# THE DETECTION OF PCM/FM[1]

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**Summary** The calculation of the error rate resulting from the detection of PCM/FM is described. Emphasis is placed on characterizing the effect of the FM demodulator (FMD).

The works of Schilling(l), Salz(2), and Klapper(3), are summarized and extended to yield error rate expressions when using an FM discriminator, Phase Locked Loop, or Frequency Demodulator Using Feedback (FCF). The "Integrate-and-Dump" and the Filter Detector are compared.

The results indicate that in certain regions the error rate is due primarily to "smooth noise", and the FM demodulator followed by a PCM detector yields an error rate comparable to that obtained with a Matched-Filter (MF) detector. In other regions the error rate is shown to be due primarily to the spikes present at the FM demodulator output.

The use of "Spike Detection and Correction" is discussed. It is shown that this technique results in reduced error rate in the "spike regions".

**Introduction** The PCM/FM detection system is shown in Fig. 1. The PCM signal is shaped using a premodulation filter, then frequency modulated and transmitted through a channel with additive gaussian noise. The received signal and noise are shaped by a predetection filter. The PCM waveform is recovered using an FM demodulator such as the discriminator, PLL or FCF, and then passed through a PCM Detector. Two PCM detectors are considered in this paper: the integrate and dump filter(2,3) and a gaussian-shaped filter(1) detector.

The FM demodulator, which is the basic element of the PCM/FM detection system has until recently been considered as a "linear" demodulator. The output noise power of this device was considered to be normally distributed, and the detector of the demodulated

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KM waveform was often tested using a binary PCM signal embedded in white gaussian noise.

Although the character of the FM noise: smooth and impulsive, was recognized in the early days of FM, an analytic treatment using impulsive noise was not attempted until 1958 when Cohn(4) showed that in addition to the FM noise (non-white gaussian noise) present at the output of an FM discriminator, there exists a train of positive and negative pulses-spikes-which influences the response of the FM discriminator.

Rice (5) characterized these spikes as impulse functions, having an area of $2\pi$ radians, and being poisson distributed (This assumption was later verified experimentally(6)). He then determined the output SNR as a function of the CNR at the input to the discriminator and obtained an analytical expression for the threshold of the FM discriminator. Schilling observed that the output of a PLL and FCF consisted of smooth noise and spike noise, and proposed that Rice's noise analysis be extended to the calculation of error rates of KM/FM signals (7).

Theoretical and experimental investigations of the PLL, FCF, and the FM discriminator indicate that the noise at the output of each device consists of smooth noise and spike noise. The FM noise is approximately the same in each device; differences are due to the demodulation mechanism of each. The spikes present at the output of each device have the same characteristics, an area of $2\pi$ and a random time occurrence which is poisson distributed. The major difference between the FM demodulators considered, is the expected number of spikes occurring per second. Assuming that the expected number of positive N+, and negative N-, spikes occurring per second are equal, we can write:

$$N+ = N- \sim \frac{1}{2\pi} |e_m(t)|\, e^{-\gamma\, CNR} \tag{1}$$

where

$e_m(t)$ is the PCM waveform. (see Fig.1) which is assumed to be negative when a positive spike occurs and positive when a negative spike occurs[1-2-3-5]

CNR is the input carrier to noise ratio measured at the output of the predetection filter,

and $\gamma$ is a parameter which is dependent on the FM demodulator used. When using the discriminator, $\gamma = 1$, when using the PLL or FCF, N+(=N-) decreases, and hence $\gamma > 1$ (it is here that the PLL and FCF controversy arises; i.e, what value of $\gamma$ should be used; rather than enter this controversy we leave the value of $\gamma$ unspecified except to bound $\gamma$ between 1 & 10).

## 1. Analysis Background

The received PCM/FM signal is corrupted by white gaussian noise, passed through an IF filter having a bandwidth $B_{IF}$, and demodulated using an FM demodulator as shown in Fig 1. The output of the IF filter is

$$e_i(t) = A\cos\left[\omega_0 t + \int^{t} e_m(\lambda)\,d\lambda\right] + x(t)\cos\omega_0 t - y(t)\sin\omega_0 t \tag{2a}$$

where $e_m(t)$ is the PCM signal transmitted) and x(t), y(t) are independent gaussian processes with Zero mean, and noise power, $N_i = B_{IF.}$

The demodulated output $e_o(t)$, is

$$e_o(t) = n_g(t) + n_s(t) \tag{2b}$$

where $n_g(t)$ is the smooth noise:

$$n_g(t) = \frac{d}{dt}\left[\frac{y(t)\cos s(t) - x(t)\sin s(t)}{A}\right] \equiv \frac{d\psi}{dt} \tag{3a}$$

$$s(t) = \int^{t} e_m(\lambda)\,d\lambda \tag{3b}$$

and $n_s(t)$, is the output spike noise,

$$n_s(t) = \sum_{i=1}^{\infty} 2\pi\delta(t-t_i) - \sum_{j=1}^{\infty} 2\pi\delta(t-t_j) \tag{4}$$

The calculation of the probability of an error in the detection of the demodulated signal due to the smooth and spike noise, assumes that at low probability of error (less than $10^{-2}$) the errors caused by each type of noise can be calculated separately. The errors due to the smooth noise is calculated using standard techniques since this noise is essentially gaussian. To utilize the spike concept to calculate the probability of error, two characteristics of the spikes are used extensively: the spike area is $2\pi$ radians, and its duration is much smaller than the bit length. Since the bandwidth of the low pass filter is much narrower than that required to pass a spike, a spike when it occurs, can be represented by an impulse of area $2\pi$ at the demodulator output. Knowing the probability of occurrence of a spike (see Eq.1), the probability of the spike amplitude exceeding the signal at the sampling instant can be calculated.

## 2. Error Rates Due to Smooth Noise

The integrate and dump detector can be considered as a filter with impulse response h(t). Then, the output of the PCM detector at the sampling instant ts, is (neglecting spikes):

$$e_d(t_s) = \int_{-\infty}^{t_s} e_m(\lambda)\,h(t_s-\lambda)\,d\lambda + \int_{-\infty}^{t_s} \frac{d\psi(\lambda)}{d\lambda}\,h(t_s-\lambda)\,d\lambda \tag{5}$$

The probability of error due to the smooth noise is found[2-3] from the formula:

$$P_{eg} \sim \frac{1}{2}\,\mathrm{erfc}\left[\frac{e_{mo}(t_s)}{\sqrt{2N_{go}(t_s)}}\right] \tag{6}$$

where (see Fig 1),

$Ngo(t_s)$ is the noise power (of the “filtered” smooth noise component) at the sampling instant, and $e_{mo}(t_s)$ is the “filtered” PCM waveform.

Using the integrate and dump filter we have (See ref 3-Eq 6, and ref 2 Eq 28; care must be taken when using ref. 2 as the results are intended for multilevel FSK not binary FSK (PCM)):

$$P_{eg} = \frac{1}{2}\,\mathrm{erfc}\sqrt{3(\Delta f)^2 T^2 \frac{E_s}{\eta}} \tag{7}$$

where

$\Delta\omega = |e_m|$ (the PCM waveform is assumed to be a rectangular pulse lasting T seconds),

**Es is the input energy/bit** $= \dfrac{A^2 T}{2}$

and

$\eta$ is the spectral density of the white noise at the input to the predetection filter.

Using a gaussian-shaped filter detector, having the characteristic

$$|H(\omega)|^2 = e^{-0.7(2Tf)^2} \tag{8a}$$

the probability of error due to the smooth noise component was found to be, (see ref 1-Eq 23):

$$P_{eg} \sim \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{8(T\Delta f)^2}{3f_o(T\Delta f)}\cdot\frac{E_s}{\eta}} \tag{8b}$$

where fo(TΔf) is shown in Fig 2.

Care must be taken when comparing Eqs 7 and since Eq 7 assumes a rectangular shaped PCM waveform while Eq 8b assumes that a rectangular premodulation filter, is employed, having a bandwidth $\frac{1}{2T}$ . The shape of the premodulation filter affects $e_{mo}(ts)$. Note, however that the results are similar showing that the type of detector used does not greatly affect Peg.

### 3. Error Rates Due to Spike Noise

The calculation of an error due to the occurence of a negative spike is illustrated in Fig 3, which shows a demodulated positive bit of arbitrary shape and a negative spike occurring $\tau$ seconds after the start of the bit (Fig 3a), the filtered bit (Fig 3b), and the filtered negative spike (Fig 3c). The output of the filter detector, neglecting the smooth noise is (at the sampling instant $t_s$):

$$e_d(t_s) = e_{mo}(t_s) - 2\pi h(t_s - \tau) \quad (9)$$

The filtered spike height exceeds the filtered bit height (at the sampling instant) when

$$\tau + t_1 \le t_s \le \tau + t_2 \quad (10a)$$

If the sampling instant occurs in this bounded region an error occurs. Thus, an error occurs if

$$t_s - t_2 \le \tau \le t_s - t_1 \quad (10b)$$

The probability of a negative spike causing an error is then (using Eq 9)

$$P_{es}\left(\frac{-\text{error}}{e_m > 0}\right) = \int^{T} d\tau \, N_+ \Big[ U(\tau - [t_s - t_2]) - U(\tau - [t_s - t_1]) \Big] \quad (11a)$$

where,U (t) is a unit step function. When the intervals ($t_s$ - $t_2$ ) and ($t_s$ -$t_1$) lie in the bit interval 0 to T, Eq 16a can be written as

$$P_{es}\left(-\text{error}/e_m > 0\right) = \int_{t_s - t_2}^{t_s - t_1} d\tau \, [N_+] \quad (11b)$$

The time intervals $t_s$ - $t_2$ and $t_s$ - $t_1$ are easily calculated for any filtered PCM waveform $e_m(t)$, using Fig 3. N+ is given by Eq 1.

If the positive and negative bits are equiprobable, the total probability of an error due to spikes is also given by Eq 11b.

The above analysis neglects the possibility of a spike occurring during one bit causing an error in the following adjacent bit. This term is, however, directly calculated using the above approach. Since one does not want the spike to cause errors in adjacent bits, output filters which do not "ring" should be used. Thus, a rectangular filter detector should not be employed as it will increase the error rate due to spikes by a significant amount.

## The Gaussian Filter Detector

The error rate caused by spike noise when the PCM signal was premodulation filtered as before was calculated (see ref 1-Eqs 26 and 28) and shown to be:

$$P_{es(\text{due to a single spike})} \sim \frac{4T\Delta f}{\pi} e^{-\gamma \frac{E_s}{\eta}\left(\frac{1}{TB_{IF}}\right)} \begin{cases} \sin\frac{\pi x}{2} & 0\le x\le 1 \\ 1 & x>1 \end{cases} \qquad (12a)$$

where

$$x = \begin{cases} 0.755\left[\ln \frac{3}{\sqrt{2}\,T\Delta f}\right]^{1/2} & T\Delta f \le \frac{3}{\sqrt{2}} \\ 0 & T\Delta f > \frac{3}{\sqrt{2}} \end{cases} \qquad (12b)$$

and

$B_{1F}T$ is proportional to the bandwidth expansion obtained using FM (see Eiq 1).

It is seen (Eq 12b) that if $T\Delta f > 3/\sqrt{2}$ , the spike height cannot exceed the height of the PCM waveform. Hence the presence of a signal spike will not result in an error. However, multiple spikes can cause errors.

It is shown(1) that spikes cause the majority of the errors when

$$0.37 < T\Delta f < \frac{3}{\sqrt{2}} \qquad (13a)$$

while the smooth-noise causes the majority of the errors when

$$T\Delta f < 0.37 \qquad (13b)$$

## The Integrate and Dump Filter Detector

When using an integrate and dump filter detector Eq 9 becomes:

$$e_d(t_s) = T\Delta\omega - 2\pi T \qquad (14a)$$

Thus, if

$$\Delta f < 1 \qquad (14b)$$

the occurrence of a spike results in an error. If $\Delta f > 1$ a single spike in a bit will not cause an error. Thus, using Eqs 1 and 11b: (2,3)

$$P_{es(\text{due to a single spike})} = T\Delta f\, e^{-\gamma \frac{E_s}{\eta}\left(\frac{1}{TB_{IF}}\right)} \qquad (15)$$

Comparing Eqs 12 and 15 we see that the error rate resulting from the occurrence of a single spike in a bit is not “sensitive” to the type of detector employed.

## 4. The Probability of Error

The probability of error for the FM discriminator ($\gamma = 1$)-integrate and dump detector, is shown in Fig 4 (this is a reproduction of Fig 5 of ref 2). The vertical drops in the probability of error $P_e$, curves do not occur in practice and result from neglecting the smooth noise component of Eq 7.

The probability of error obtained using an FM discriminator-gaussian filter detector, is compared with that obtained using the matched filter detector in Fig 5 (This is a reproduction of Fig 5 of ref 1). Notice that in the region of small T $\Delta$f, the matched filter and FMD display comparable error rate. This is in the smooth noise error region of the discriminator. When the error rate using ttie FMD results principally from spikes, ($0.37 < T \Delta f < 3/\sqrt{2}$ ) the matched filter is seen to have a substantially lower probability of error. The PLL, and FCF which have lower probability of occurrence of a spike than the discriminator ( $\gamma > 1$), yield Drobability of error approaching that of the matched filter in the spike region.[9]

It is seen from this figure that the minimum probability of error using the MF detector occurs when $T \Delta f \simeq 1$. Using the FMD,the minimum $P_e$ occurs when $T \Delta f = ½$

Comparing the results shown in Figs 4 & 5 indicate that the $P_e$ is significantly less in Fig 4. This is not due to the use of an integrate and dump detector, but is a result of the decreased bit energy resulting from premodulation filtering.

## 5. Spike Detection and Correction

Figure 6 shows the output of an FM Discriminator in the smooth noise and spike regions. Sinusoidal modulation is applied to the FM carrier. Note the distinct character of the spike. This character has led us to develop spike detecting systems which are capable of detecting spikes and correcting the data. Digital computer simulations as well as laboratory models using the output of the FM Discriminator and PLL have been constructed. An elementary spike detection system is described in Ref 1. Figure 7 shows that the errors in the spike region can be reduced by a factor of 3 using elementary spike detector.

The digital computer simulation[9] indicates that when using spike correction for digital signals substantial improvement can be obtained over that shown in Fig 7.

**Conclusions** Rice's mathematical model for the smooth noise and spike noise can be used to analyze the response of other FM demodulators.

The results obtained show that a PCM detector should not be tested with a white noise input since for many applications it is the spike noise which results in most of the errors observed. The input noise to the PCM detector consists of smooth noise (which is non-white) and spike noise. Therefore there is no reason to use the integrate and dump filter (which is optimum only for pulses in white gaussian noise).

Anyone who has spent one hour watching the spikes at the output of a PLL, FCF or discriminator will agree that the shape of the spike is distinctive. A great deal of work must be done to determine the optimum spike detector, for here is the key to decreased error rate (as well as low threshold signals).

Acknowledgement

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## References

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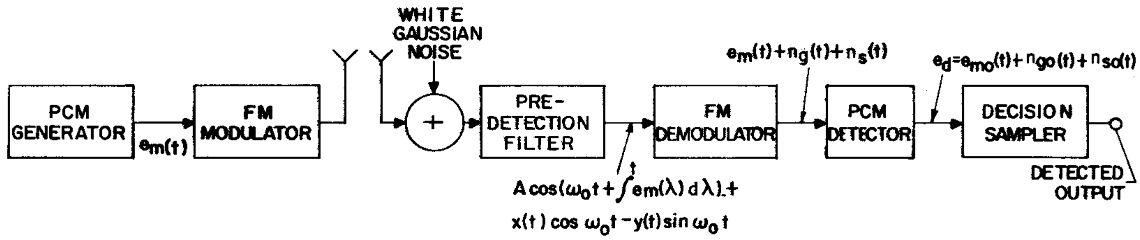
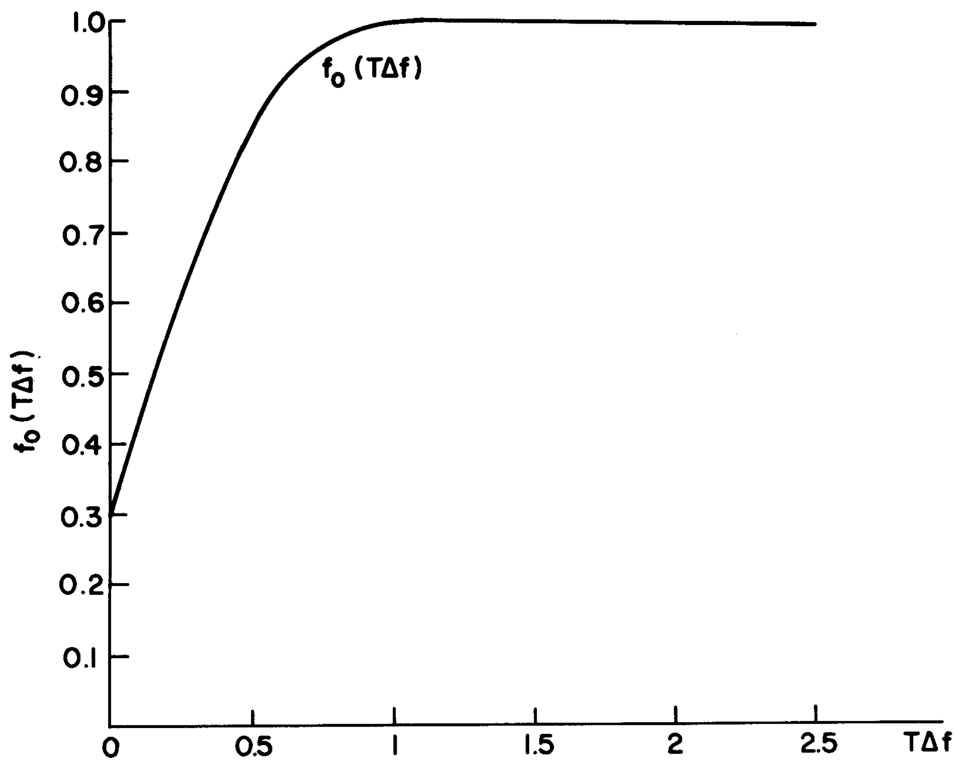
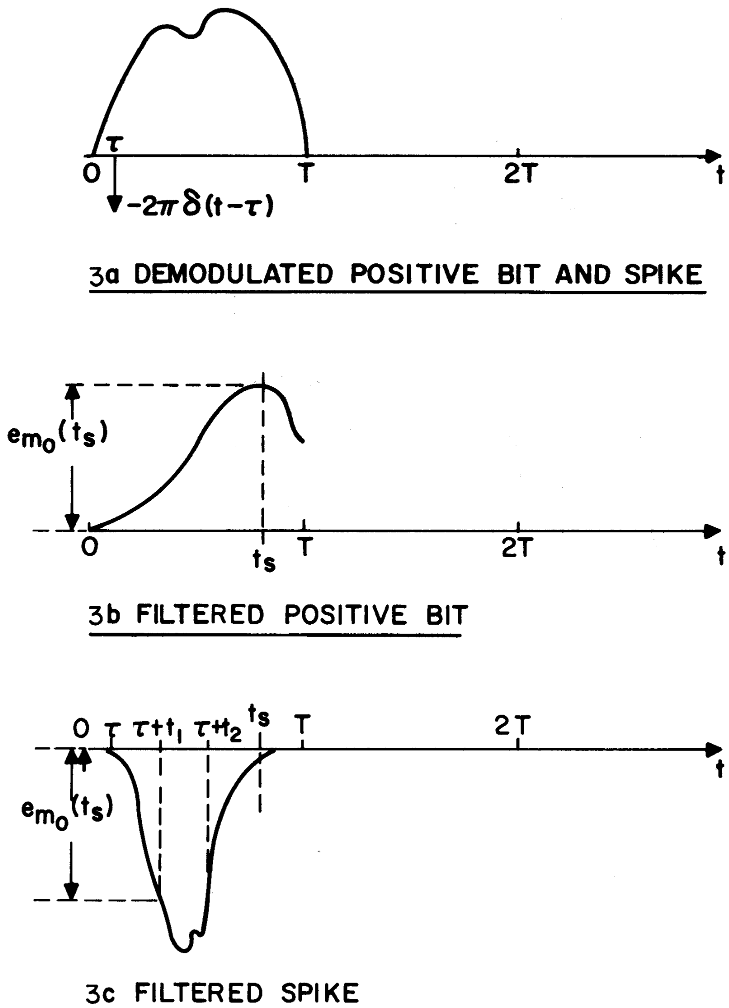


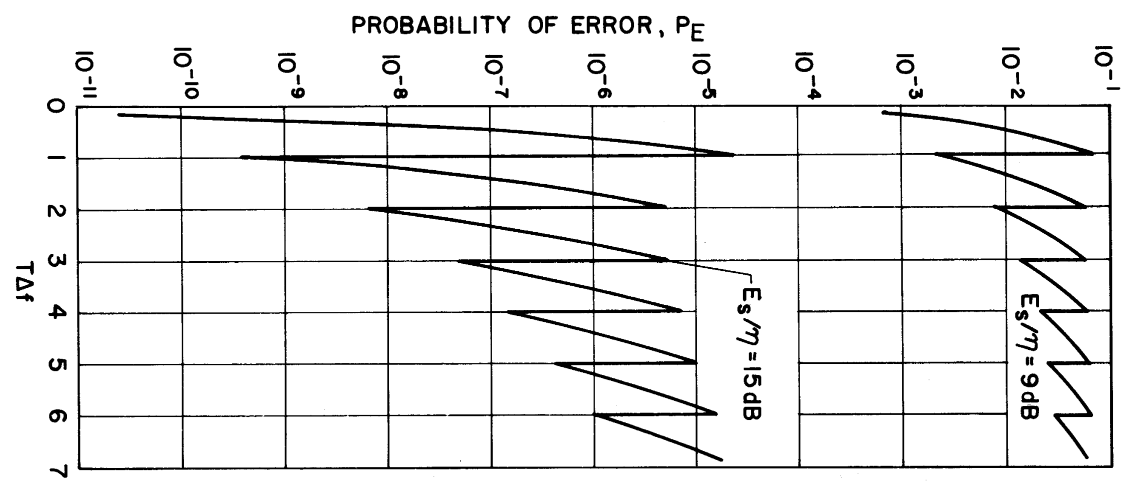

**1. PCM/FM Detection**



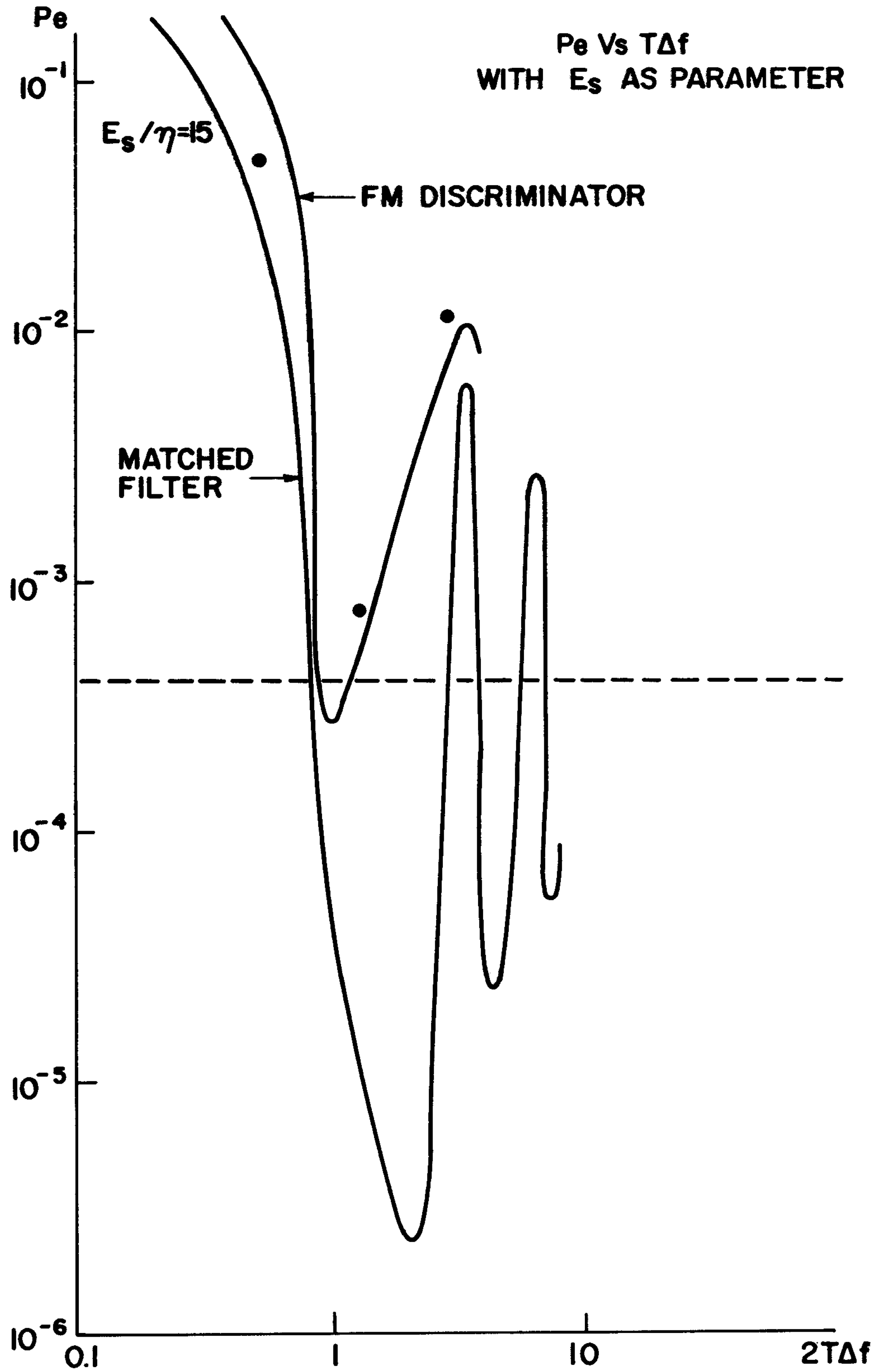

**2. Variation of** $f_o$ (TΔf)



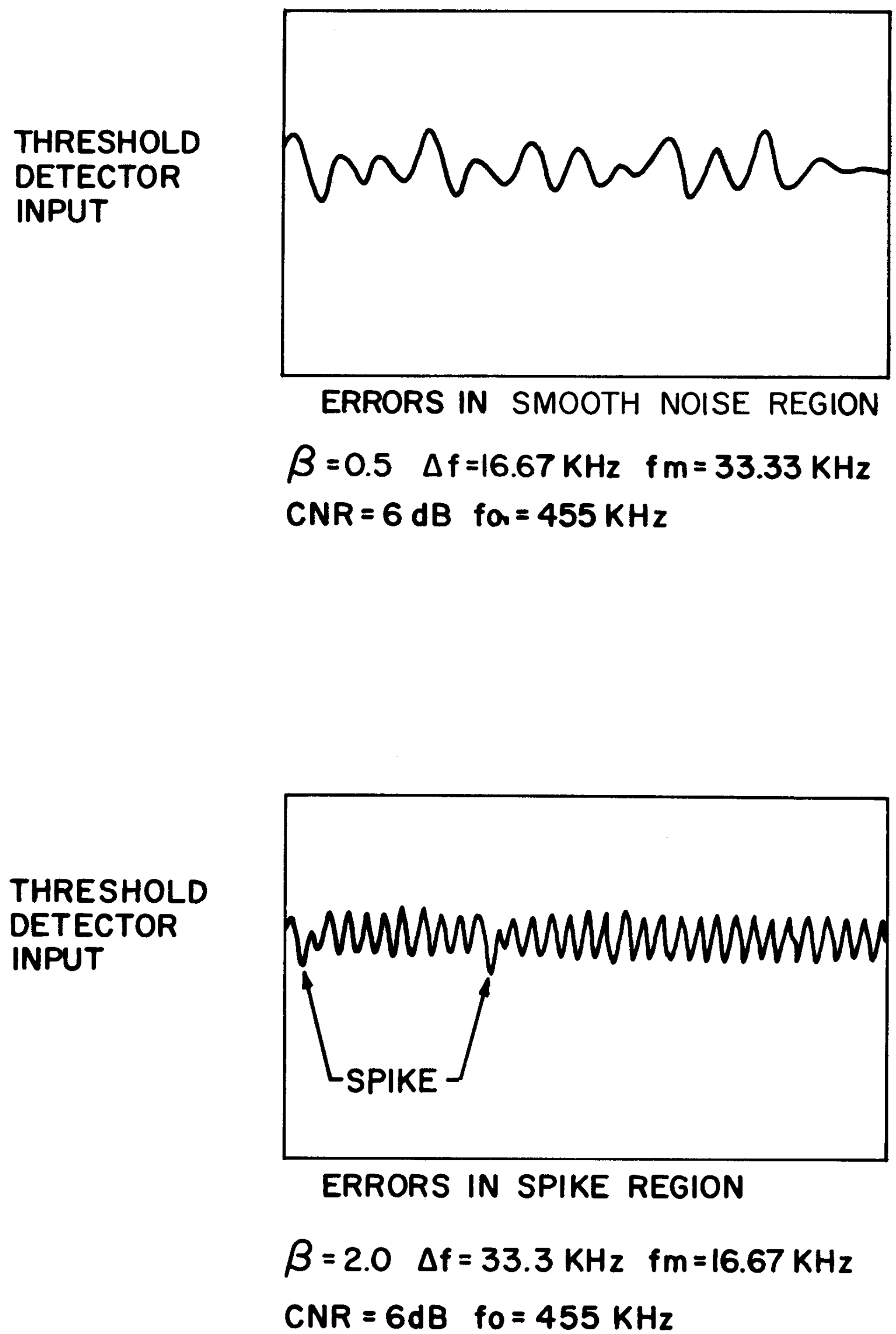

**3. An Error Caused by a Spike**



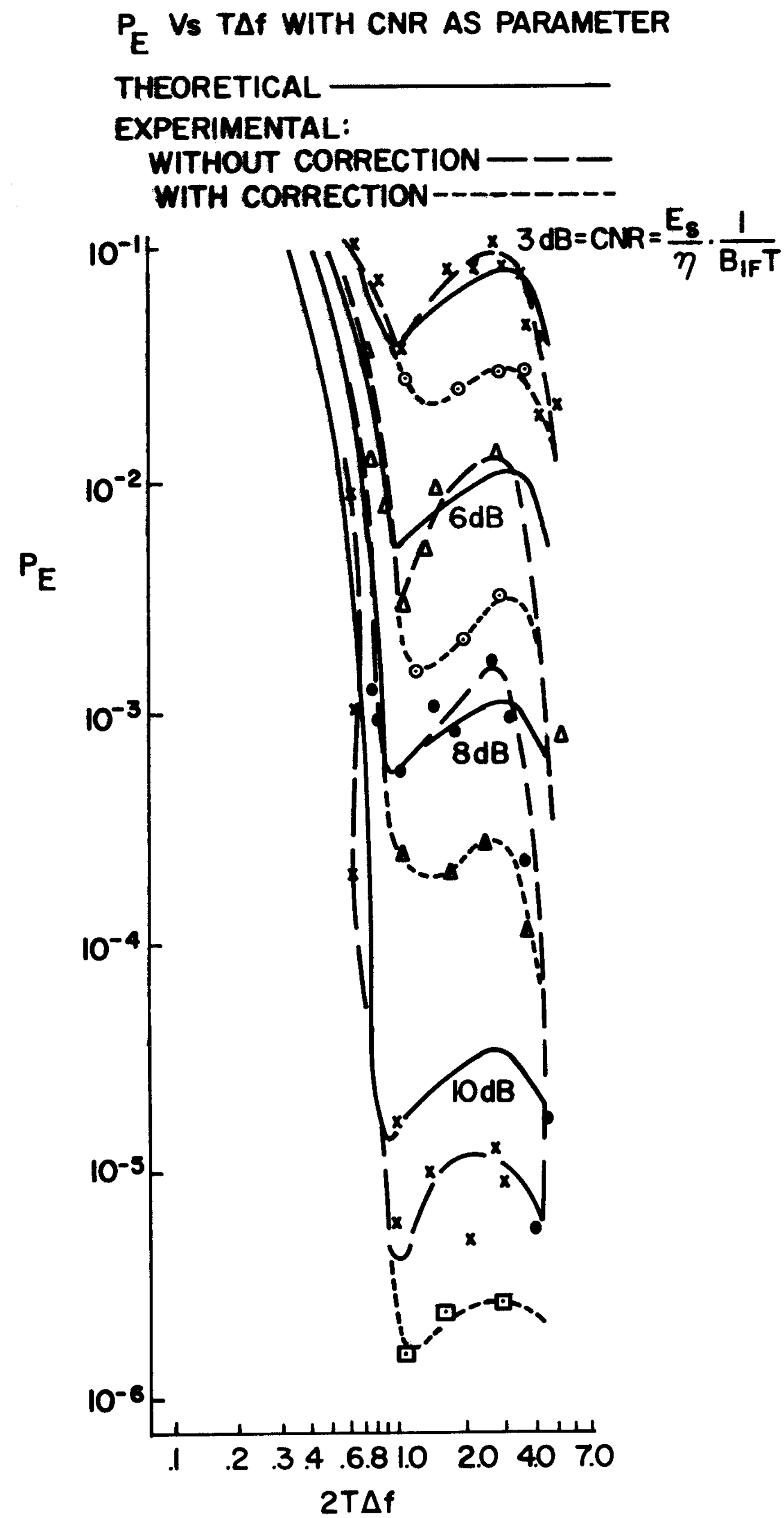

**4. Probability of Errors Caused by Spikes versus TΔf**




5. Comparison of Probability of an Error Obtained When Using an FMD and a MF




**6. Errors in Smooth and Spike Regions**




**7. Improvement Achieved Using Spike Detection and Correction**