SPECTRALLY EFFICIENT CONCATENATED CONVOLUTIONAL CODES WITH CONTINUOUS PHASE MODULATIONS

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ABSTRACT

We develop bandwidth-efficient serially concatenated coded (SCC) continuous phase modulation (CPM) techniques for aeronautical telemetry. The concatenated code consists of an inner and an outer code, separated by an interleaver, and is decoded using relatively simple near-optimum iterative decoding algorithms. CPM waveforms such as shaped-offset quadrature phase shift keying (SOQPSK) and pulse code modulation/frequency modulation (PCM/FM), which are currently used in military satellite and aeronautical telemetry standards, can be viewed as inner codes due to their recursive nature. For the outer codes, we apply serially concatenated convolutional codes (SCCC) because of their large coding gains, high coding rates, and because their decoding algorithms are readily implemented. High-rate codes are of special interest in aeronautical telemetry applications due to recent reductions in available spectrum and ever-increasing demands on data rates. We evaluate the proposed coding schemes with a large set of numerical simulation results and make a number of recommendations based on these results.

INTRODUCTION

The primary objective of any digital communication system is to effectively transmit information over a channel, while efficiently utilizing power, bandwidth and complexity. For this to be done, the selected modulation scheme must match the channel characteristics. Moreover, efficiency of data transmission is increased with well-chosen combinations of channel coding and modulation techniques. The introduction of *turbo codes* in 1993 [1] led to a flurry of research effort in parallel concatenated convolutional codes (PCCC) separated by a random interleaver and decoded iteratively. Turbo codes yield bit error rates (BER) around 10^{-5} at rates well beyond the channel cutoff rate [1].

Another equally powerful code configuration with comparable performance to turbo codes is serially concatenated convolutional codes (SCCC) separated by a random interleaver and decoded iteratively [2]. Although the use of channel codes provides protection against errors introduced by the channel and increases the power efficiency of data transmission, their use also reduces the bandwidth efficiency of the overall communication system.

In recent years, bandwidth efficiency has become a major concern in aeronautical telemetry. PCM/FM (pulse code modulation/frequency modulation), which is a rather spectrum *inefficient* modulation, has been the dominant carrier for aeronautical telemetry since the 1970s. Spectrum reallocations of frequency bands in 1997 prompted a migration away from PCM/FM and gave rise to the Advanced Range Telemetry (ARTM) program [3]. Size, weight, and power supply constraints forced the use of fully saturated, nonlinear RF power amplifiers. As a consequence, the search for more bandwidth efficient waveforms was limited to constant envelope waveforms, in particular, continuous phase modulations (CPMs). By 2004, a pair of interoperable waveforms were adopted in the IRIG 106 standard as “ARTM Tier I” modulations [4]. The first is a version of Feher-patented QPSK (FQPSK) [5], which is a licensed technology. The second is a version of shaped offset quadrature phase shift keying, known as “SOQPSK-TG” [6], which is an unlicensed technology that has also been used in military satellite communication standards [7]. These waveforms achieve twice the spectral efficiency of PCM/FM even when nonlinear amplifiers are used [8].

This paper treats the development of bandwidth-efficient serially concatenated coded (SCC) techniques for PCM/FM and SOQPSK-TG. Forward error correction (FEC) schemes for aeronautical telemetry have received only preliminary attention to date. The only published results on this subject are found in [9], which discussed a combination of turbo-product codes (TPCs) with PCM/FM and SOQPSK-TG using a non-CPM based *ad hoc* approach.

In this paper, we propose SCC schemes for aeronautical telemetry that take full advantage of the fact that PCM/FM and SOQPSK-TG are *coded modulations* and can be treated as inner codes in SCC schemes [10, 11, 12, 13, 14]. In particular, we develop high-rate SCCC schemes. We also develop *coherent* and *noncoherent* soft-input soft-output (SISO) demodulators for use with these codes in an iterative demodulation and decoding architecture. We present a set of numerical simulation results that compare the resulting SCC schemes in terms of 1) SOQPSK-TG vs. PCM/FM, 2) coherent demodulation vs. noncoherent demodulation, and 3) SCCC vs. TPC. The numerical results indicate that SOQPSK-TG is an excellent choice due to its spectrum efficiency advantage over PCM/FM and its large coding gains. The results also show that SCCCs yield larger coding gains than TPCs and that noncoherent demodulation offers attractive performance in light of its simplified synchronization requirements.

The paper is organized as follows. The next section gives a brief description of the CPM schemes used in aeronautical telemetry and this is followed by a section which develops SCC-CPM schemes. Following this is a section on numerical simulation results of the coding schemes and finally we summarize our conclusions.

CPM SCHEMES

The term *continuous phase modulation* (CPM) refers to a large class of constant-envelope waveforms that are characterized by three parameters: the data alphabet size M (e.g. binary, quaternary), the *modulation index* h, and the shape and duration of the *frequency pulse*. These three parameters are usually selected to satisfy constraints on the limited resources of power, bandwidth, and complexity. CPMs are a natural choice for the inner codes in a SCC system. This is because they can be viewed as *recursive codes*,

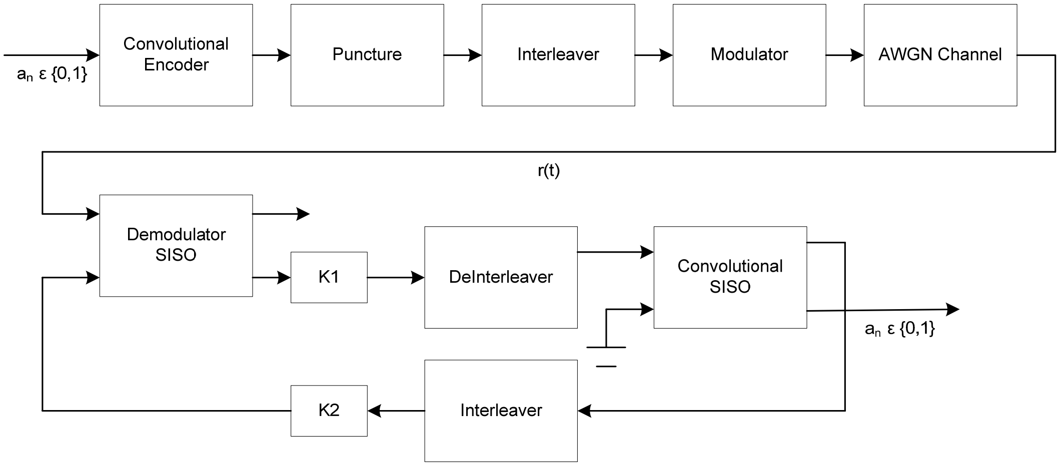

Figure 1: Serially concatenated convolutionally coded CPM (SCCC-CPM) with iterative decoding.

which are necessary to yield large interleaving gains in such systems [2]. In this paper, we consider coding schemes using two of the CPMs used in aeronautical telemetry [4]: PCM/FM and SOQPSK-TG.

PCM/FM is a binary CPM with parameters $M = 2$, $h = 7/10$, and a raised cosine frequency pulse shape with duration $L = 2$ symbol times (2RC). Among the two modulations considered here, PCM/FM has the highest detection efficiency, the lowest spectrum efficiency, and it requires moderate decoding complexity.

The other modulation technique considered here is SOQPSK-TG. SOQPSK-TG is often considered a derivative of offset quadrature phase shift keying (OQPSK) and minimum shift keying (MSK). Although OQPSK has an improved power spectrum compared to quadrature phase shift keying (QPSK) when using nonlinear amplifiers, it still has waveform envelope fluctuations due to instantaneous transitions between adjacent phase states. SOQPSK-TG is a *constant envelope* generalization of OQPSK. It is more spectrally efficient than OQPSK and MSK, in exchange for slightly lower detection efficiency. The SOQPSK transmitter consists of a special binary-to-ternary *precoder*—which converts the binary information symbols to ternary channel symbols that are constrained to follow OQPSK data transitions—followed by a standard CPM modulator. In the case of SOQPSK-TG, the CPM modulator is configured with $h = 1/2$ and uses the custom frequency pulse shape specified in [4]. SOQPSK-TG can be described with a 4 state trellis [15], which requires low decoding complexity. Compared to PCM/FM, SOQPSK-TG has twice the spectral efficiency and has lower detection efficiency.

As mentioned above, we use PCM/FM and SOQPSK-TG as inner codes of a SCC system over an additive white Gaussian noise (AWGN) channel. The inner demodulator/decoders for these codes are based on the soft-input soft-output (SISO) algorithm [16, 11] and were designed and implemented in [17]. We now develop SCC systems that combine a number of different outer codes with these inner modulations/codes.

Table 1: Map of deleting bits for high rate punctured convolutional codes derived from basic rate 1/2 convolutional codes with constraint lengths $k = 2$ and $k = 4$.

Coding Rate	$k = 2$	$k = 4$	N	S
$1/2$	$1(5)$ $1(7)$	$1(27)$ $1(31)$	2048	32
$2/3$	10 11	11 10	1536	27
$3/4$	101 110	101 110	1364	26
$4/5$	1011 1100	1010 1101	1280	25
$5/6$	10111 11000	10111 11000	1230	24
$6/7$	101111 110000	101010 110101	1197	24
$7/8$	1011111 1100000	1010011 1101100	1168	24
$8/9$	10111111 11000000	10100011 11011100	1152	24
$9/10$	101111111 110000000	111110011 100001100	1140	23

SERIALLY CONCATENATED CONVOLUTIONALLY CODED CPM

The SCC scheme we consider is serially concatenated convolutionally coded CPM (SCCC-CPM), a block diagram of which is shown in Fig. 1. This scheme consists of an outer convolutional encoder and an inner CPM modulator, which are separated by an S-random interleaver [18]. We select two rate 1/2 convolutional codes as candidates for the outer code:

- CC1: constraint length $k = 2$ and generators $(5, 7)$;[1] and
- CC2: constraint length $k = 4$ and generators $(27, 31)$.

The encoders for CC1 and CC2 are explained in [19] and are non-recursive with a free distance of 5 and 7, respectively. These codes completely satisfy the design criteria for outer codes stated in [2].

We use *puncturing* [20] as a low-complexity method of achieving coding rates higher than the basic rate of 1/2. This approach allows rate flexibility while requiring only a single encoding/decoding algorithm for all rates. Table 1 specifies the puncturing patterns and interleaver parameters for CC1 and CC2 with rates 1/2, 2/3, $\cdots$, 9/10. The higher coding rates are of particular interest in the present aeronautical telemetry application due to the shortage of available spectrum.

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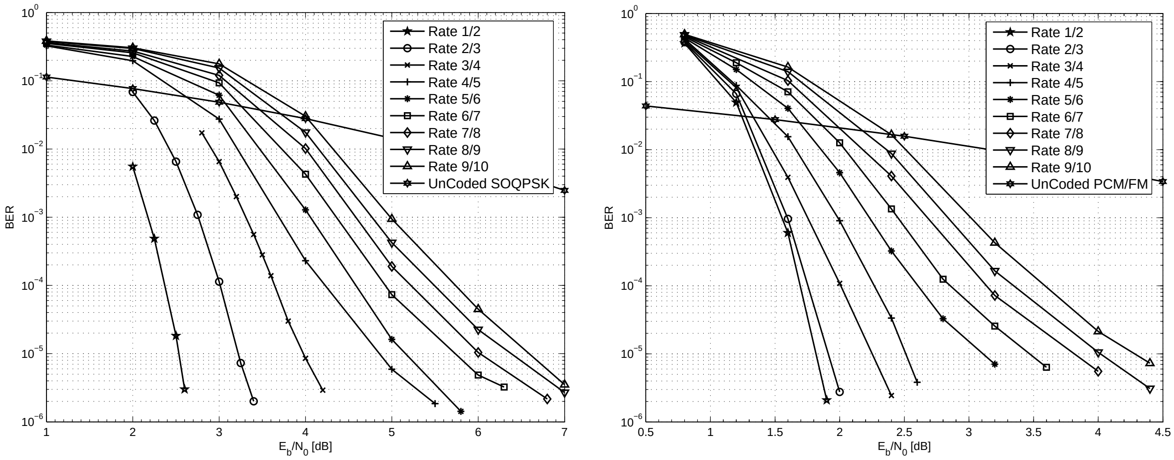
[1] We use the standard *octal* notation [19] for specifying the generators.



(a) Simulation results for CC1 with coherent SOQPSK-TG.

(b) Simulation results for CC1 with coherent PCM/FM.

Figure 2: Simulation Results for CC1 with CPM under Coherent Demodulation.

In Fig. 1, the received signal is demodulated using the PCM/FM and SOQPSK-TG SISO algorithms explained in [17]. The two SISO modules are “max-log” versions of the ones in [11, 16] and the soft probabilities are in the form of log-likelihood ratios (LLRs). The demodulator SISO takes as its inputs 1) the received signal $r(t)$ and 2) a soft input on the probability of the coded symbols. The demodulator SISO outputs an updated soft probability of the coded symbols. This output is deinterleaved and used as a soft input to the convolutional SISO decoder. The lower soft input to the convolutional SISO decoder in Fig. 1 has a numerical value of zero, which corresponds to the assumption that the information bits (ones and zeros) are equally likely to occur. The outer SISO decoder produces updated versions of its inputs, one of which is interleaved and fed back as an input to the CPM SISO demodulator.

Probabilities are exchanged between the inner demodulator and the outer decoder in this manner for a predefined number of iterations. These probabilities are scaled by constants $K1$ and $K2$ to improve the overall performance [21]. In the case of SOQPSK-TG we select $K1 = 0.75$ and $K2 = 0.75$; In the case of PCM/FM we select $K1 = 0.65$ and $K2 = 0.65$; these values were determined by simulation. At the end of predefined number of iterations (in this case five iterations), the lower soft output of the convolutional SISO decoder is hard limited and constitutes the final output of the decoder. The performance of this system under coherent and noncoherent demodulation is presented in the next section.

SIMULATION RESULTS

We now present simulation results with comparisons of different aspects of the above-mentioned SCC-CPM schemes.

A. SOQPSK-TG vs. PCM/FM

The bit error rate (BER) performance of coded SOQPSK-TG and PCM/FM with CC1 and various coding rates is shown in Figs. 2(a) and 2(b), respectively. We measure the coding gains of these two schemes at the BER $= 10^{-5}$ crossing point. By way of reference, uncoded SOQPSK-TG crosses BER $=$ 10^{-5} at $E_b/N_0 = 10.56$ dB and uncoded PCM/FM crosses BER $= 10^{-5}$ at $E_b/N_0 = 8.44$ dB [17]. (E_b/N_0 denotes the bit energy to noise power spectral density ratio.) From these two figures, we can see larger coding gains in the case of coded SOQPSK-TG. A code rate 1/2 SCCC-SOQPSK-TG scheme yields a coding of 8.0 dB whereas a similar rate SCCC-PCM/FM scheme yields a coding gain of 6.6 dB. However, as the code rate increases (e.g. rate 7/8), the gains produced by both the coded modulations are approximately the same. Extensive simulations confirm similar performance gains by SOQPSK-TG over PCM/FM when the modulations are coupled with CC2 and this is shown in Figs. 4(a) and 4(b). In addition to coding gain performance, SOQPSK-TG has the advantage of having twice the bandwidth efficiency of PCM/FM, which makes it the better choice of the two for coded aeronautical telemetry.

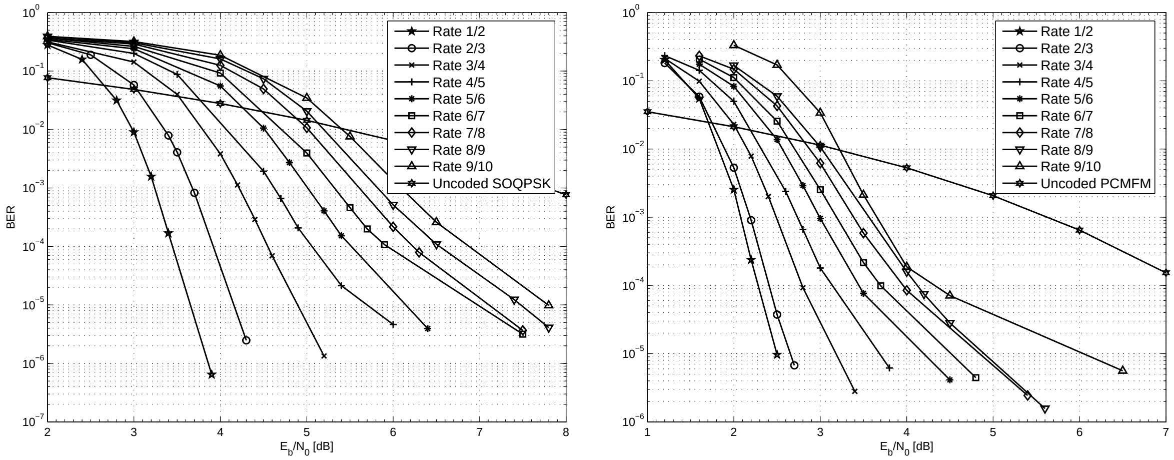
B. COHERENT DEMODULATION vs. NONCOHERENT DEMODULATION

The BER performance comparison between coherent and noncoherent demodulation of SCCC-CPM is shown in Figs. 2, 4 and Figs. 3, 5, respectively. The *forgetting factor* parameter of the noncoherent demodulator is selected as 0.875 [17]. For instance consider Figs. 2(a) and 3(a), from these figures we see that noncoherent demodulation of SCCC-SOQPSK-TG is about 1 dB worse than coherent demodulation of SCCC-SOQPSK-TG at BER $= 10^{-5}$. We can see this clearly if we look at rate 1/2 SCCC-SOQPSK-TG with coherent demodulation which yields a coding gain of 8.0 dB, which is 1 dB more than the gain produced by a similar system with noncoherent demodulation. Similar differences in performance are also evident with other SCC-CPMs considered and also at higher code rates. This is a good tradeoff between complexity and performance since noncoherent demodulation reduces the synchronization complexity of the receiver.

C. CC1 vs. CC2

Figs. 2(a) and 4(a) show the BER performance of SCCC-SOQPSK-TG with CC1 and CC2, respectively. At code rate 1/2, the performances of the two SCCCs with SOQPSK-TG are similar, where both codes yield a gain of 8.0 dB. As the code rate increases, the coding gain with CC2 becomes larger than the coding gain with CC1. For instance, at code rate 4/5, CC2 yields a coding gain of 6.6 dB, which is 0.9 dB better than the gain produced by CC1 with SOQPSK-TG.

Similarly, the BER performance of SCCC-PCM/FM with CC1 and CC2 is shown in Figs. 2(b) and 4(b), respectively. From these figures we see that at lower code rates, given PCM/FM, CC1 performs slightly better than CC2. For instance, at code rate 1/2, CC1 with PCM/FM yields a gain of 6.6 dB, which is 0.4 dB better than the gain produced by CC2 with PCM/FM. However, as observed with SOQPSK-TG, the relative performance of CC2 with respect to CC1 improves as the code rate increases.



(a) Simulation results for CC1 with noncoherent SOQPSK-TG.

(b) Simulation results for CC1 with noncoherent PCM/FM.

Figure 3: Simulation Results for CC1 with CPM under NonCoherent Demodulation.

D. SCCCs vs. TPCs

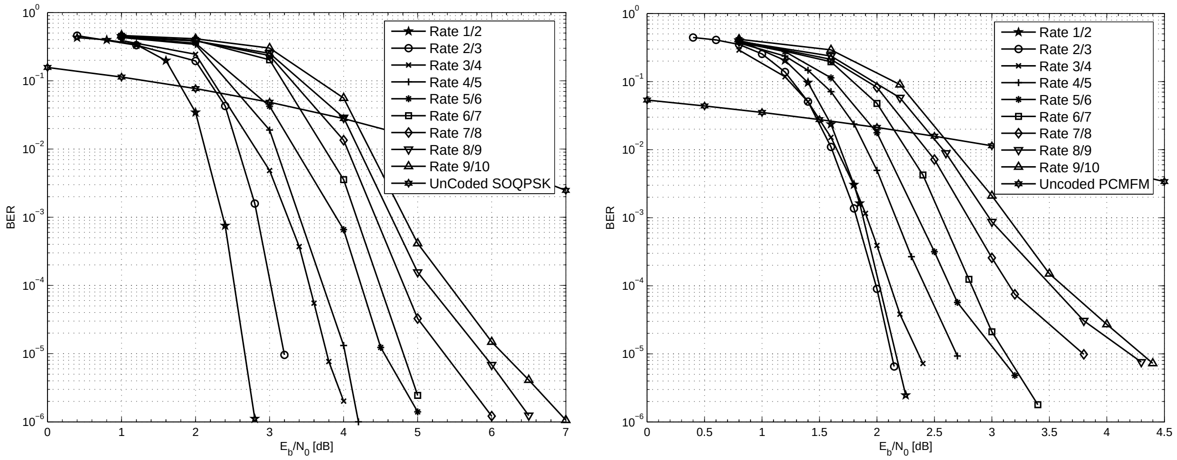
In general, the performance of SCCCs with SOQPSK-TG and PCM/FM is better than the performance of TPC with these modulations. As shown in Figs. 2(a) and 4(a), SCCCs with SOQPSK-TG at code rate 4/5 realize coding gains of 5.7 dB and 6.6 dB, respectively. A code rate 0.7932 TPC with SOQPSK-TG, shown in [9], yields a coding gain of only 3.7 dB, which underlines a superior performance shown by SCCC-SOQPSK-TG considered here.

Similarly, the performance of SCCCs with PCM/FM in Figs. 2(b) and 4(b) at code rate 4/5 shows a coding gain of 5.9 dB and 5.7 dB, respectively. This is better performance than rate 0.7932 TPC with PCM/FM, which yields a gain of only 3.4 dB, as shown in [9]. The above results clearly indicate that SCCCs are the better choice than TPC in terms of coding gains for the CPMs considered.

CONCLUSION

This paper considered SCC-CPM systems with iterative demodulation and decoding, where the inner modulation was SOQPSK-TG or PCM/FM. These systems consist of SISO algorithms each for the inner modulation and the outer code, which pass soft probabilities to each other in an iterative manner.

Based on our numerical simulation results, we conclude that SOQPSK-TG yields larger coding gains than PCM/FM. This advantage comes in addition to the fact that SOQPSK-TG has twice the spectrum efficiency of PCM/FM. Our numerical results also demonstrated that noncoherent demodulation results in a small loss on the order of 1 dB, which is an attractive trade for simplified synchronization requirements at the receiver. We also compared the relative performances of two SCCCs with each other and with a turbo product code. We found that the convolutional codes resulted in larger coding gains than the turbo product code. We also found that the relative ranking between the convolutional codes was a function



(a) Simulation results for CC2 with coherent SOQPSK-TG.

(b) Simulation results for CC2 with coherent PCM/FM.

Figure 4: Simulation Results for CC2 with CPM under Coherent Demodulation.

of code rate. In general, we found that SCC-CPM using the modulations currently used in aeronautical telemetry resulted in large coding gains and could be implemented with practical levels of complexity.

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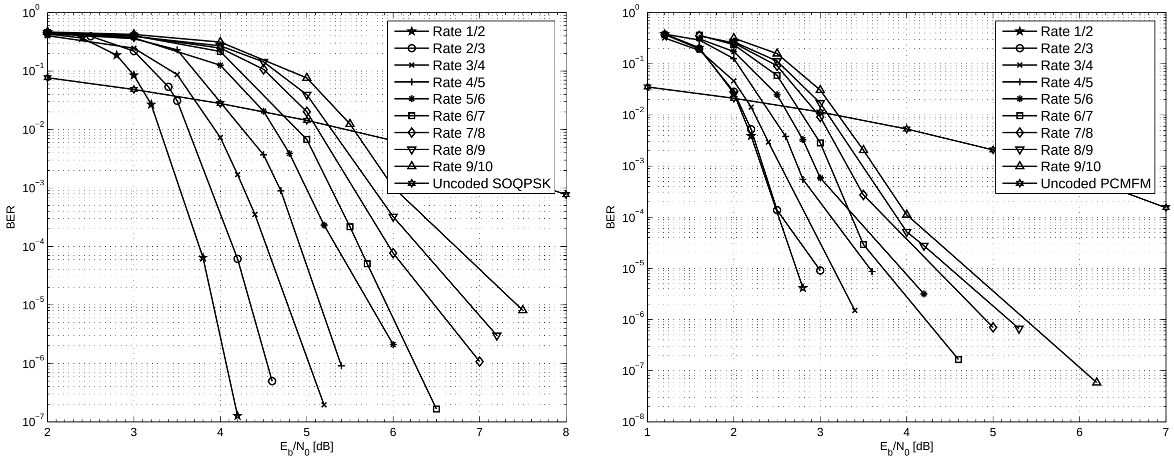

(a) Simulation results for CC2 with noncoherent SOQPSK-TG.

(b) Simulation results for CC2 with noncoherent PCM/FM.

Figure 5: Simulation Results for CC2 with CPM under NonCoherent Demodulation.

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