

# RE-ENGINEERING PCM/FM AS A PHASE MODULATION SCHEME

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## ABSTRACT

Historically, (PCM/FM) receivers have used simple detection schemes yielding low performance. Using multi-symbol detection methods, PCM/FM can be received with better error performance than either SOQPSK or multi-h CPM. We present an approximation by which PCM/FM can be reinterpreted as a phase modulation scheme, allowing the use of coherent detection techniques. This is backward compatible with existing receivers. We also present an extension by which the error performance of the approximated PCM/FM can be improved even further with no change to the spectral properties. This improved waveform can be used in systems where compatibility with existing frequency allocation schemes is required.

## I. INTRODUCTION

Telemetry transmissions require a constant envelope modulation scheme for power efficiency. ARTM Tier-0 (PCM/FM) is a frequency shift keying scheme that has been used in telemetry applications for decades [1]. Traditionally, PCM/FM reception was based on a simple limiter-discriminator based approach. In recent years, other constant envelope modulation schemes (Tier-1 SOQPSK-TG, Tier-2 multi-h CPM) have been adopted that have better spectral efficiency. These modulation schemes also have better error performance than the traditional PCM/FM receivers but have worse error performance than newer PCM/FM reception techniques using optimal sequence detection.

Thus PCM/FM as implemented using modern reception techniques has a better error performance but worse spectral efficiency than the ARTM Tier-1 and Tier-2 modulation schemes. This is illustrated in Figure 1.

In many applications better reliability is of greater importance than spectral efficiency. It is certainly possible to use error coding with the Tier-1 and Tier-2 schemes and improve their performance. However, there are several existing telemetry applications where it is difficult

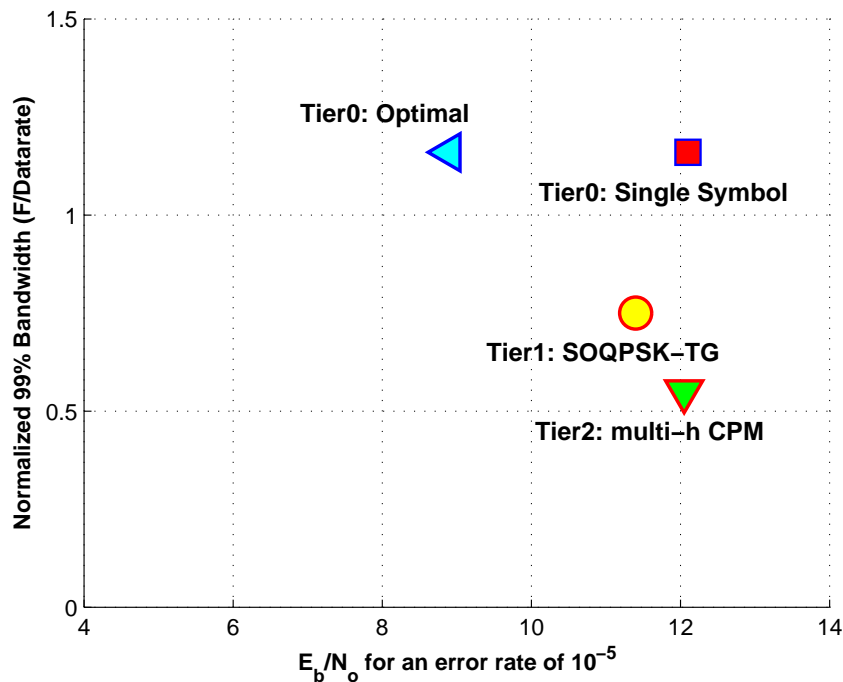


Fig. 1. Comparison of the ARTM modulation schemes.

to logistically change the frequency allocation scheme to accommodate the new modulation schemes, which have different spectral properties.

We present a modification of PCM/FM that maintains the spectral properties and improves error performance. The advantage of this approach over using the Tier-1 and Tier-2 schemes with coding, is that this modification can be transparent to the user of the transmitter (no change in input data rate) and is also compatible with the existing frequency allocation schemes.

Section II provides a historical perspective on the improvements in reception of PCM/FM. Section III shows how PCM/FM can be approximated as a phase-modulation scheme which is still compatible with existing PCM/FM receivers. Section IV shows how a transparent coding scheme can be used with this approximation to improve performance. Section V places the improvements in the context of the existing transmission schemes.

## II. A HISTORY OF IMPROVEMENTS IN PCM/FM ERROR PERFORMANCE

Analog telemetry systems used frequency modulation of analog waveforms for RF transmission. This provided much better performance than amplitude modulation and also allowed to trade-off bandwidth occupancy for performance. With the transition to digital data representation, frequency modulation of filtered digital non-return-to-zero (NRZ) data provided a natural upgrade path. In many cases, the analog FM transmitter could be reused by feeding its input with filtered digital data (instead of an analog information waveform). This required minimal infrastructure

change, and usually had no impact on the frequency allocation schemes. Instead of a bandwidth to SNR trade-off as in the fully analog FM case, the modulation index for the digital transmission determined the data rate that could be sustained within a given RF bandwidth. Early receivers used a limiter-discriminator type receiver. The error performance of this was approximated as

$$p_b = 0.5e^{-kE_b/N_0}, \quad (1)$$

where,  $k$ , was estimated by curve fitting to be 0.7 [2]. In many telemetry systems the modulation index was chosen to be 0.7 (peak deviation of  $0.35R_b$ ) because experimentally this was shown to provided good performance. Since, the phase of the output RF signal is continuous from bit to bit, this waveform can also be treated as a Continuous Phase Frequency Shift Keying (CPFSK) scheme. The more well-known modulation techniques from this family are Minimum Shift Keying (MSK) (where the modulation index,  $h$ , is 0.5 and is considered to provide best orthogonality), and Gaussian Minimum Shift Keying (GMSK), a Gaussian filtered version of MSK which is used in the GSM cellular telephony standard. The error rates for CPFSK can be obtained directly from the minimum squared Euclidean distance (MSED). The MSED is a measure of the difference between the waveforms corresponding to a zero and to a one. The more different these waveforms, the greater the MSED, and the lower the error rate. Very early on, it was known that for single symbol detection, the minimum squared Euclidean distance as a function of modulation index was given by [3]

$$d_{min}^2 = 2 \left[ 1 - \frac{\sin 2\pi h}{2\pi h} \right]. \quad (2)$$

The error rate is minimized when  $d_{min}^2$  is maximized. This occurs when  $h = 0.715$ , resulting in  $d_{min}^2 = 2.434$ . The analytical error rate for CPFSK (and thus PCM/FM) is,

$$P_e = Q \left( \sqrt{\frac{1}{2} \frac{E_b}{N_0} d_{min}^2} \right) = Q \left( \sqrt{\frac{1}{2} \frac{E_b}{N_0} 2.43} \right). \quad (3)$$

However, it was assumed that Minimum Shift Keying with  $h = 0.5$  and an observation interval of two bits would provide better performance since the  $d_{min}^2$  for this case is four [3]. A seminal study into Continuous Phase Shift Keying (CPFSK) systems by Aulin and Sundberg numerically showed that better performance could be obtained by observing the received signal over multiple bit periods [4]. An analytical formula for the minimum squared Euclidean distance of CPFSK for arbitrary modulation index was finally presented by Ekanayake [5]. For  $h = 0.7$ , the result, which is a simple extension of Eqn 2, is

$$d_{min}^2 = 4 \left[ 1 - \frac{\sin 2\pi h}{2\pi h} \right]. \quad (4)$$

This effectively provides an analytical result for the best achievable error rate for CPFSK and PCM/FM with  $h = 0.7$ ,

$$P_e = Q \left( \sqrt{\frac{1}{2} \frac{E_b}{N_0} 4.86} \right). \quad (5)$$

The original Tier-0 PCM/FM telemetry receivers used the limiter-discriminator approach and their

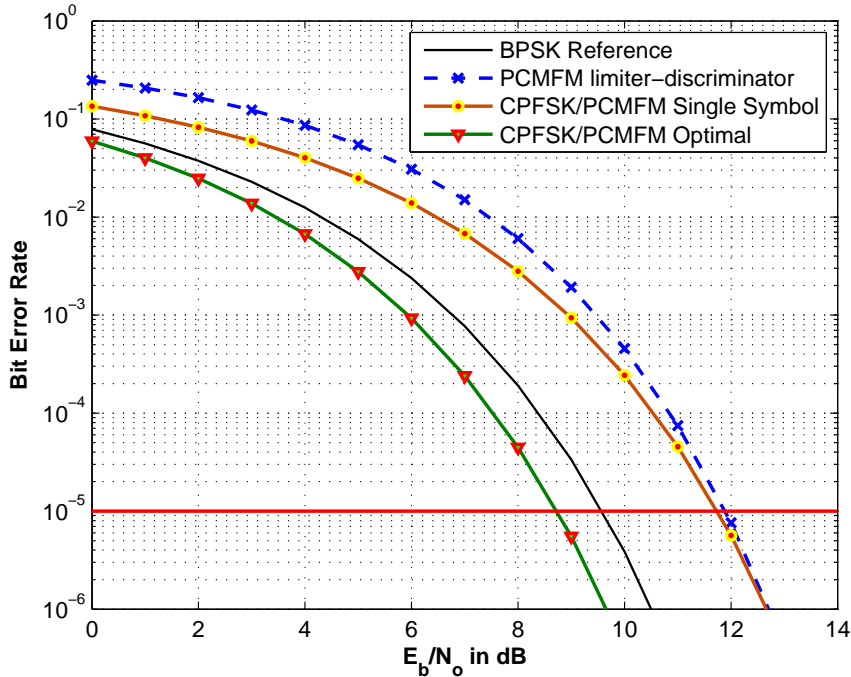


Fig. 2. Performance comparison of different demodulation methods for PCM/FM.

performance tends to be similar to the single-symbol detection method shown in Figure 2, just a little worse. First generation Tier-0 receivers are based on the multi-symbol correlator approach that has been described in the literature [6], [7]. This improves performance dramatically over the limiter-discriminator case. Second generation Tier-0 receivers perform very close to the theoretical limit for optimum sequence detection. Note that cumulatively this has increased performance over the original receiver approach by approximately 3.5 dB (at a BER of  $10^{-5}$ ).

### III. APPROXIMATING PCM/FM AS A TERNARY PHASE MODULATION SCHEME

Most implementations of PCM/FM schemes use a modulation index of  $h \approx 0.7$  which provides good error performance (it is quite close to the optimum of  $h = 0.715$ ). This is also the modulation index used in the IRIG/ARTM scheme. The phase trajectory of PCM/FM with  $h = 0.7$  is shown in Figure 3. At every bit period, the phase advances or retards by  $h\pi$ . On a unit circle, there are twenty unique phase-points used by this modulation scheme. Thus this can be viewed as a twenty point phase modulation scheme, with the constraint that the phase at any bit instant can only be  $\phi_{prev} \pm h\pi$ , where  $\phi_{prev}$  is the accumulated phase at the end of the previous bit period.

At this point, let us make an approximation that will simplify the phase constellation for  $h = 0.7$ . By approximating  $h = 2/3 = 0.67$ , we can reduce the modulation scheme to a simple three point constellation. with the constraint that at each bit period, the phase changes. Even though the value of  $h = 2/3$  is less optimal than  $h = 0.7$ , the actual difference in performance is marginal.

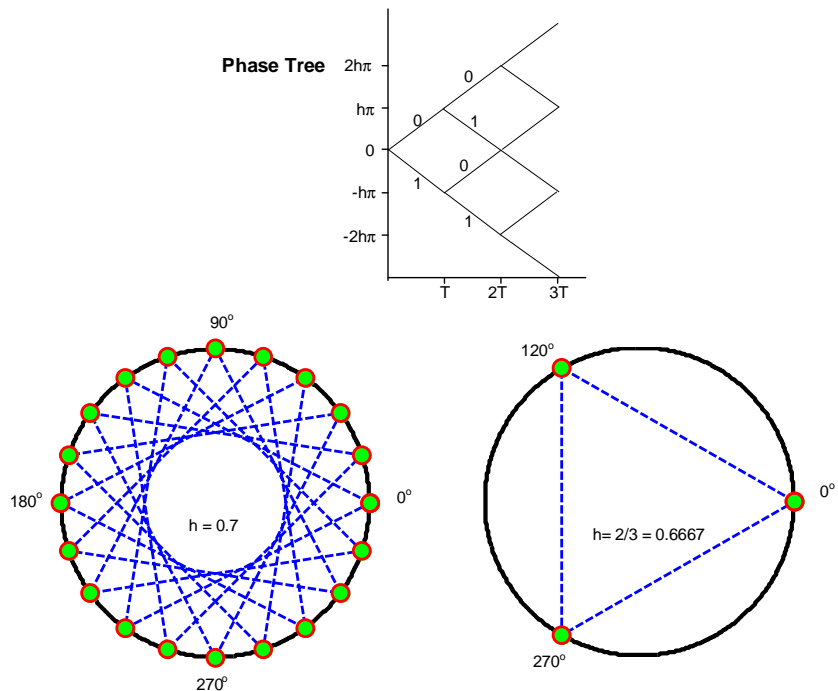


Fig. 3. Phase-tree for PCM/FM with  $h = 0.7$  and  $h = 2/3$ . The dashed lines show the sequential phase-targets not the envelope trajectory. For a constant envelope modulation scheme, the trajectory lies on the circle.

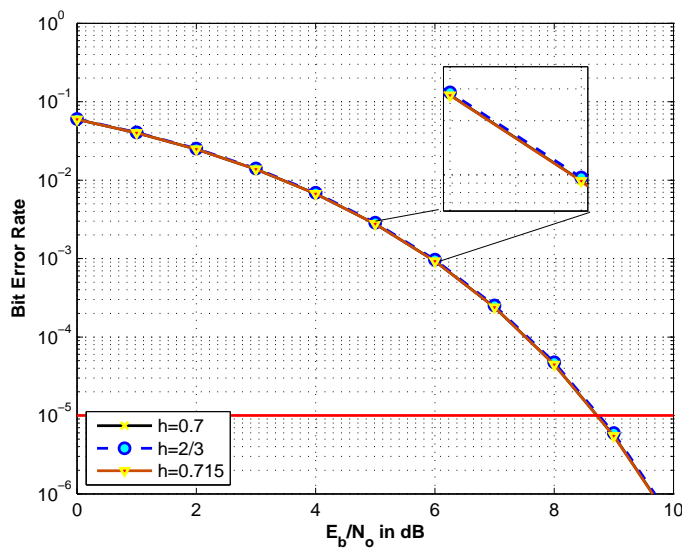


Fig. 4. Performance comparison for CPFSK with  $h = 0.7$ ,  $h = 2/3$ ,  $h = 0.715$ .

As can be seen in Figure 4 both of these modulation indices have insignificant degradation from the optimum index of  $h = 0.715$ .

At this point, it is worth asking if a 3-PSK modulation scheme is identical to a CPFSK modulation scheme with  $h = 2/3$ . This question can be answered by looking at the definition of the distance

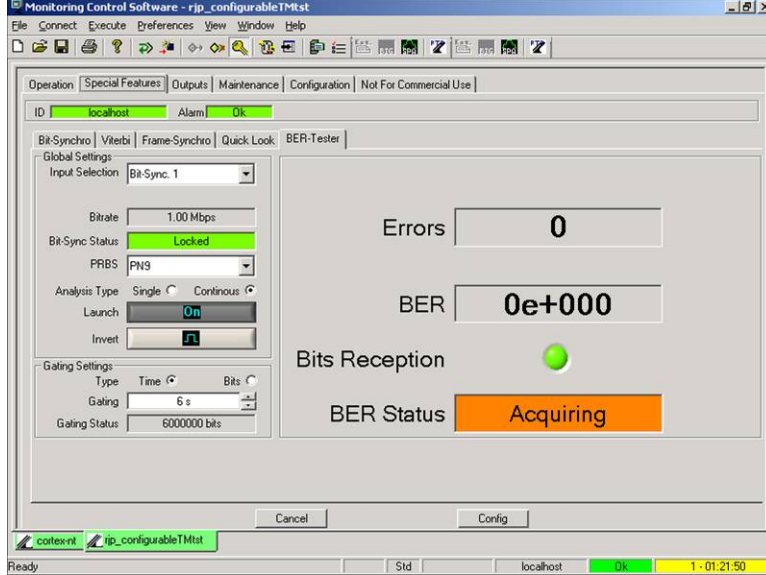


Fig. 5. A Lumistar LS-24 receiver correctly demodulating the CPFSK  $h = 2/3$  waveform.

metric [8], i.e.,

$$d_{min}^2 \triangleq \int_0^{kT} [s_i(t) - s_k(t)]^2 dt. \quad (6)$$

A theoretical m-PSK modulation scheme maintains a certain phase for the duration of a symbol, then abruptly transitions to a different phase for the next symbol. Thus, the distance metric reduces to a geometric distance between constellation points. However, in practice, abrupt transitions are avoided by filtering. With the appropriate filter, the 3-PSK modulation scheme (with the constraint that the same symbol is never repeated) will look like a CPFSK signal with  $h = 2/3$ . So from this point, we will refer to a CPFSK signal with  $h = 2/3$  as a ternary phase shift keying scheme (with the appropriate filtering).

It is important to note that the CPFSK signal with  $h = 2/3$  can be easily demodulated with existing Tier-0 receivers (see Figure 5). The small difference in modulation index is well within the boundaries of variations in existing transmitters. Also, the power spectrum of this modulation scheme is almost exactly the same as the power spectrum for a modulation index of  $h = 0.7$ .

#### IV. IMPROVING PERFORMANCE OF THE APPROXIMATED TERNARY PHASE MODULATION SCHEME

We have shown how the ARTM Tier-0 modulation scheme can be viewed as a ternary phase modulation scheme by approximating the modulation index as  $h = 2/3$ . The input to our modulator has only two possible input choices, zero or one. The ternary phase shift keying scheme can have three possible output phase points. For the CPFSK scheme with  $h = 2/3$ , only two possible options are used for the output phase, either an increase or decrease by  $2\pi/3$  from the

previous phase. The zero phase change option is unused. This constraint, which is a form of coding, improves the error performance of the Tier-0 modulation, since the minimum Euclidean distance is over two bit periods (Figure 3).

This third phase option can be utilized to improve performance further. The modulator has an effective output information bit rate of  $\log_2(3) = 1.5850$ . This provides a mechanism for coding without having to change the input data rate. An appropriate code that takes dibits as inputs and outputs tribits can be used for this purpose. These type of codes are uncommon since they are not linear and do not lend themselves easily to analysis. One such code has been published in the literature by Nakamura and Torii [9]. It takes as input dibits and outputs tribits. It can be described as a convolutional code, except that all operations are performed over  $GF(3)$  instead of  $GF(2)$ . The generator polynomial is

$$g(D) = 1 + 2D + D^2 + D^4 + D^5 + D^6. \quad (7)$$

By utilizing the unused phase option, we can increase performance. Since the phase trajectory is changed, the spectrum of the modulated signal also changes. However, this is to our benefit. Since, we are adding an additional state where the phase does not change, the spectral occupancy actually gets better not worse. This can be seen in Figure 6. The IRIG standard does not specify the exact filter to use for pulse-shaping, just its characteristics, but a Bessel filter is commonly used [1]. This is a carry-over from the analog FM days, since a Bessel filter does not distort the output phase characteristics. For our digital modulation, we can pick a filter that is already used in the ARTM modulation schemes. In particular, the SOQPSK pulse shaping filter has four parameters that allow flexibility in the waveform definition. For this purpose, we can use the SOQPSK shaping filter with the parameters,  $\rho = 0$ ,  $B = 0$ ,  $T_1 = 0$ ,  $T_2 = 1$ . This results in a simple raised cosine type of filter that does not cause inter symbol interference (ISI).

Figure 7 shows the increased performance obtained by using this ternary code. The gain of approximately 1.3 dB is good considering that this code also has the following properties:

- 1) Does not increase the input data rate.
- 2) Does not change the spectral occupancy.
- 3) Is a convolutional code and therefore does not incur extensive decoding delay.
- 4) Automatically performs a binary to ternary conversion.

This is just one example of the performance gain to be obtained from such a code. There may well be other codes that provide the same properties with even better performance.

## V. CONCLUSION

The ARTM Tier-0 PCM/FM waveform, when decoded optimally, has better performance than the Tier-1 or Tier-2 waveforms but has worse spectral efficiency. The analytical expression for the error probability was given in Section II. The error performance of the Tier-1 and Tier-2 schemes can be improved by coding. This would require either a decrease in the input data

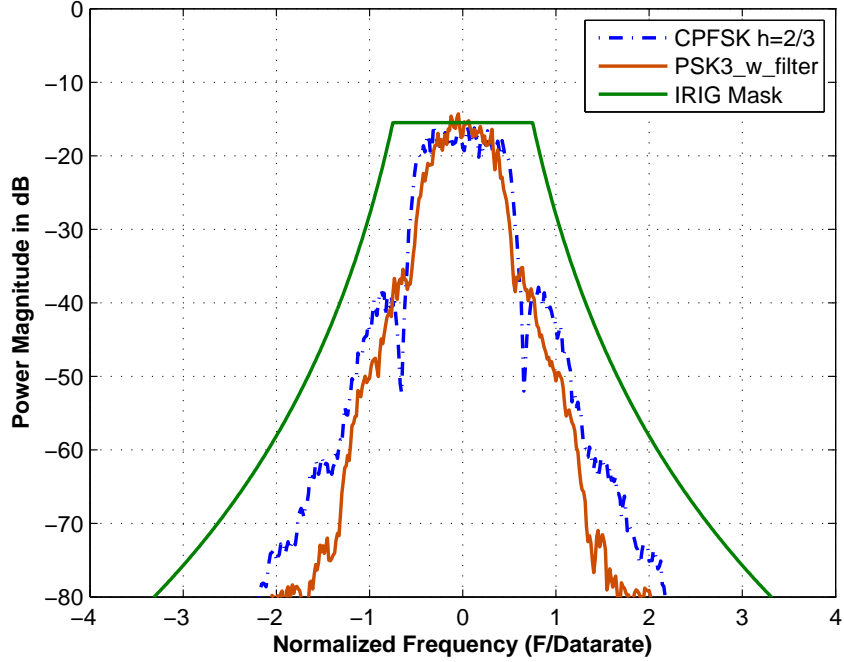


Fig. 6. Spectrum of CPFSK with  $h = 2/3$  and 3-PSK. The 3-PSK scheme allows the phase to transition to any of three outputs, rather than the two for CPFSK.

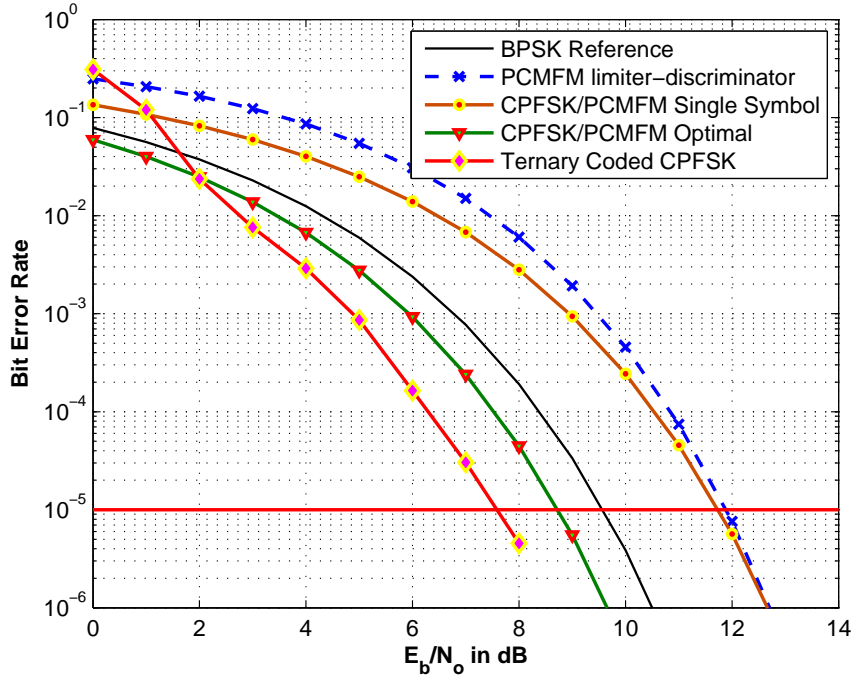


Fig. 7. Performance improvement by using 3-PSK with coding.

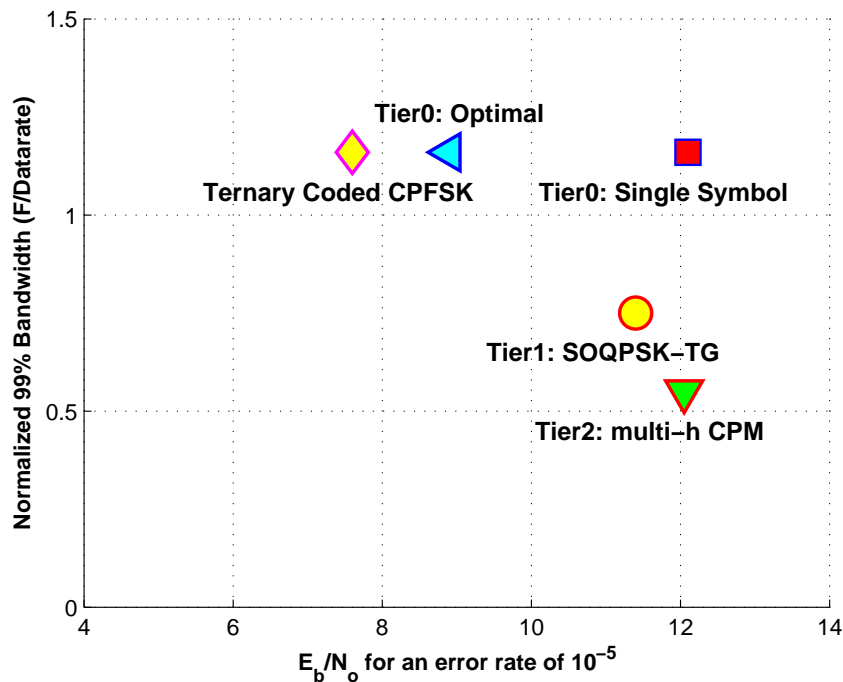


Fig. 8. Comparison of IRIG modulation schemes in perspective.

rate or a change in the output spectral mask. There are some telemetry applications, where it is logistically difficult to change some frequency allocations.

We have presented a way to approximate the Tier-0 PCM/FM waveform as a phase-shift keying scheme with three constellation points. This can reduce the complexity of receivers without affecting performance. This is also backwards compatible with existing receivers. At the same time, this approximation opens the door for increasing the performance of PCM/FM without changing the input data rate to the modulator and without changing the output spectral mask. This transparent coding can be used to increase performance while being compatible with existing frequency allocation schemes. An example of this was shown in Section IV. Figure 8 compares the performance of this ternary coded CPFSK signal to the existing ARTM waveforms. This waveform has the same spectral properties as traditional PCM/FM, but it provides even better error performance. This approach can be of aid in many applications where reliability is more important than spectral efficiency and where compatibility with existing frequency allocation schemes is required.

## VI. ACKNOWLEDGMENTS

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