EVALUATION OF REDUNDANCY REDUCTION ALGORITHMS

DR. R. S. SIMPSON
Department of Electrical Engineering
University of Alabama

1.0 Introduction The concept of redundancy reduction in PM telemetry data prior to transmission is an attractive idea which has received much attention in the literature. The potential gain is in the conservation of transmitter power and spectral bandwidth through more effective utilization of the available channel capacity.

In this paper an important group of redundancy reduction algorithms is classified, which yields a basis for establishing a nomenclature to describe the algorithms. Also, it is shown that several algorithms can cause the unnecessary transmission of samples because of variations in the data which are small with respect to the tolerance range. Finally, the algorithms are compared relative to performance by computer simulation using both real and synthesized data and relative to implementation difficulty.

2.0 Sources of Redundancy and Errors Associated with Redundancy Reduction
Redundancy in sampled data is chiefly the result of sampling at a higher rate than is necessary to reconstruct the time domain waveform of a signal to the accuracy required. The oversampling is caused by two factors. First, there may be insufficient knowledge of the signal spectrum to allow a precise a priori determination of the sampling rate, which frequently results in the rate being overestimated. Second, the necessary sampling rate is often a function of time, because data signals can be non-stationary. Hence, a sampling rate adequate to prevent undersampling will at times cause oversampling. Since redundancy reduction systems simply remove the unnecessary samples due to oversampling, in a broad sense such systems result in adaptive sampling.

Redundancy can also result if the data user is supplied with a signal waveform when he needs only information concerning certain features of the waveform or spectrum. In such a case the problem is not one of complete signal reconstruction from samples, but only the extraction of needed information. Additionally, complete signal reconstruction is probably unwise if the sampled signal is perturbed by noise. In this case it is desirable to reduce both the redundancy and noise content of the sampled signal and to reconstruct only the unperturbed signal.

In a strict sense the removal of redundant samples will result in no degradation of the reconstructed waveform. However, in reality redundancy cannot be precisely measured; hence the approximate methods, which are used to determine if a particular sample is redundant and to be discarded, can result in the discarding of information. Therefore redundancy reduction can produce an error between the reconstructed and original waveforms. Further, if an approximate interpolation technique, e.g., linear interpolation, is used, there is an additional contribution to the error. In this paper the RMS value of the total error between the linearly interpolated non-redundant samples and the original waveform will be used as a measure of the effect of redundancy reduction on data. No consideration is given to the situation where the data user does not require the original signal waveform.

3.0 Classification of Redundancy Reduction Algorithms A significant amount of effort has been expended in the investigations of redundancy reduction systems. However, a comparison of the results is often difficult because, among other factors, there are differences in the nomenclature used to describe the algorithms. This problem can be solved by establishing a consistent method of classifying the algorithms and deriving a nomenclature from the classification.

One particular classification which is applicable to a large class of algorithms consists of the specification of the order, the type corridor, and the particular transmitted sample. A nomenclature results from using a letter to denote each of the three categories of this classification.

The order of an algorithm denotes the order of the derivative used to predict subsequent samples. In this paper only zero-order (Z) and first-order (F) algorithms will be considered. The corridor, which can be fixed (F) or variable M, refers to how the prediction of subsequent samples is made. Finally, the transmitted sample determines what information is transmitted when a non-redundant sample occurs. The sample can be the non-redundant sample (N), the sample preceding the non-redundant sample (P), or an adjusted value for the preceding sample (A).

The classification is best understood by considering the specific algorithms included in the Appendix. For each algorithm the rules for redundancy reduction and first-order (linear) interpolation are given, and the maximum peak error is stated.

4.0 Qualitative Analysis of the Effect of Low-level Noise The performance of several first-order algorithms can be seriously affected if the data amplitude is relatively constant but perturbed by low-level noise or if there are small variations in the data with respect to the tolerance range.

FFN and FFP are affected as illustrated in Fig. 1. Sample 2 is outside of the first corridor; therefore, Samples I and 2 are used to establish a second corridor. Because of the slight difference in the amplitude of these samples, Sample 7 is outside of the second corridor, but plainly there is no significant change in the data relative to the tolerance range. Unless a subsequent corridor happens to be horizontal, unnecessary samples will continue to be produced, and the algorithm will, in effect, oscillate.

FFA is affected in a different but similar way as indicated in Fig. 2. Since the corridors are constructed using adjusted samples which exceed the variation in the actual samples, the oscillation can be more severe than for FFN and FFP.

In Fig. 3 the effect on FVA of small variations in the data samples is examined. Region I is the region in which the ends of the tolerance ranges of the samples fall. If the adjusted sample used to start a corridor lies in Region I, e.g., Sample A, an unnecessary transmitted sample will be produced in a manner similar to that for the other algorithms. However, if the adjusted sample used to start a corridor lies in or on the boundaries of Region II, e.g., Samples B or C, no unnecessary samples will be produced. If several subsequent adjusted samples lie in Region 1, oscillation will be produced and will continue until an adjusted sample falls in Region II. The effect of small data variations on FVA is thought not to be serious because of the small probability of obtaining adjusted samples in Region I and the large probability of obtaining such samples in Region II when the variations are small. Larger variations can cause difficulty, but in such cases it is not clear that the variations are not significant.

5.0 Evaluation of Algorithms Using Computer Simulation Redundancy reduction algorithms can be studied by digital computer simulation and by processing either synthesized data or typical flight data. An evaluation can be made by plotting the compression ratio (the total number of samples divided by the number of non-redundant samples) as a function of the RMS error (previously defined).

The RMS error is chosen as a comparison criterion because it results in a statement of the average performance of an algorithm. However, peak error, if desired, can be calculated using the relationships in the Appendix between peak error and tolerance range and realizing that tolerance range is a variable along the compression-ratio versus RMS error curve.

In Fig. 4 eight algorithms are compared using synthesized data consisting of band-limited white noise having an attenuation of 30 db/octave beyond the cut-of f frequency. The data is sampled at a rate of ten times the cut-off frequency, and the tolerance ranges used are 1, 4, 7, and 10 percent of full scale. Each curve consists of straight line segments connecting the four points corresponding to the four tolerance ranges. Observe that FFA exhibits the worst performance, and FFN and FFP do not rank very well among the other algorithms. Also, FVA performs relatively well, almost as well as FVP. For large tolerance ranges these observations can be partially explained by the analysis in Section 4.0 of small variations relative to the tolerance range. It is concluded that ZVA probably exhibits the best average performance across the range of compression ratios.

In Figs. 5 and 6 the four best algorithms from Fig. 4, ZVA, FVA, FVP, and ZFN, are compared using Saturn flight data from the SA-10 vehicle. The tolerance ranges are 1, 4, 7, and 10 percent of full scale. The comparison in Fig. 5 is based upon 42 pressure measurements sampled at a rate of 4 samples per second during the entire flight. ZVA clearly gives the best performance. In Fig. 6 the comparison is made using the same data, except that only the ignition period is considered. Again, ZVA ranks highest in performance but by a smaller margin.

From these two comparisons of the algorithms using both generalized and real data, it is concluded that ZVA exhibits the best performance.

6.0 Relative Difficulty of Implementation Consideration must be given to the relative difficulty of the implementation problems associated with the various algorithms. While the primary emphasis in this paper is on performance, the following information is helpful in evaluating the algorithms.

In Fig. 7 four of the most promising processes are compared with respect to the number of logic modules, the basic clock rate, and the number of reference memory bits necessary for an X-channel system operating with a word rate of F words per second. To give additional insight into the problem, the number of logic modules necessary for common functions such as gating and buffering is listed separately from those associated directly with the particular process.

From this comparison ZVA is seen to be more difficult to implement than ZFN, as would be expected, but easier to implement than FVA or FVP. Thus ZVA appears to be a compromise with respect to implementation difficulty.

7.0 Conclusion A nomenclature for a class of redundancy reduction algorithms has been proposed which would greatly simplify reference to the algorithms by different investigators. Also, the propensity of FFN, FFP, FFA, and FVA to oscillate has been investigated. FVA has the least tendency to sustain such oscillation, and it may be a usable algorithm, as indicated by the evaluation using computer simulation.

The most promising algorithm is ZVA. Computer simulation has shown that the performance is excellent and implementation difficulty is only moderately greater than that for the simplest algorithms.

APPENDIX

A. Zero Order, Fixed Corridor, Non-redundant Sample Transmitted (ZFN). Rules for redundancy reduction:

a) The occurrence of a non-redundant sample requires that a new corridor be established. This is accomplished by drawing lines of zero slope through the end points of the tolerance range placed around the non-redundant sample.
b) If a subsequent sample lies inside the corridor, it is a redundant sample and is discarded.
c) If a subsequent sample lies outside the corridor, it is a non-redundant sample and is transmitted.

Rules for interpolation:

a) If there are no omitted samples between transmitted samples, the latter are connected with a straight line.
b) b) If there are intermediate omitted samples, a straight line of zero slope is drawn from the first transmitted sample to the time of occurrence preceding the second transmitted sample. The end of this line is then connected to the second transmitted sample with a second line.
c) c) The maximum peak error between raw data and interpolated, reduced data is plus or minus one-half tolerance range.

These rules are demonstrated in Fig. A-1.

B. Zero Order, Variable Corridor, Preceding Sample Transmitted (ZVP). Rules for redundancy reduction:

a) The occurrence of a non-redundant sample requires that a new corridor be established. This is accomplished by drawing lines of zero slope through the end points of the tolerance range placed around the non-redundant sample.
b) If a subsequent sample lies inside the corridor, it is a redundant sample and is discarded. Each redundant sample modifies the corridor extended to the next sample in the following way. The new corridor consists of that part of the previous corridor which is overlapped by the tolerance range placed about the redundant sample.
c) If a subsequent sample lies outside the corridor, it is a non-redundant sample, but it is not transmitted. Rather the preceding sample is transmitted.

Rules for interpolation:

a) If there are no omitted samples between two transmitted samples, the latter are connected with a straight line.
b) If there are intermediate omitted samples, a straight line of zero slope is drawn from the second transmitted sample back to the time of occurrence of the first sample following the first transmitted sample. The end of this line is then connected to the first transmitted sample with a second line.
c) The maximum peak error between the raw data and the interpolated,

Processed data is plus or minus one-half tolerance range.

These rules are demonstrated in Fig. A-2.

C. Zero Order, Variable Corridor, Adjusted Preceding Sample Transmitted (ZVA).

Rules for redundancy reduction:

a) The occurrence of a non-redundant sample requires that a new corridor be established. This is accomplished by drawing lines of zero slope through the end points of the tolerance range placed around the non-redundant sample.
b) The requirement for a subsequent sample to be redundant and thus discarded is that one end of the tolerance range placed about the sample fall within the corridor. Note that the sample itself is not required to be within the corridor. Each redundant sample modifies the corridor extended to the next sample in the following way. The new corridor consists of that part of the previous corridor which is overlapped by the tolerance range placed about the redundant sample.
c) If the tolerance range placed about a sample does not overlap the corridor, the sample is non-redundant but is not transmitted. Rather, the mid-point of the corridor used to analyze this sample, actually the predicted value of the sample, is transmitted for the preceding sample. Hence, the transmitted sample is not a real data sample but an adjusted sample.

Rules for interpolation:

a) If there are no omitted samples between two transmitted samples, the latter are connected with a straight line.
b) b) If there are intermediate omitted samples, a straight line of zero slope is drawn from the second transmitted sample back to the time of occurrence of the first sample following the first transmitted sample. The end of this line is then connected to the first transmitted sample.

c) The maximum peak error between the raw data and the interpolated, processed data is plus or minus one-half tolerance range.

These rules are demonstrated in Fig. A-3.

D. First Order, Fixed Corridor, Non-redundant Sample Transmitted (FFN). Rules for redundancy reduction:

a) The occurrence of a non-redundant sample requires that a new corridor be established. This is accomplished by placing tolerance ranges around the non-redundant sample and the previous sample and by drawing two straight lines, one through the upper ends of the tolerance ranges and the other through the lower ends.
b) If a subsequent sample lies inside this corridor, it is a redundant sample.
c) If a subsequent sample lies outside the corridor, it is a non-redundant sample and is transmitted.

Rules for interpolation:

a) Transmitted samples are connected by straight lines.
b) The maximum peak error between the raw data and the interpolated, processed data approaches plus or minus full scale.

These rules are illustrated in Fig. A-4.

E. First Order, **F**ixed Corridor, Preceding Sample Transmitted (FFP). Rules for redundancy reduction:

a) The occurrence of a non-redundant sample requires that a new corridor be established. This is accomplished by placing tolerance ranges around the non-redundant sample and the previous sample and by drawing two straight lines, one through the upper ends of the tolerance ranges and the other through the lower ends.
b) If a subsequent sample lies inside this corridor, it is a redundant sample.
c) If a subsequent sample lies outside the corridor, it is a non-redundant sample, but it is not transmitted. Rather the preceding sample is transmitted.

Rules for interpolation:

a) Transmitted samples are connected by straight lines.
b) The maximum peak error between the raw data and the interpolated analyzed data approaches plus or minus one tolerance range.

These rules are illustrated in Fig. A-5.

F. First Order, Fixed Corridor, Adjusted Preceding Sample Transmitted (FFA).

Rules for redundancy reduction:

a) The occurrence of a non-redundant sample requires that a new corridor be established. This is accomplished by placing tolerance ranges around the non-redundant sample and the adjusted preceding sample, which is transmitted, and by drawing two straight lines, one through the upper ends of the tolerance ranges and the other through the lower ends.
b) If a subsequent sample lies inside this corridor, it is a redundant sample and is discarded.
c) If a subsequent sample lies outside the corridor, it is a non-redundant sample, but it is not transmitted. Rather, an adjusted value for the previous sample is transmitted. This value is the mid-point of the corridor for the previous sample, which is the predicted value for that sample.

Rules for interpolation:

a) Transmitted samples are connected by straight lines.
b) The maximum peak error between the raw data and the interpolated, processed data is plus or minus one-half tolerance range.

These rules are illustrated in Fig. A-6.

G. First Order, Variable Corridor, Preceding Sample Transmitted (FVP).

Rules for redundancy reduction:

a) The occurrence of a non-redundant sample requires that a new corridor be established. This is accomplished by placing a tolerance range around the non-redundant sample and drawing two straight lines. The first passes through the previous sample and the upper end of the tolerance range; the second passes through the same sample and the lower end of the tolerance range.
b) If a subsequent sample lies inside this corridor, it is a redundant sample and is discarded. Each redundant sample modifies the corridor extended to the next sample in the following way. If a boundary line of the corridor does not pass through the tolerance range placed around the redundant sample, a new boundary is established which passes through the nearest end point of the tolerance range. Either or both boundary lines can be affected.

c) A sample falling outside the corridor is a non-redundant sample, but it is not transmitted. Rather the preceding sample is transmitted.

Rules for interpolation:

a) Transmitted samples are connected by straight lines.
b) The maximum peak error between the raw data and the interpolated analyzed data is plus or minus one-half tolerance range.

These rules are illustrated in Fig. A-7.

H. First Order, Variable Corridor, Adjusted Preceding Sample Transmitted (FVA).

Rules for redundancy reduction:

a) The occurrence of a non-redundant sample requires that a new corridor be established. This is accomplished by placing a tolerance range around the non-redundant sample and drawing two straight lines. The first passes through the adjusted preceding sample, which is transmitted, and the upper end of the tolerance range; the second passes through the same sample and the lower end of the tolerance range.
b) The requirement for a subsequent sample to be redundant and discarded is that one end of the tolerance range placed around the sample fall within the corridor. Note that the sample itself is not required to be within the corridor. Each redundant sample modifies the corridor extended to the next sample in the following way. If one of the boundary lines of the corridor does not pass through the tolerance range placed around the redundant sample, a new boundary is established which passes through the nearest end point of the tolerance range.
c) If the tolerance range placed about a sample does not overlap the corridor, the sample is non-redundant but is not transmitted. Rather an adjusted value for the preceding sample is transmitted, which is simply the predicted value for the previous sample. This value is the mid-point of the corridor at the time corresponding to the previous sample, where the corridor is the one used for analysis of the nonredundant sample.

Rules for interpolation:

a) Transmitted samples are connected by straight lines.
b) The maximum peak error between the raw data and the interpolated, processed data is plus or minus one-half tolerance range.

These rules are illustrated in Fig. A-8.

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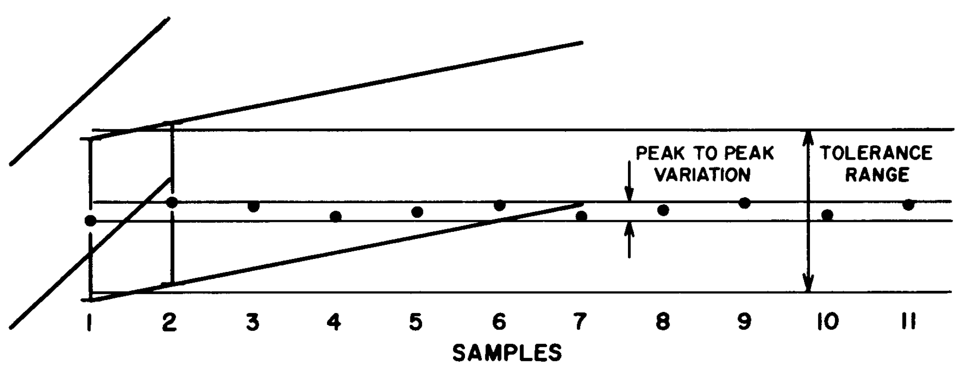

Fig. 1 - Effect of Small Variations on FFN and FFP.

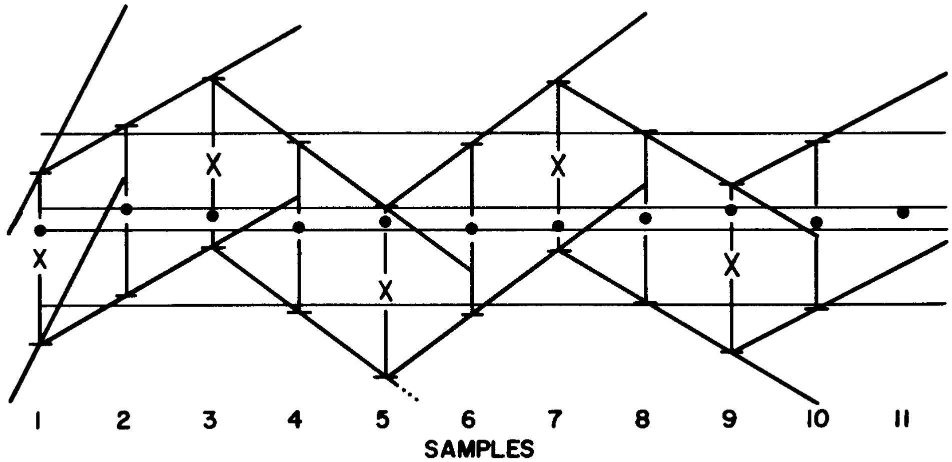

Fig. 2 - Effect of Small Variations on FFA.

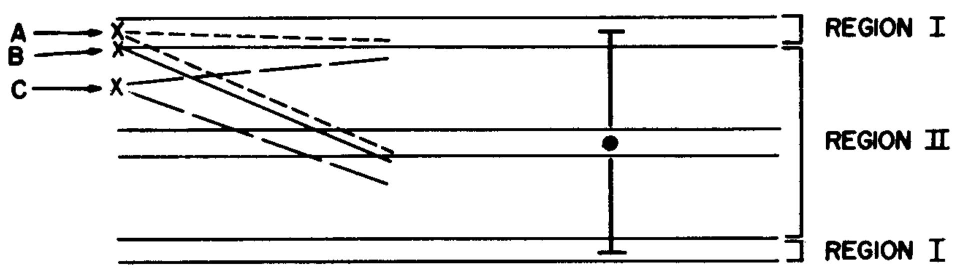

Fig. 3 - Effect of Small Variations on FVA.

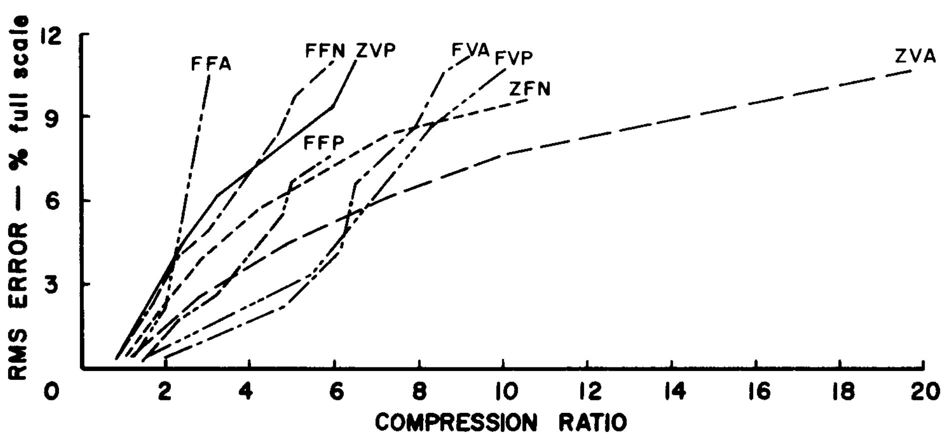

Fig. 4 - Evaluation of Algorithms Using Synthesized Data.

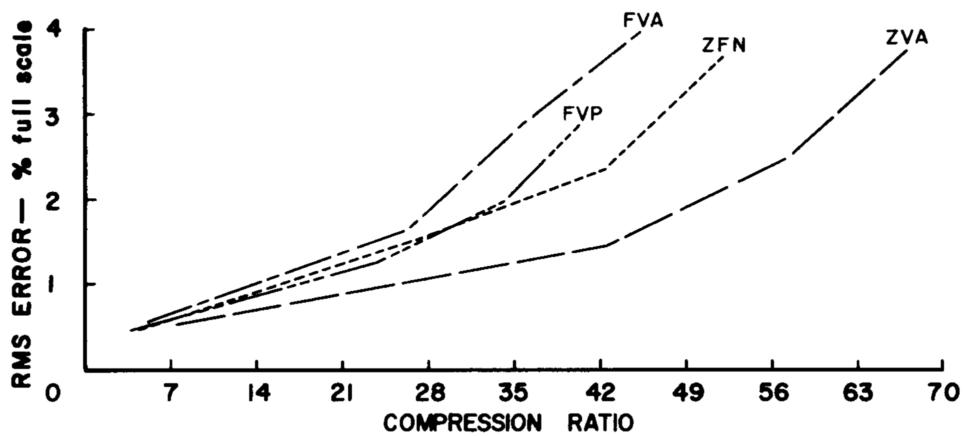

Fig. 5 - Evaluation of Algorithms During Entire Flight of SA-10 Using Pressure Measurement Data.

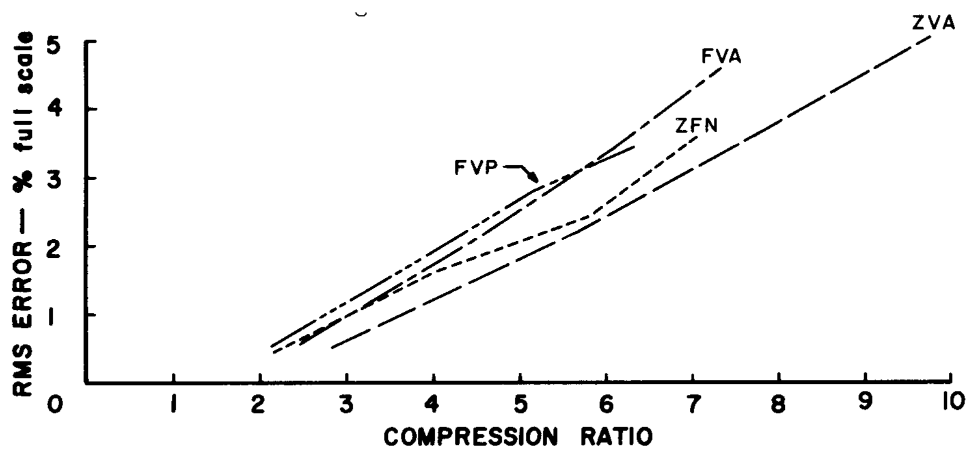

Fig. 6 - Evaluation of Algorithms During Ignition of SA-10 Using Pressure Measurement Data.

		ZFN	ZVA	FVA	FVP
Number Logic Modules	Common Circuitry	N	N	N	N
	Particular Circuitry	.3N	.6N	.9N	N
Basic Clock Rate		20F	150F	200F	200F
Number Reference Memory Bits		15X	20X	40X	50X

Fig. 7 - Number of Logic Modules Required for Implementation of X-channel System with Word Rate of F.

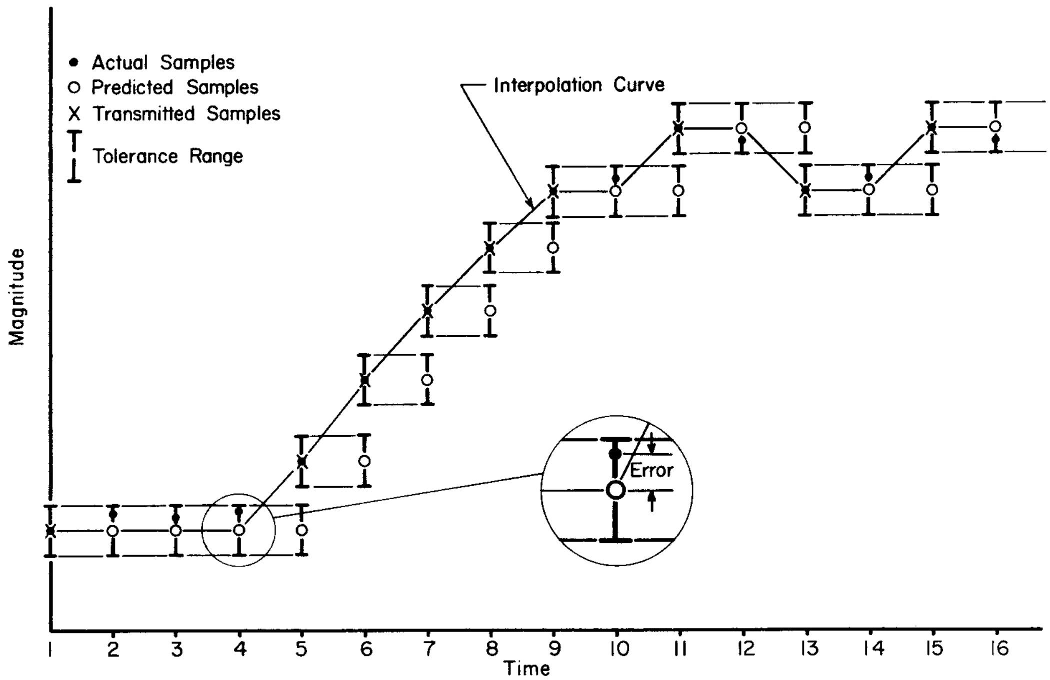

Fig. A-1 - Zero Order, Fixed Corridor, Non-redundant Sample Transmitted (ZFN).

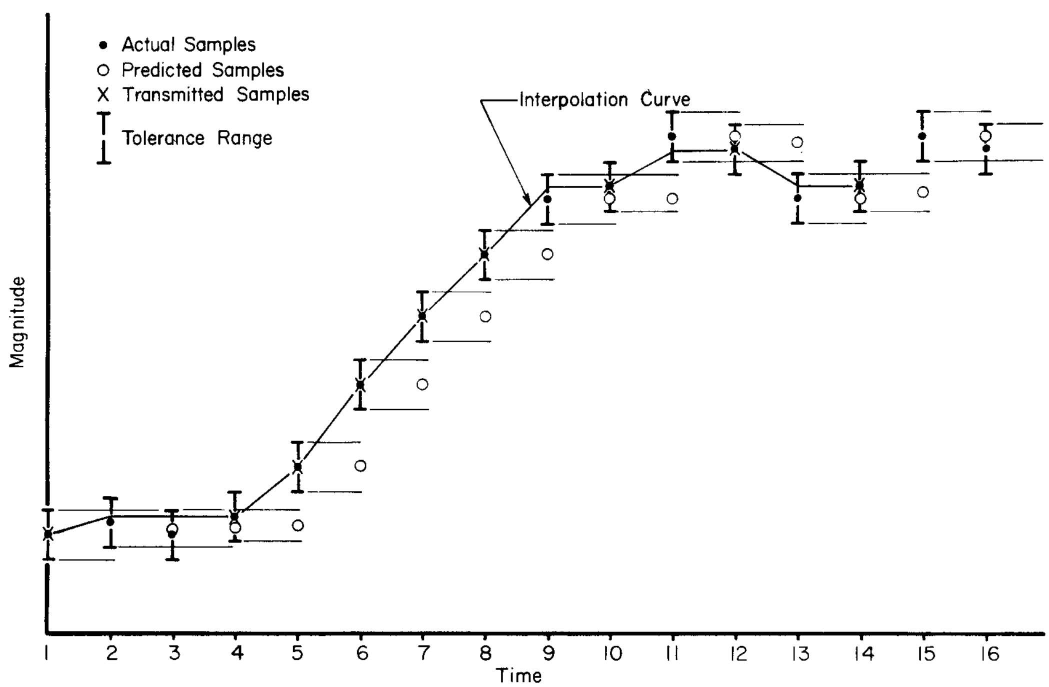

Fig. A-2 - Zero Order, Variable Corridor, Preceding Sample Transmitted (ZVP).

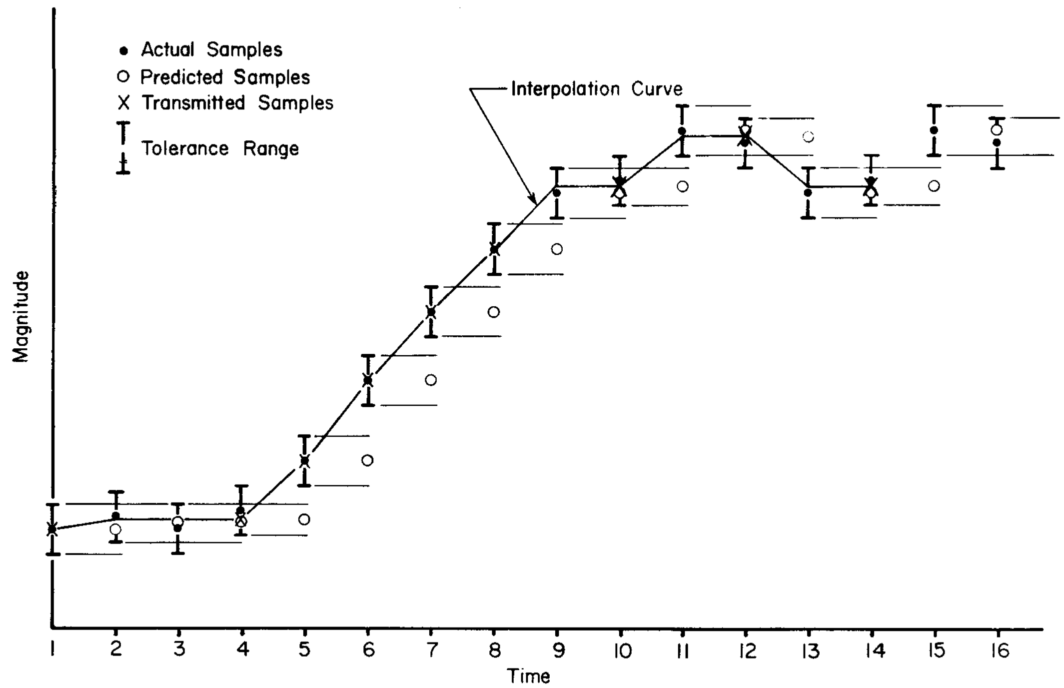

Fig. A-3 - Zero Order, Variable Corridor, Adjusted Preceding Sample Transmitted (ZVA).

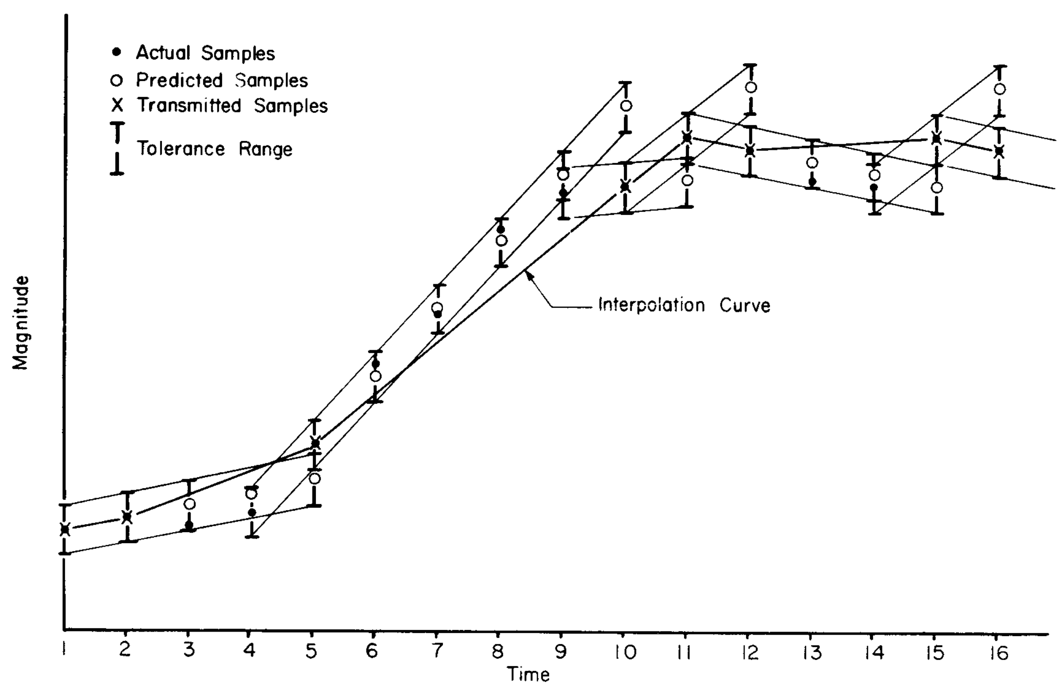

Fig. A-4 - First Order, Fixed Corridor, Non-redundant Sample Transmitted (FFN).

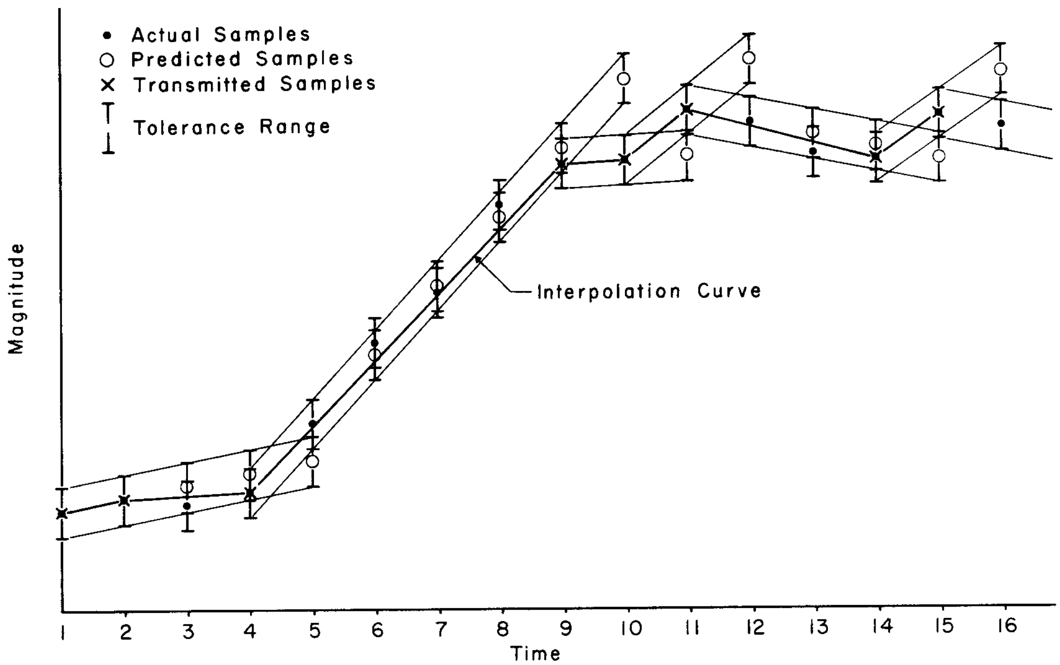

Fig. A-5 - First Order, Fixed Corridor, Preceding Sample Transmitted (FFP).

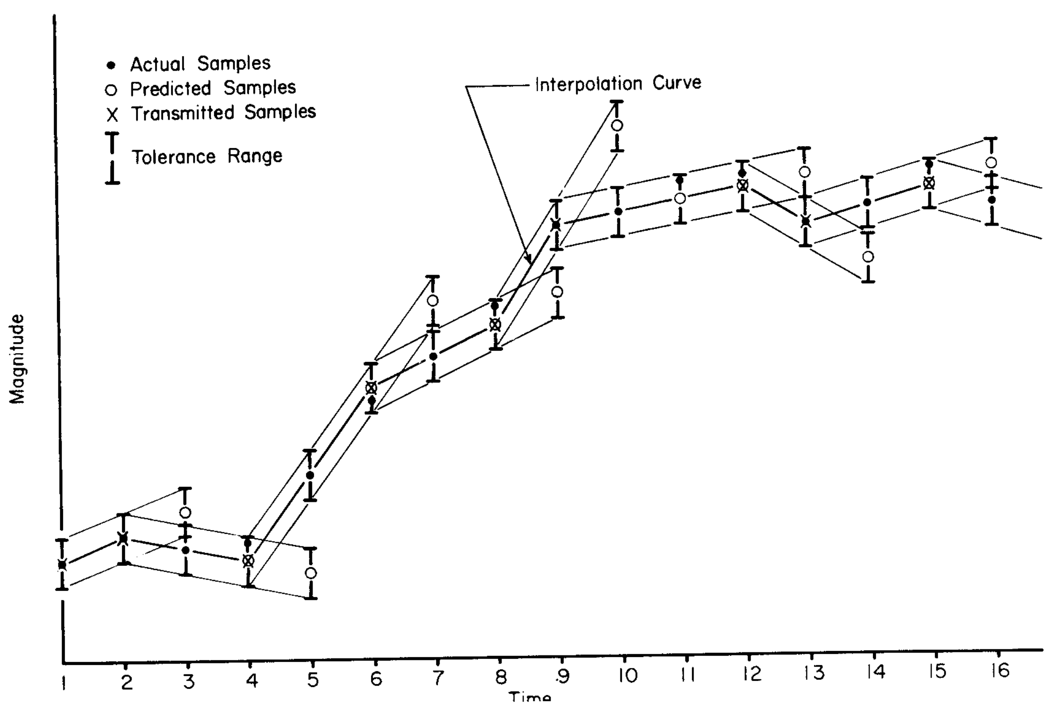

Fig. A-6 - First Order, Fixed Corridor, Adjusted Preceding Sample Transmitted (FFA).

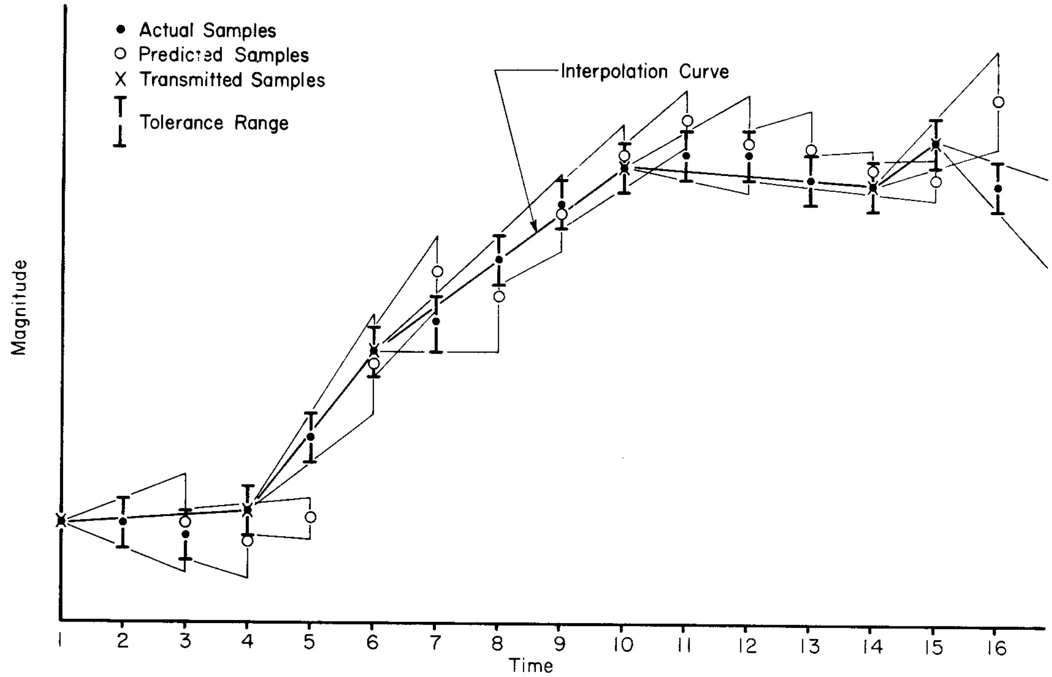

Fig. A-7 - First Order, Variable Corridor, Preceding Sample Transmitted (FVP).

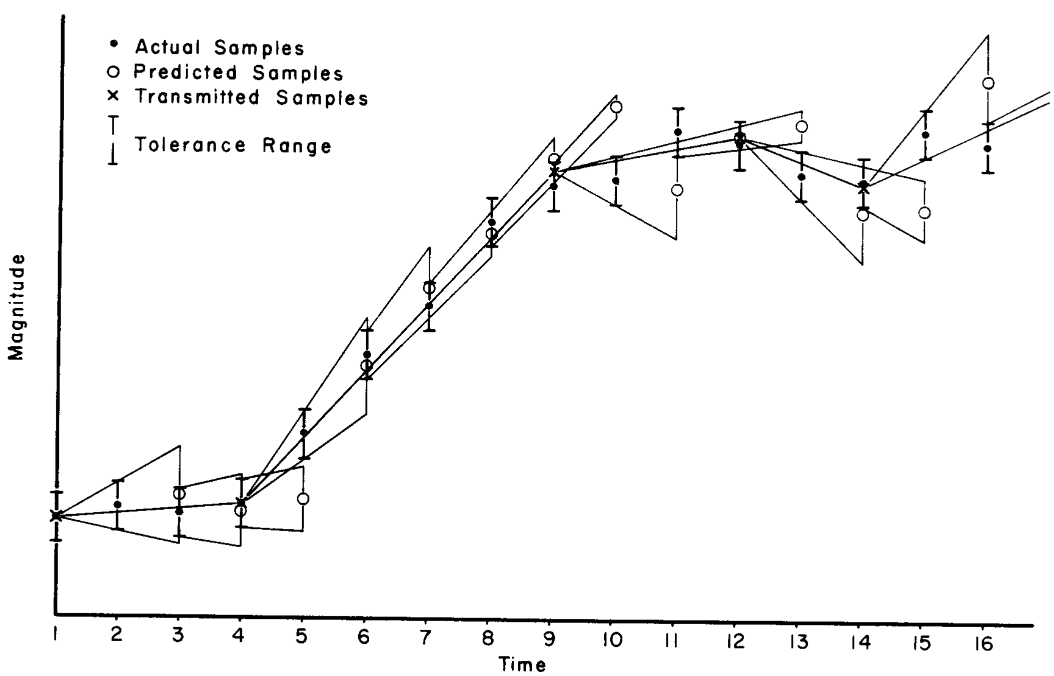

Fig. A-8 - First Order, Variable Corridor, Adjusted Preceding Sample Transmitted (FVA).