+1	0	0	+1	X*	+1	0	0
0	0	+1	-π/2	-1	0	0	+1	+π/2	+1
0	+1	+1	+π/2	+1	0	+1	+1	-π/2	-1
+1	0	0	+π/2	+1	+1	0	0	-π/2	-1
+1	+1	0	-π/2	-1	+1	+1	0	+π/2	+1

Fig. 3. SOQPSK Symbol Encoder table. *Don't care.

The data bits, d_k, at time, k, are transformed into differentially encoded values by the differential encoder (Eqn 1).

$$\begin{aligned} \delta_{2k} &= d_{2k} \oplus \bar{\delta}_{2k-1} \\ \delta_{2k+1} &= d_{2k+1} \oplus \delta_{2k} \end{aligned} \tag{1}$$

These values are then passed through the pre-coder to produce a ternary output, α_k (Eqn 2).

$$\alpha_k = (-1)^{k+1}\left[\frac{a_{k-1}\left(a_k - a_{k-2}\right)}{2}\right], \tag{2}$$

where a_k is the polar(± 1) representation of the bit δ_k, i.e., $a_k = 2\delta_k - 1$.

The α_k values, which are ternary (+1,-1,0), are used to modulate the carrier with a phase change of 90, -90, or 0 degrees. The combined operation of the differential encoder and pre-coder result in the seemingly complicated input-to-output relationship given in Figure 3 [1]. However, it turns out to be much simpler than this.

B. The Simplified View

We can derive a much simpler model for the symbol encoder by combining and analyzing the the differential encoder and pre-coder. Let us use d'_k which is the polar(± 1) representation of the

d_{k-1}	d_k	α_k
0	0	+1
0	1	0
1	0	0
1	1	-1

Fig. 4. Simplified view of the SOQPSK Symbol Encoder.

input bit sequence d_k. We can re-write the differential encoder into a simple equation simply as

$$a_k = (-1)^{k+1}(-d'_k a_{k-1}). \tag{3}$$

Note that for a polar representation, $p \in \pm 1$, of a bit, the following identities hold:

$$p^2 = 1. \ \textit{and} \ \ 1/p = p. \tag{4}$$

We can describe a_{k-1} and a_{k-2} as

$$\begin{aligned} a_{k-1} &= (-1)^{k+1}(-d'_k a_k) \\ a_{k-2} &= (-1)^{k}(-d'_{k-1} a_{k-1}) = -d'_{k-1} d'_k a_k. \end{aligned} \tag{5}$$

Substituting this into the pre-coder equation, gives us a very elegant result:

$$\alpha_k = (-1)^{2k+2}(-1) d'_k a_k (a_k + d'_{k-1} d'_k a_k) = -\frac{1}{2}(d'_k + d'_{k-1}). \tag{6}$$

This is a simple result stating that an input bit transition from 0 to 1 or 1 to 0, results in an output symbol of 0. Two successive 0s result in an output of $+1$. Two successive 1s result in an output of -1 (Fig 4). This is a much simpler description of the SOQPSK symbol encoder than the table given in the IRIG standard (Fig 3).

III. USING A TERNARY CODE FOR THE SYMBOL ENCODER IN SOQPSK

As seen in the previous section, the standard SOQPSK symbol encoder takes bits, d_k, as inputs and produces ternary symbols, α_k (-1,0,1), as outputs. This is essentially a binary to ternary code with a 1 to 1 ratio (for each binary input bit, we have a ternary output bit). If we use an improved binary to ternary code with a 1 to 1 ratio, we could possibly improve the SOQPSK error performance with the increased coding gain of the ternary code.

One such code is described in ”Ternary Phase Shift Keying and Its Performance” by Makoto Nakamura and Hideyuki Torii [4]. In the paper, a ternary convolutional encoder is derived which is described in Figure 5. The encoder was developed over GF(3) instead of GF(2) to give a ternary output. The generator polynomial of the code is:

$$g(D) = 1 + 2D + D^2 + D^4 + D^5 + D^6 \tag{7}$$

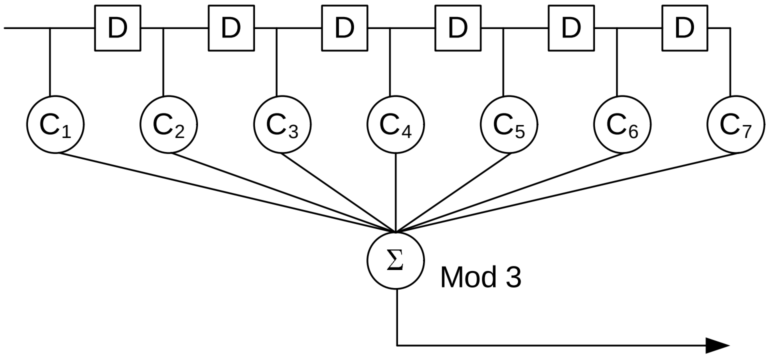

Fig. 5. Ternary Convolutional Encoder [4].

Constraint Length	Generator Polynomial	Minimum Free Distance
7	1 2 1 0 1 1 1 2 1 2 0 2 2 2	6 6

Fig. 6. Best Code of the Ternary Convolutional Encoder [4].

The code has a constraint length of 7 and a minimum free distance of 6 which is shown in Figure 6. In the next section we will see the results of replacing the symbol encoder with this code for SOQPSK modulation.

IV. SIMULATION RESULTS USING THE TERNARY CODE IN SOQPSK

Using the Ternary convolutional encoder described in Section III as the symbol encoder instead of the current differential encoder and pre-coder, we can try to improve the error performance of SOQPSK while minimally affecting the spectral properties. The ternary encoder does not significantly extend the re-sync time after dropped bits since the constraint length of the code is of the same magnitude as the current symbol encoder.

For the simulations, we analyzed SOQPSK using Matlab and Simulink. We created a custom trellis structure in Matlab to use in custom trellis blocks in Simulink. These trellis blocks are utilized in the modulator and demodulator. The demodulator also uses optimal Viterbi decoding to achieve a lower error rate. The channel noise is modeled using Gaussian distributed noise which is added to each symbol.

In Figure 7 we see the performance of several different modulation schemes including ternary encoded SOQPSK, SOQPSK, and QPSK. The SOQPSK parameters we used are as follows: $\rho = 0.7, B = 1.25, T_1 = 1.5, T_2 = 0.5$. We see that the proposed ternary encoded SOQPSK has better error performance than SOQPSK with the old symbol encoder by around 1 dB at a bit error rate of 10^{-5}.

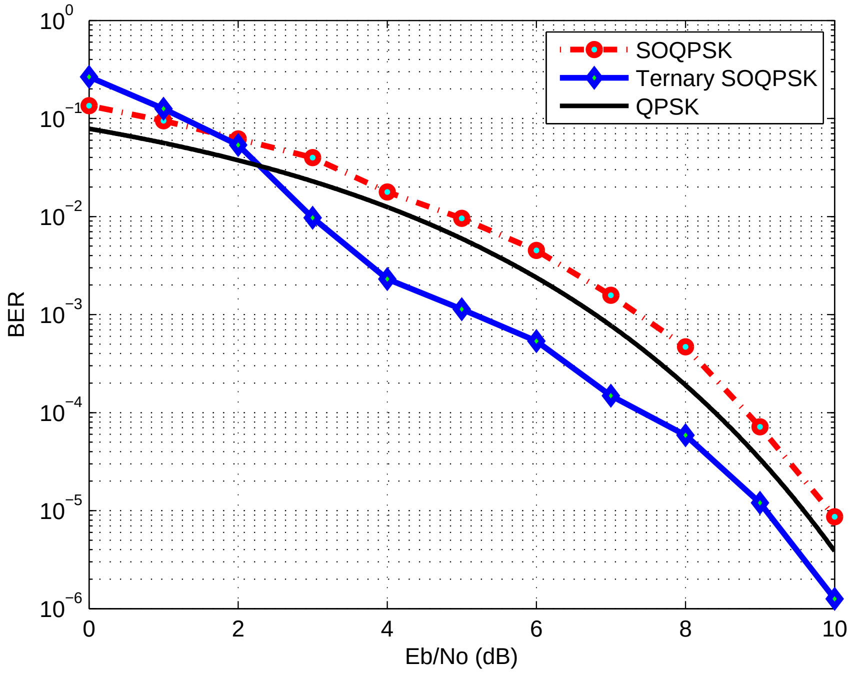

Fig. 7. Error performance of ternary encoded SOQPSK compared with SOQPSK and QPSK.

V. SPECTRAL EQUIVALENCE

Besides looking at the error performance improvement of the proposed ternary code, we must also look at the effect the ternary code has on the spectrum. This is important because if the spectrum is not within the IRIG spectral mask, there will be added channel interference and reduced spectral efficiency.

The spectrum of the ternary encoded SOQPSK signal should be very similar to the standard SOQPSK signal. We would expect minor differences since in the standard SOQPSK signal, the 0 ternary output is twice as likely as the $+1$ or -1 symbols, i.e., it is twice as likely to have zero phase change as a phase increase or a phase decrease. With the new symbol encoder, either of $+1$, 0, -1 are equally likely. The spectrums (99% power bandwidth) are shown in Figure 8. We can see that the spectrum of the ternary encoded SOQPSK is a little larger than the SOQPSK spectrum, however, it still conforms to the IRIG mask and will therefore not add channel interference or reduce the spectral efficiency when used with the IRIG standards.

VI. CONCLUSION

In this paper we first studied the symbol encoder, or differential encoder and pre-coder, used in SOQPSK and presented a simplified view of these blocks by encapsulating the logic in a simple truth table shown in Figure 4.

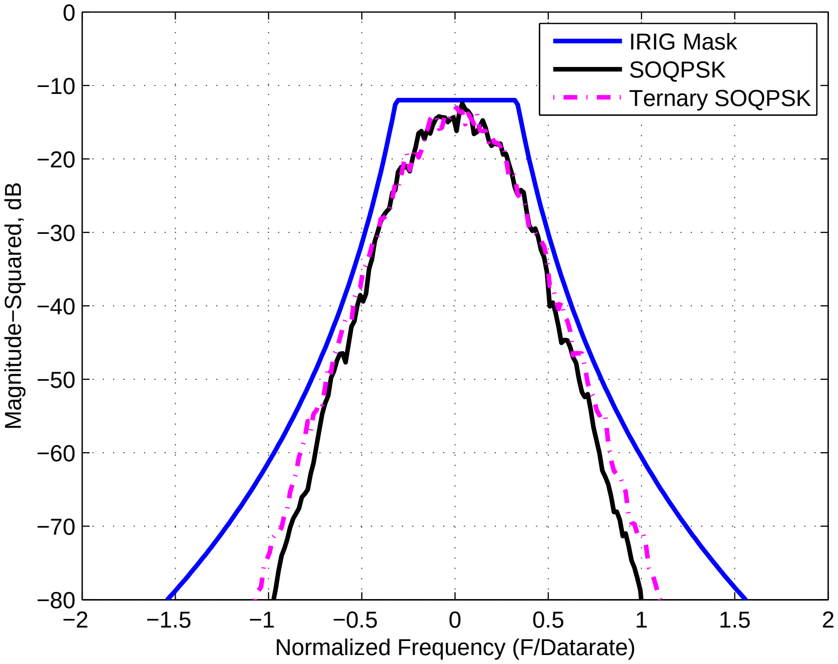

Fig. 8. Ternary encoded SOQPSK spectrum compared with SOQPSK and the IRIG mask.

We proposed an improved symbol encoder using the ternary convolutional code described in Section III. In Section IV we demonstrated that we are able to achieve a performance gain over the current SOQPSK modulation scheme by about 1 dB at a bit error rate of 10^{-5}. These results are shown in Figure 7.

We then continue to analyze ternary encoded SOQPSK by looking at the spectrum and confirming it conforms to the IRIG spectral mask and is very similar to the spectrum of standard SOQPSK.

As a result we now have introduced a coding scheme to improve error performance while minimally affecting the spectral properties for SOQPSK modulation. The new scheme can be implemented with firmware updates to the SOQPSK transmitter and receiver.

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