TURBO PRODUCT CODE WITH CONTINUOUS PHASE MODULATION

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ABSTRACT

A basic problem statement in aeronautical telemetry has been to develop communication systems with good detection efficiency coupled with good spectral efficiency. Shaped-offset quadrature phase shift keying (SOQPSK) is a spectrally more efficient form of continuous phase modulation (CPM) as opposed to pulse code modulation/frequency modulation (PCM/FM). With these modulation techniques, we propose concatenated turbo product code (TPC) with CPM as a solution to our above problem statement. The performance of this turbo product coded CPM (TPC-CPM) system is simulated under coherent and non-coherent demodulation. Finally we present simulation results showing impressive coding gain performance of TPC-CPM over the AWGN channel.

INTRODUCTION

Effective information transfer over a communication channel depends upon an efficient use of power, available bandwidth and complexity. This efficiency is further increased with an optimum combination of modulation and error correction coding technique. While most of the digital communication systems use a simple carrier phase modulation such as binary phase shift keying (BPSK), finding a channel code that works well with the selected modulation scheme has been an area of active research. In 1993, introduction of *turbo codes* [1] marked the birth of a class of codes whose probability of error decreased exponentially with a little algebraic increase in decoding complexity. This kindled tremendrous research interest on parallel concatenated convolutional codes (PCCC) separated by a random bit interleaver and decoded iteratively.

Forney [2] in 1996 developed serially concatenated convolutional codes (SCCC) which consists of a cascade of an inner code and an outer code. Similar to turbo codes, constituent convolutional codes in SCCCs are separated by a random bit interleaver and decoded iteratively. As shown by Benedetto in his 1998 paper [3], the performance of SCCCs is generally comparable and in most cases superior to the performance shown by turbo codes. However, unlike a lot of authors who focused on concatenated convolutional codes, a few authors have considered serially concatenated block codes [4] [5]. Unfortunately, the first algorithms proposed [6] [7] to decode these codes gave poor results as they were based on hard-input/hard-output decoders and they lacked soft-input/soft-output (SISO) decoders. However with the introduction of SISO decoders for all block codes [8] [9], the performance of serially concatenated block codes (SCBC) have been shown to be comparable to SCCCs. In this paper, we study the effects of combining block turbo code (BTC) or turbo product code (TPC) with advanced modulation techniques like partial response CPM.

Even as channel codes provide protection against errors introduced by the channel and as they increase the efficiency of reliable transmission, they tend to reduce the overall bandwidth efficiency of a communication system. Moreover in aeronautical telemetry there has been an ever increasing demand for higher data rates. With the available spectrum being limited, advanced modulation techniques like CPM are required to improve spectral efficiency. In this paper we consider SOQPSK, an important bandwidth efficient constant envelope modulation technique. For real time simulations we use an even better bandwidth efficient version of SOQPSK known as SOQPSK-TG which was adopted from IRIG $106-04$ [10], an aeronautical telemetry standard.

In this paper we develop TPC-CPM which is based on the results given in [11]. CPM was subjected to vast research in the late 1970s and in early 1980s. Since mid 1980s coded CPM was investigated as a way to improve performance while maintaining constant envelope [12] [13]. Since these codes showed better performance than any other previous concatenated coded system, a lot of research was put into code searches for various CPM systems.

The objective of this paper is to study the potential benefits of combining TPC with CPM. Apart from combining TPC with SOQPSK-TG, we try to study the performance of TPC with legacy PCM/FM. Then the performance of TPC-SOQPSK-TG is compared against the performance of TPC-PCM/FM. Additionally the performance of these systems under non-coherent demodulation is simulated and compared against the performance of similar systems under coherent demodulation. In the following section we give a brief description on encoding and decoding TPCs. Then we continue describing the CPM techniques considered in this paper followed by a section on the TPC-CPM simulation model. Finally we present the simulation results and draw up a few concluding facts.

TURBO PRODUCT CODE

Among different channel coding techniques, TPCs are highly important owing to their simple structure, large coding gains, rate flexibility and simple synchronization requirements. Moreover encoder and decoder of such TPCs are commercially available in the form of integrated circuits. Fig. 1 shows a simple illustration of a TPC. Different TPCs such as $(32,26)\times(32,26)$, $(64,57)\times(64,57)$, $(128,120)\times(128,120)$ product codes can be visualized similar to the illustration shown in Fig. 1. In general, TPC by itself is considered to be a serially concatenated code with two or more shorter block codes [9]. Hence these codes are decoded using the chase algorithm [9] which is a near-optimumal (practical) iterative decoding algorithm.

Consider two systematic linear block codes C^1 and C^2 with parameters (n_1,k_1,δ_1) and (n_2,k_2,δ_2), where n_i, k_i, δ_i are codeword length, information block length, and minimum hamming distance, respec-

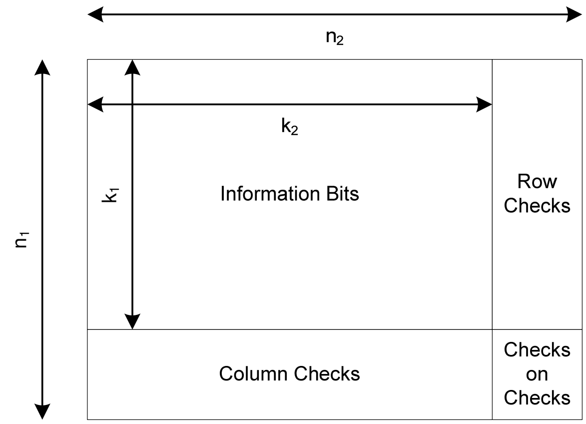


Figure 1: A Simple Turbo-Product Code Example.

tively. Now product code P, as depicted in Fig. 1, is obtained by arranging k_1 information bits along rows and k_2 information bits along columns and then coding k_1 rows using code C^2 and n_2 columns using code C^1 [14]. The resulting product code P has dimensions $n = n_1 \times n_2$, $k = k_1 \times k_2$, $\delta = \delta_1 \times \delta_2$ with code rate R given by $R = R_1 \times R_2$, where R_1 and R_2 are code rates of individual systematic linear block codes [14]. Thus long TPCs with large minimum hamming distance can be produced by simply multiplying short systematic block codes with small minimum hamming distance. Once encoded, n coded bits are modulated and sent over an AWGN channel.

At the receiver end, the received signal is demodulated and then decoded sequentially along rows followed by columns. Though this decoding procedure results in good performance, optimum performance can be obtained when SISO chase decoders [9] are used to decode rows and columns of P. With the received signal matrix R, the chase decoder initially operates on rows and updates soft information along the rows of matrix R. Similar operations are then performed along columns of matrix R and this process is iteratively done until the decoder converges on best results. Among different issues which affect the decoder performance, some important issues include channel characteristics, code parameters, soft decision quality, type of modulation and number of iterations.

CPM SCHEMES

Among the many CPMs developed by Range Commanders Council (RCC) [10] some of them are extremely popular owing to the needs of their applications like spectral efficiency, decoding complexity and power efficiency. Thus aeronautical telemetry have identified two popular modulation techniques with their unique properties to work with UHF frequencies. They are SOQPSK-TG and PCM/FM, which this paper utilizes to develop a TPC-CPM system.

PCM/FM has been used in aeronautical telemetry standards since 1970's. PCM/FM being a binary CPM is specified by CPM parameters $h = 7/10, M = 2, 2RC$ pulses, where h is the modulation index, M indicates the binary nature of this CPM and RC stands for raised cosine frequency pulses. Among the modulation types considered here, PCM/FM has the highest detection efficiency and has moderate

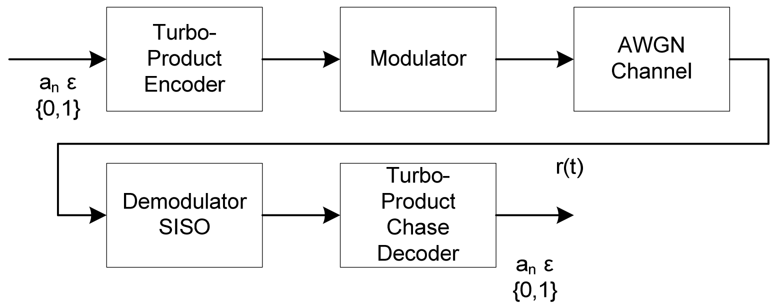


Figure 2: Turbo Product Codes with CPM.

decoding complexity. However it is the least spectrum efficient modulation technique among the two modulations considered here. The other advantage of using PCM/FM is that it is least sensitive to phase noise and hence it is easily synchronized.

The other modulation technique considered here is SOQPSK-TG. Although offset quadrature phase shift keying (OQPSK) has an improved power spectrum compared to quadrature phase shift keying (QPSK), it still suffers waveform envelope fluctuations due to instantaneous transitions between adjacent phase states. SOQPSK-TG, often considered as a derivative of OQPSK and minimum shift keying (MSK), is a constant envelope advanced modulation technique. It is spectrally more efficient than OQPSK and MSK, but it suffers from slightly lower detection efficiency. SOQPSK-TG has a 4 state trellis [15] with small decoding complexity. SOQPSK-TG uses a precoder [16] to convert binary information to ternary symbols which are modulated using a CPM modulator with a modulation index $h = 1/2$. The precoder imposes OQPSK-like properties on SOQPSK-TG while a constant envelope results from the CPM modulator. Among the CPM's considered here, SOQPSK-TG is moderately sensitive to phase noise. These CPM modulation techniques are alternatively used in our TPC-CPM system over an AWGN channel. Either a coherent or a non-coherent demodulator corresponding to a CPM modulator utilizes SISO decoding algorithm and is based on the demodulators built in [17].

TURBO PRODUCT CODED CPM

The potential benefits of coupling TPC with CPM are studied and compared against the results given in [11]. Fig. 2 shows the block diagram of TPC-CPM. Information bits are turbo product encoded. Either a basic $(64, 57) \times (64, 57)$ TPC or other TPCs like $(32, 26) \times (32, 26)$ or $(128, 120) \times (128, 120)$, explained in [14] [9], are assumed to be an encoder. An interleaver between TPC and CPM is not considered here because there was no noticeable difference in performance of TPC-CPM with or without an interleaver. Similar to SCCC-CPM built in [18], encoded bits are mapped into symbols for CPM with higher order signaling using natural or gray mapping. This modulated signal is transmitted over an AWGN channel.

The received signal is CPM demodulated using a SISO demodulator, which is identical to the demodulator developed in [17] [15] [18]. Then the demodulated signal is fed to an iterative chase decoder [9]. Before proceeding to the simulation results, we shall now give the parameters used in the simulation of the Chase decoder which are exactly the same as the parameters specified in [9].

• The number of test patterns is 16 and are generated by the four least reliable bits;
• $\alpha = [0.0, 0.2, 0.3, 0.5, 0.7, 0.9, 1.0, 1.0]$;
• $\beta = [0.2, 0.4, 0.6, 0.8, 1.0, 1.0, 1.0, 1.0]$;
• The maximum iteration number is 4, which is equivalent to eight decoding steps.

Since TPCs are effectively concatenated block codes, the chase decoder iteratively decodes the demodulated signal. At the end of pre-defined number of iterations (in this case 4 iterations), the output from this decoder is hard limited and presented at the output.

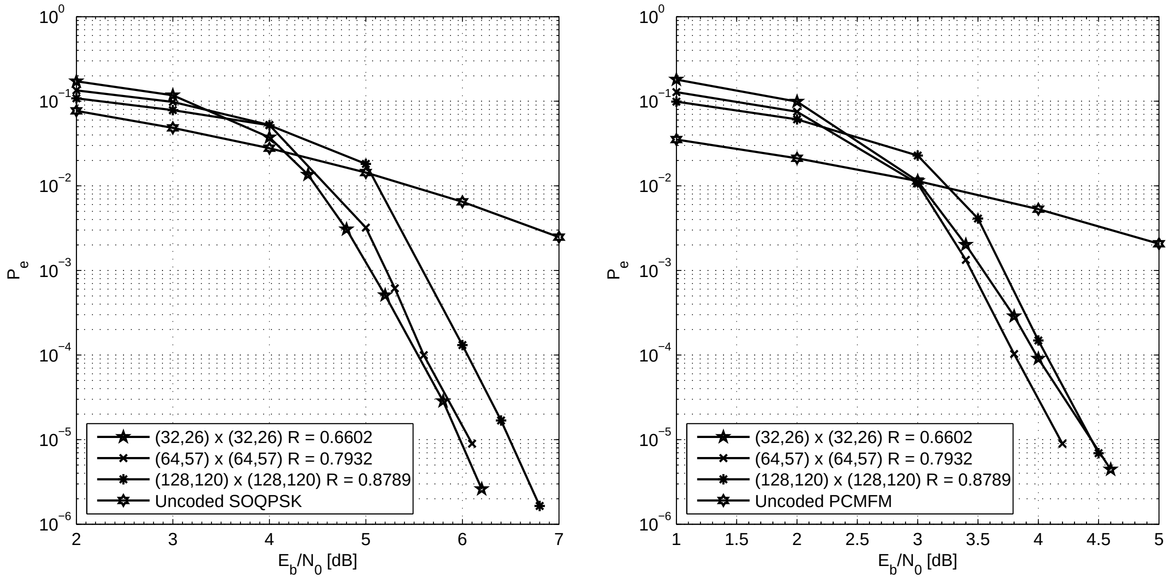
Although the way in which the above system works is quite similar to the working of a system explained in [11], bit error rate (BER) curves shown in the next section indicate a better performance compared to the results listed in [11]. This is largely because we use a near-optimal SISO algorithm for CPM demodulation in contrast to an adhoc soft output CPM demodulator used in [11].

SIMULATION RESULTS

In this paper, we considered three TPCs, the first TPC is $(32, 26) \times (32, 26)$ with rate 0.6602, the next code is $(64, 57) \times (64, 57)$ with rate 0.7932 and finally we chose $(128, 120) \times (128, 120)$ with rate 0.8789. The performance of TPCs with CPM were evaluated on the AWGN channel. The different simulation parameters calibrating TPCs and CPMs are similar to the parameters found in [9] and [15] respectively.

The performance comparison between SOQPSK-TG and PCM/FM with TPCs is shown in Fig. 3(a) and Fig. 3(b). We measure the coding gains of these two schemes at the $P_e = 10^{-5}$ crossing point. By way of reference, uncoded SOQPSK-TG crosses $P_e = 10^{-5}$ at $E_b/N_0 = 10.56$ dB and uncoded PCM/FM crosses $P_e = 10^{-5}$ at $E_b/N_0 = 8.44$ dB [17]. (E_b/N_0 denotes the bit energy to noise power spectral density ratio and P_e denotes the probability of error.) From these figures it becomes clear that the coding gain performance realized with SOQPSK-TG, is better compared to a similar coded PCM/FM. At a code rate 0.6602, TPC-SOQPSK-TG yields a gain of 4.6 dB which is 0.7 dB better than the gain realized with TPC-PCM/FM. This is shown in Fig. 3(a) and Fig. 3(b) respectively. Extensive simulations confirm similar performance results at various code rates considered. This performance of SOQPSK-TG combined with its high spectral efficiency makes it an ideal choice for aeronautical telemetry.

Fig. 4(a) shows the performance of TPC with SOQPSK-TG under non-coherent demodulation. To simulate such a performance we adopt non-coherent demodulators for CPM from [17]. A detailed description on non-coherent demodulation of SOQPSK-TG and PCM/FM can be found in [17] [15]. In this section we consider non-coherent demodulation with a forgetting factor of 0.875 and a 2° standard deviation of phase noise. Comparing this performance of SOQPSK-TG to its performance under coherent demodulation, shown in Fig. 3(a), it is easily seen that the non-coherent demodulators perform 1 to 2 dB worse than coherent demodulators. For instance, consider a code rate 0.7932 TPC-SOQPSK-TG under coherent demodulation. This yields a coding gain of 4.6 dB compared to a gain of 3.2 dB produced by a similar system under non-coherent demodulation. Similar performance difference is also realized with coded PCM/FM under coherent and non-coherent demodulations. Fig. 3(b) and Fig. 4(b) shows the performance of coded PCM/FM under coherent and non-coherent demodulation respectively. At a code rate 0.7932 coherent TPC-PCM/FM yields a coding gain of 4.2 dB which is 0.8 dB better than the gain realized



(a) Simulation Results for Coherent TPC-SOQPSK-TG. (b) Simulation Results for Coherent TPC-PCM/FM.

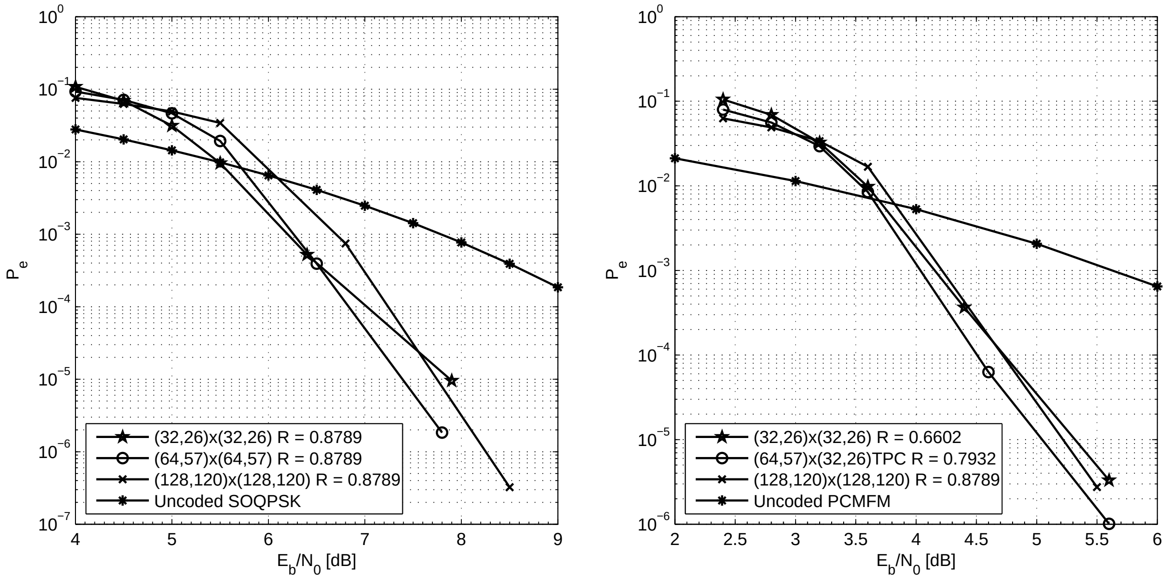
Figure 3: Simulation Results for TPC-CPM under Coherent Demodulation.

by a non-coherent TPC-PCM/FM system. However this is an appreciative trade off between complexity and performance since non-coherent demodulators reduce the complexity of receivers without much loss in performance.

The performance comparison between our TPC-CPM system and a similar system built in [11] are shown in Fig. 3(a) and Fig. 3(b). With our TPC-SOQPSK-TG, at a code rate 0.7932, the coding gain realized is 4.6 dB. This performance, shown in Fig. 3(a), is 0.8 dB better that the gain realized by a similar system built in [11]. Similarly as shown in Fig. 3(b), at the same code rate, our TPC-PCM/FM realizes a coding gain of 4.2 dB which again is 0.8 dB better than the gain reported in [11]. This additional coding gain is mainly due to a near-optimal SISO decoding algorithm which we use for CPM demodulation.

CONCLUSION

In this paper we have developed a TPC-CPM with a SISO CPM demodulator and an iterative TPC chase decoder. We have also simulated the performance of various code rate TPCs with important CPM schemes like SOQPSK-TG and PCM/FM. We have studied and reported the performance of these systems under coherent and non-coherent demodulations. We have also shown that this system achieves better coding gains compared to gains reported earlier for similar TPC-CPM systems.



(a) Simulation Results for Non-Coherent TPC-SOQPSK-TG.

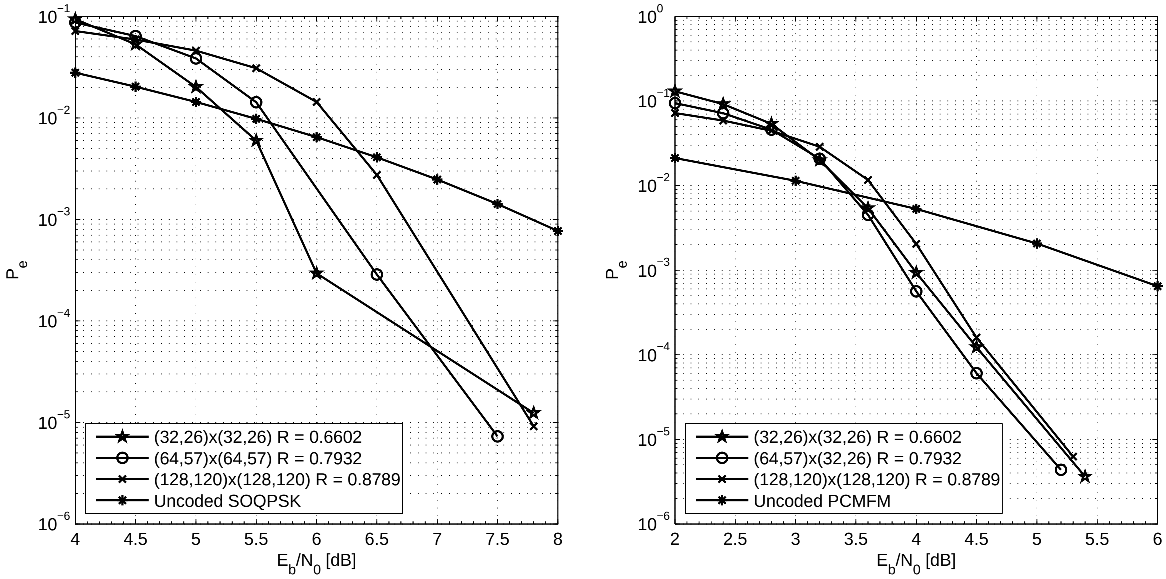
(b) Simulation Results for Non-Coherent TPC-PCM/FM.

Figure 4: Simulation Results for TPC-CPM under Non-Coherent Demodulation with a Forgetting Factor of 0.875 and a 2° Standard Deviation of Phase Noise.

APPENDIX

In this appendix we show the performance of TPC-CPM under non-coherent demodulation with varying forgetting factors and phase noise standard deviations. Fig. 5(a) and 5(b) shows the BER performance of TPCs with SOQPSK-TG and PCM/FM under non-coherent demodulation with a forgetting factor of 0.9375 and a 2° standard deviation of phase noise. The coding gains realized with this system is tabulated in Table 1. For instance consider a code rate 0.7932 TPC-SOQPSK-TG shown in Fig. 5(a), this system yields a coding gain of 3.2 dB which is 1.2 dB less compared to the coding gain produced by the same system under coherent demodulation. Similarly a code rate 0.7932 TPC-PCM/FM system produces a gain of 3.4 dB which is 0.8 dB less than the gain produced under coherent demodulation. Similar performance difference between coherent and non-coherent demodulation is evident with other code rate TPC-CPMs.

Now we consider the performance of TPCs with SOQPSK-TG and PCM/FM under non-coherent demodulation with a forgetting factor of 0.875 and a 5° standard deviation of phase noise. This is shown in Fig. 6(a) and 6(b) respectively. Table 2 tabulates the coding gains produced by this TPC-CPM. As shown in Fig. 6(b), a code rate 0.7932 TPC-PCM/FM yields a coding gain of 2.9 dB which is 1.3 dB less than the gain produced a similar rate system under coherent demodulation. From Fig. 6(a) we can see the performance of TPC-SOQPSK-TG at a 5° standard deviation of phase noise. As seen from this figure the performance of TPC-SOQPSK-TG is rather very poor. This shows the impact of increased phase noise on TPC-CPM systems. However as seen in Fig. 4(a) and Fig. 5(a), TPC-SOQPSK-TG performs close to coherent demodulation at a moderate 2° standard deviation of phase noise. From Fig. 4 and 5 we can also see that, given a 2° standard deviation of phase noise, the performance of TPC-CPM systems does not



(a) Simulation Results for Non-Coherent TPC-SOQPSK-TG.

(b) Simulation Results for Non-Coherent TPC-PCM/FM.

Figure 5: Simulation Results for TPC-CPM under Non-Coherent Demodulation with a Forgetting Factor of 0.9375 and a 2° Standard Deviation of Phase Noise.

vary much due to varying forgetting factors (0.875 and 0.9375). This is evident from almost equal coding gains reported by these systems under consideration.

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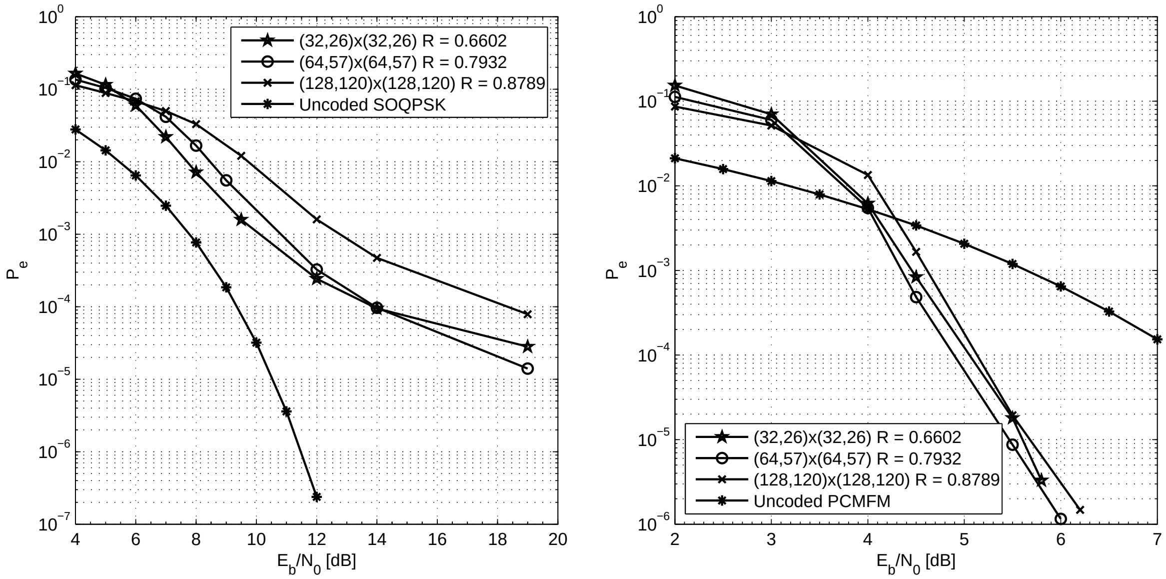
Table 1: Performance of TPC-CPM under Non-Coherent Demodulation with a Forgetting Factor of 0.9375 and a 2° Standard Deviation of Phase Noise.

Modulation	TPC	Coding Rate	Coding Gain
SOQPSK-TG	$(32, 26) \times (32, 26)$	0.6602	2.8 dB
SOQPSK-TG	$(64, 57) \times (64, 57)$	0.7932	3.2 dB
SOQPSK-TG	$(128, 120) \times (128, 120)$	0.8789	2.8 dB
PCM/FM	$(32, 26) \times (32, 26)$	0.6602	3.2 dB
PCM/FM	$(64, 57) \times (64, 57)$	0.7932	3.4 dB
PCM/FM	$(128, 120) \times (128, 120)$	0.8789	3.2 dB



(a) Simulation Results for Non-Coherent TPC-SOQPSK-TG.

(b) Simulation Results for Non-Coherent TPC-PCM/FM.

Figure 6: Simulation Results for TPC-CPM under Non-Coherent Demodulation with a Forgetting Factor of 0.875 and a 5° Standard Deviation of Phase Noise.

REFERENCES

[1] C. Berrou, A. Glavieux, and P. Thitimajshima, “Near shannon limit error correcting coding and decoding: Turbo-codes,” in *Proc. IEEE Int. Conf. Commun.*, vol. 2, pp. 1064–1070, May. 1993.

[2] G. D. Forney, “Concatenated codes,” *MIT Press.*, 1996.

[3] S. Benedetto, D. Divsalar, G. Montorsi, and F. Pollara, “Serial concatenation of interleaved codes: Performance analysis, design, and iterative decoding,” *IEEE Trans. Inform. Theory*, vol. 44, pp. 909–926, May 1998.

[4] J. Lodge, R. Young, P. Hoeher, and J. Hagenauer, “Separable MAP ’filters’ for the decoding of product and concatenate codes,” in *Proc. IEEE Int. Conf. Commun.*, pp. 1740–1745, May 1993.

Table 2: Performance of TPC-CPM under Non-Coherent Demodulation with a Forgetting Factor of 0.875 and a 5° Standard Deviation of Phase Noise.

Modulation	TPC	Coding Rate	Coding Gain
SOQPSK-TG	$(32, 26) \times (32, 26)$	0.6602	–
SOQPSK-TG	$(64, 57) \times (64, 57)$	0.7932	–
SOQPSK-TG	$(128, 120) \times (128, 120)$	0.8789	–
PCM/FM	$(32, 26) \times (32, 26)$	0.6602	2.7 dB
PCM/FM	$(64, 57) \times (64, 57)$	0.7932	2.9 dB
PCM/FM	$(128, 120) \times (128, 120)$	0.8789	2.7 dB

[5] J. Hagenauer, E. Offer, and L. Papke, “Iterative decoding of binary block and convolutional codes,” *IEEE Trans. Inform. Theory*, vol. 42, pp. 429–445, Mar. 1996.

[6] S. M. Reddy and J. P. Robinson, “Random error and burst correction by iterated codes,” *IEEE Trans. Inform. Theory*, pp. 182–185, Jan. 1972.

[7] S. M. Reddy, “On decoding iterated codes,” *IEEE Trans. Inform. Theory*, pp. 624–627, Sept. 1970.

[8] R. Pyndiah, A. Glavieux, A. Picart, and S.jacq, “Near optimum decoding of product codes,” in *Proc. IEEE Global Telecommun. Conf.*, (San Francisco, CA), pp. 339–343, Nov.-Dec. 1994.

[9] R. Pyndiah, “Near-optimum decoding of product codes: Block turbo codes,” *IEEE Trans. Commun.*, vol. 46, pp. 1003–1010, Aug. 1998.

[10] Range Commanders Council Telemetry Group, Range Commanders Council, White Sands Missile Range, New Mexico, *IRIG Standard 106-04: Telemetry Standards*, 2004. (Available on-line at http://www.ntia.doc.gov/osmhome/106.pdf).

[11] M. Geoghegan, “Exprimental results for PCM/FM, tier 1 SOQPSK and tier 2 multi-h CPM with turbo-product codes,” in *Proc. Int. Telemetering Conf.*, (Las Vegas, NV), Oct. 2003.

[12] G. Lindell, *On coded continuous phase modulations*. PhD thesis, Dept. of Telecomm. Theory, Univ. of Lund., Sweden, 1985.

[13] S. V. Pizzi and S. G. Wilson, “Convolutional coding combined with continuous phase modulation,” *IEEE Trans. Commun.*, pp. 20–29, Jan. 1985.

[14] S. Lin and D. Costello, *Error Control Coding*. New York: Prentice Hall, 2004.

[15] D. Kumaraswamy and E. Perrins, “On reduced complexity techniques for bandwidth efficient continuous phase modulations in serially concatenated coded systems,” in *Proc. Int. Telemetering Conf.*, (Las Vegas, NV), Oct. 2007.

[16] E. Perrins and M. Rice, “Reduced-complexity approach to iterative detection of SOQPSK,” *IEEE Trans. Commun.*, vol. 55, pp. 1354–1362, Jul. 2007.

[17] D. Kumaraswamy, “Simplified detection techniques for serially concatenated coded continuous phase modulations,” Master’s thesis, Dept. Elect. Eng. Comp. Sci., Univ. Kansas, Lawrence, KS, July. 2007.

[18] K. Damodaran and E. Perrins, “Serially concatenated high rate convolutional codes with continuous phase modulation,” in *Proc. Int. Telemetering Conf.*, (Las Vegas, NV), Oct. 2007.