A MAXIMUM LIKELIHOOD BIT SYNCHRONIZER

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1. Summary

A method of implementing a maximum likelihood synchronizer for baseband signals in Gaussian noise is presented along with analysis and measurements of its noise performance. Results are given showing the noise jitter of the synchronizer as a function of the energy per bit to noise power density ratio for various parameters of the synchronizer system. The Cramer Rao inequality is used to give a qualitative description of the system noise performance in terms of the signal structure. Finally the noise performance of this technique is compared with several other techniques which are currently used to synchronize baseband PCM signals.

2. Introduction

Since the quality of PCM data is heavily dependent on quality of synchronization, the method of synchronization becomes an important consideration in telemetry system design. The maximum likelihood philosophy was adopted for two reasons. First it can be shown that under certain conditions the maximum likelihood estimate coincides with the minimum mean squared error estimate and secondly it is one of the few approaches which can be analyzed. The specific problems treated here are to implement the maximum likelihood estimate of the bit phase parameter of a baseband PCM signal and to analyze and measure the noise performance.

The likelihood function is implemented by approximating the derivative of the likelihood function and using this derivative as the error signal for a servo system which continuously sets that derivative to zero. This approach is quite straightforward and does not present any hardware design difficulties.

The Cramer Rao inequality is used to give a qualitative indication of system performance as a function of signal-to-noise ratio and signal structure. If the Cramer Rao inequality held with the equal sign then the resulting expression indicates that a perfectly “rectangular” signal would hive no phase error due to noise. However, investigation of the necessary conditions for equality for the Cramer Rao inequality revel that zero phase error cannot occur in the presence of noise. These conditions also imply that in the absence of noise the error cannot be zero for a random PCM waveform. The synchronizer is analyzed for a rectangular PCM waveform. The results correlate well with measured data.

The test data for the maximum likelihood system is compared with an older model PCM synchronizer (See Figure 3). The tests indicate that the new approach is better. Also, on a theoretical basis, the maximum likelihood system appears to have better large signal noise performance than a synchronizer which gets its phase information by integrating across a bit transition.

3. The Synchronization System Model

For obtaining a mathematical Model of the synchronization process it is assumed that the received waveform is observed in the interval $-T \leq t \leq 0$ and that the synchronization is based on this observation. The observed waveform is represented by:

$$y(t) = s(t,\alpha,\tilde{a}) + n(t), \quad -T \leq t \leq 0 \tag{1}$$

where n(t) is a realization from a stationary Gaussian noise ensemble with mean

$$\langle n(t) \rangle = 0 \tag{2}$$

and continuous covariance function

$$R(\tau) = \langle n(t)n(t+\tau) \rangle \quad . \tag{3}$$

The signal portion of the observed waveform is

$$s(t,\alpha,\tilde{a}) = \sqrt{S} \sum_{k=-\infty}^{\infty} a_k \, g(t-\alpha-kt_o) \tag{4}$$

where S is the signal power, t_o is the known binary symbol period and α is the synchronization parameter for which an estimate is to be made. The α satisfies $|\alpha| \leq t_o/2$. The binary message sequence is represented by the stochastic sequence $\tilde{a} = (\cdots a_{-1}, a_o, a_1, \cdots)$ where the random variables a_k are independent and each equally likely to take on the values 1 and -1. The g(t) is the known binary symbol pulse and satisfies the relation

$$(1/t_o) \int_{-\infty}^{\infty} |g(t)|^2 \, dt = 1. \tag{5}$$

When the noise, n(t), becomes white it can be shown[1] that the maximum likelihood estimate, $\hat{\alpha}$, of the synchronization parameter satisfies

$$\Lambda(y(t),\hat{\alpha}) = \max_{\alpha} \Lambda(y(t),\alpha) \tag{6}$$

where the statistic $\Lambda(y(t),\alpha)$ is defined by

$$\Lambda(y(t),\alpha) = \left\langle \exp\left(\frac{1}{N_o} \int_{-T}^{0} y(t)\, s(t,\alpha,\tilde{a})\, dt \right) \right\rangle_{\tilde{a}} \tag{7}$$

where $\langle\ \rangle_{\tilde{a}}$ is the average over all the ensemble $\tilde{a}$. There are two different conditions under which this average can be made easily. The first condition is the weak signal case, where a_k is equally likely to be 1 or -1. In this case after substituting for $s(t,\alpha,\tilde{a})$

$$\Lambda(y(t),\alpha) = \prod_{k=-\infty}^{\infty} \cosh\left(\frac{\sqrt{S}}{N_o} \int_{-T}^{0} y(t)\, g(t-\alpha-kt_o)\, dt \right). \tag{8}$$

Letting $T = Kt_o$ this expression becomes very nearly

$$\Lambda(y(t),\alpha) = \prod_{k=-K}^{0} \cosh\left(\frac{\sqrt{S}}{N_o} \int_{-\infty}^{\infty} y(t) g(t-\alpha-kt_o) dt \right) \tag{9}$$

In the strong signal case the approximation is made that with probability unity

$$a_k = \mathrm{sgn}\left(\frac{\sqrt{S}}{N_o} \int_{-\infty}^{\infty} y(t)\, g(t-\alpha-kt_o)\, dt \right) \tag{10}$$

where $\mathrm{sgn}(x) = x/|x|$. With this assumption and assuming that $T = Kt_o$ the result for strong signals is

$$\Lambda(y(t),\alpha) = \prod_{k=-K}^{0} \exp\left(\left| \frac{\sqrt{S}}{N_o} \int_{-\infty}^{\infty} y(t)\, g(t-\alpha-kt_o)\, dt \right| \right). \tag{11}$$

Since 1nx is a monotonic function of x we have

$$\ln \Lambda\ (y(t),\hat{\alpha}) = \max_{\alpha} \ln \Lambda\ (y(t),\alpha)$$

and the expressions for (9) and (11) can be replaced by

$$\ln \Lambda(y(t),\alpha) = \sum_{k=-K}^{0} \ln \cosh \left(\frac{\sqrt{S}}{N_0} \int_{-\infty}^{\infty} y(t) g(t-\alpha-kt_0) dt \right) \tag{12}$$

and

$$\ln \Lambda(y(t),\alpha) = \sum_{k=-K}^{0} \left| \frac{\sqrt{S}}{N_0} \int_{-\infty}^{\infty} y(t)\; g(t-\alpha-kt_0)\; dt \right| . \tag{13}$$

These last two equations form the basis for the design of the synchronizer in the next section.

4. Instrumentation of the Likelihood Function

In order to design a system which adjusts the phase parameter a to maximize the likelihood functions in (12) and (13) it is assumed that these expressions have derivatives. These derivatives are then used as the error signal in a servo system which continually sets the derivities to zero. The derivitie of (12) and (13) is approximated in the following way:

$$\frac{\partial \ln\Lambda(y(t),\alpha)}{\partial \alpha} = \frac{\ln\Lambda(y(t),\alpha+\Delta/2) - \ln\Lambda(y(t),\alpha-\Delta/2)}{\Delta} \tag{14}$$

for small Ä. The resulting equation in the weak signal case is

$$\frac{\partial \ln\Lambda(y(t),\alpha)}{\partial \alpha} = \sum_{k=-K}^{0} \Delta^{-1} \ln \cosh \left(\frac{\sqrt{S}}{N_0} \int_{-\infty}^{\infty} y(t) g(t-\alpha+\Delta/2-kt_0) dt \right)$$

$$- \sum_{k=-K}^{0} \Delta^{-1} \ln \cosh \left(\frac{\sqrt{S}}{N_0} \int_{-\infty}^{\infty} y(t) g(t-\alpha-\Delta/2-kt_0) dt \right). \tag{16}$$

The feedback system which uses this expression as an error signal is shown in Figure 1. The integrations in 15 are performed by the matched filters. The ln cosh operation is approximated by a full wave rectifier. The summation from -K to 0 is approximated by th averaging effect of the servo loop. The advance and delay by Ä/2 is done by the timing of the samples taken from matched filters 1 and 2. It should be noted that the difference between the strong signal and weak signal synchronizer is in the characteristics of the full wave rectifier.

As can be seen the instrumentation is quite straightforward and can be adapted to any bit pulse form by an appropriate change in the matched filters. In the work done here the bit pulses are nearly rectangular so-the matched filtering is done with integrate and dump circuitry.

5. Noise Performance

The analysis of the noise performance of this system can only be done for the strong signal condition. Two approaches have been taken. First, the Cramer Rao inequality has been used to give qualitative answers on the relation between phase error, signal-to-noise ratio and signal structure. Second, the large signal instrumentation indicated by equations (13) and (14) has been analyzed for perfectly rectangular pulses. The result expressed as equation 38, correlates well with measured test results.

It can be shown that under certain regularity conditions the mean squared difference between the phase parameter, á, and the estimate of the phase parameter, $\hat{a}$, obeys the inequality

$$\langle(\hat{\alpha}-\alpha)^2\rangle \geq \frac{1}{\left\langle\left(\frac{\partial \log \Lambda^*(y(t),\alpha)}{\partial \alpha}\right)^2\right\rangle} \qquad 9160$$

where $\Lambda^*(y(t),\alpha) = \Lambda(y(t),\alpha)\ \exp\ (-ST/2N_o)$.

Using equation 13 and assuming that the polarity of the expression inside the absolute value sign is the same as a_k the resulting equation after substituting for y(t) is

$$\frac{\partial \log \Lambda^*(y(t),\alpha)}{\partial \alpha} = \sum_{k=-K_1}^{0} \frac{\sqrt{S}}{N_o}\ a_k \frac{\partial}{\partial \alpha} \left\{ \int_{-\infty}^{\infty} \left(\sum_{j=-\infty}^{\infty} \sqrt{S}\ a_j g(t-\alpha_o-jt_o) + n(t)\right) g(t-\alpha-kt_o)\ dt\ \exp(-ST/2N_o) \right. \qquad (17)$$

Since in this expression á is taken as the actual phase parameter the terms arising from the signal by itself are equal to zero. After squaring, averaging, inverting the order of integration and using the identity $R(t_1 - t_2) = \langle n(t_1)n(t_2)\rangle$ and neglecting the cross terms the result is

$$\left\langle\left(\frac{\partial \log \Lambda(y(t),\alpha)}{\partial \alpha}\right)^2\right\rangle = \frac{S}{N_o^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \sum_{k=-K_1}^{0} R(t_1-t_2)g'(t_1-\alpha-kt_o)\, g'(t_2-\alpha-kt_o)dt_1dt_2\ \exp(-ST/2N_o), \qquad (18)$$

where the prime (') denotes differentiation with respect to á. Under certain conditions this last expression can be expressed in terms of Fourier integrals to give

$$<\left(\frac{\partial \log\Lambda(y(t),\alpha)}{\partial\alpha}\right)^2> = \sum_{k=-K_1}^{0} \frac{Se^{(-ST/2N_0)}}{2\pi N_0^2}\int_{-\infty}^{\infty} R(\omega)g'(\omega)g'(-\omega)d\omega \qquad (19)$$

where R(ù) and g'(ù) denote the Fourier integral of R(ô) and g'(t) respectively. Substituting equation 19 into 16 the result is

$$<(\hat{\alpha}-\alpha)^2> \geq \frac{(2\pi N_0^2/S)\exp(ST/2N_0)}{(K_1+1)\int_{-\infty}^{\infty} R(\omega)g'(\omega)g'(-\omega)d\omega} . \qquad (20)$$

If g(t) is a pulse of unity aptitude inside the interval $0\leq t\leq t_o$ and zero elsewhere then

$$g'(\omega) = -j\omega\int_{0}^{t_0} e^{-j\omega t}\,dt$$

$$= -2j\,\exp(-j\omega t_0/2)\,\sin(\omega t_0/2) \qquad (21)$$

so that

$$g'(\omega)g'(-\omega) = 4\sin^2(\omega t_0/2). \qquad (22)$$

Therefore, if the signal and noise were passed through an ideal low pass filter which cuts off at ù $_o$ rad/sec. the mean square error would obey the inequality

$$<(\hat{\alpha}-\alpha)^2> \geq \frac{(2\pi N_0/S)\ \exp\ (ST/2N_0)}{4(K_1+1)\int_{-\omega_0}^{\omega_0} \sin^2(\omega t_0/2)d\omega} . \qquad (23)$$

The right hand member goes to zero as ù $_o \rightarrow \infty$. This might lead to the speculation that for perfectly square pulses there would be zero error for a properly instrumented synchronizer. This speculation is reinforced by equation (38) which indicates that as the approximation to a derivative in equation (14) becomes better (Ä → 0) the error due to noise goes to zero. In order to answer this question consider the condition for equality in equation (16) which is that there is equality if and only if /\(y(t),á) satisfies the relation

$$\frac{\partial \log \Lambda(y(t),\alpha)}{\partial \alpha} = K(\alpha,\tilde{a})\ (\hat{\alpha}-\alpha) \tag{24}$$

where $K(\alpha,\tilde{a})$ is independent of $n(t)$. In order to investigate whether this condition holds expand the left hand side of the above equation in a stochastic Taylor series, and ignore all but the first two terms. To do this $(\alpha-\hat{\alpha})$ is assumed small. The resulting expression is

$$\frac{\partial \log \Lambda(y(t),\alpha)}{\partial \alpha} = \frac{\partial \log \Lambda(y(t),\alpha)}{\partial \alpha}\Bigg|_{\alpha=\hat{\alpha}} + (\alpha-\hat{\alpha})\ \left(\frac{\partial^2 \log \Lambda(y(t),\alpha)}{\partial \alpha^2}\Bigg|_{\alpha=\hat{\alpha}}\right). \tag{25}$$

The first term is zero since it is evaluated at $\alpha=\hat{\alpha}$. From equation (17) it follows that the coefficient of $\alpha-\hat{\alpha}$ can be written

$$\frac{\partial^2 \log \Lambda(y(t),\alpha)}{\partial \alpha^2}\Bigg|_{\alpha=\hat{\alpha}}$$

$$= \frac{\sqrt{S}}{N_o} \sum_{k=-K_1}^{0} a_k \int_{-\infty}^{\infty} \Big(\sum_{j=-\infty}^{\infty} a_j \sqrt{S}\ g(t-\alpha_o-jt_o)$$

$$+ n(t)\ \Big) \ \frac{\partial^2 g(t-\alpha-kt_o)}{\partial \alpha^2}\, dt\Bigg|_{\alpha=\hat{\alpha}} . \tag{26}$$

Here $\acute{a}_o$ is taken as the actual phase parameter of the received signal. If it is assumed that $g(t) = 0$ for $t > 3t_o/2$ and $-t_o/2>t$ the right hand side of the above equation becomes

$$\frac{\sqrt{S}}{N_o} \sum_{k=-K_1}^{0} \sqrt{S} \int_{-\infty}^{\infty} \frac{\partial^2 g(t-\alpha-kt_o)}{\partial\alpha^2} \left[g(t-\alpha_o-kt_o) \right.$$

$$+ a_k a_{k+1} \; g(t-\alpha_o-(k+1)t_o)$$

$$\left. + a_k a_{k-1} \; g(t-\alpha_o-(k-1)t_o) \right] dt$$

$$+ \frac{\sqrt{S}}{N_o} \sum_{k=-K_1}^{0} a_k \int_{-\infty}^{\infty} n(t) \frac{\partial^2 g(t-\alpha-kt_o)}{\partial\alpha^2} dt \bigg|_{\alpha=\hat{\alpha}} . \qquad (27)$$

In order to simplify the above expressions, make the definitions

$$G_o(\hat{\alpha},\alpha_o) = \int_{-\infty}^{\infty} \frac{\partial^2 g(t-\alpha-kt_o)}{\partial\alpha^2} g(t-\alpha_o-kt_o)dt \bigg|_{\alpha=\hat{\alpha}}$$

and

$$G_1(\hat{\alpha},\alpha_o) = \int_{-\infty}^{\infty} \frac{\partial^2 g(t-\alpha-kt_o)}{\partial\alpha^2} g(t-\alpha_o-(k+1)t_o)dt \bigg|_{\alpha=\hat{\alpha}}$$

and

$$G_{-1}(\hat{\alpha},\alpha_o) = \int_{-\infty}^{\infty} \frac{\partial^2 g(t-\alpha-kt_o)}{\partial\alpha^2} g(t-\alpha_o-(k-1)t_o)dt \bigg|_{\alpha=\hat{\alpha}} , \qquad (28)$$

then expression (25) becomes

$$\frac{\partial \log \Lambda(y(t),\alpha)}{\partial \alpha} = (\hat{\alpha}-\alpha) \{ \frac{S}{N_o} (K_1+1) \; G_o(\hat{\alpha},\alpha_o)$$

$$+ \frac{S}{N_o} \sum_{k=-K_1}^{o} [a_k a_{k+1} \; G_1(\hat{\alpha},\alpha_o)$$

$$+ a_k a_{k-1} \; G_{-1} \; (\hat{\alpha},\alpha_o) \;]$$

$$+ \frac{\sqrt{S}}{N_o} \sum_{k=-K_1}^{o} a_k \int_{-\infty}^{\infty} n(t) \; \frac{\partial^2 g(t-\alpha-kt_o)}{\partial \alpha^2} \; dt \Big|_{\alpha=\hat{\alpha}} \; \}. \tag{29}$$

Examination of equation (29) reveals two aspects of this synchronization technique. First, the derivative of $\Lambda(y(t),\alpha)$ is a function of $\tilde{a}$. This results in a variable gain in the sync servo which leads to the well known phenomenon of larger phase errors in the presence of random data as compared to periodic data. The second aspect is the last term in (29), which depends on the signal and the noise. In order to tell whether the phase estimate of this technique obeys equation (24), it is necessary to examine the behavior of this last term in (29) as g(t) becomes a rectangular pulse. Therefore, take this term, square, average, invert the order of averaging and integrating and neglect the cross terms to get the result

$$\sigma^2 = \frac{S}{{N_o}^2} \sum_{k=-K_1}^{o} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R(t_1-t_2) \; \frac{\partial^2 g(t_1-\alpha-kt_o)}{\partial \alpha^2}$$

$$\frac{\partial^2 g(t_2-\alpha-kt_o)}{\partial \alpha^2} \; dt_1 dt_2 \Big|_{\alpha=\hat{\alpha}} \; . \tag{30}$$

Again under suitable circumstances the above integral can be replaced by Fourier integrals to give

$$\sigma^2 = \frac{S}{{N_o}^2} \sum_{k=-K_1}^{o} \frac{1}{2\pi} \int_{-\infty}^{\infty} R(\omega) \; g''(\omega) \; g''(-\omega) \; d\omega \tag{31}$$

where $R(\omega)$ and $g''(\omega)$ are the Fourier integrals of $R(\tau)$ and $\partial^2 g(t)/dt^2$ respectively. A development similar to the one leading up to equation (23) reveals that

$$\sigma^2 = \frac{S}{N_o} \sum_{k=-K_1}^{o} \frac{1}{2\pi} \int_{-\omega_o}^{\omega_o} \omega^2 \sin^2 \frac{t_o \omega}{2} d\omega \quad . \tag{32}$$

This indicates that as the pulse becomes perfectly rectangular the conditions for equality are not met. ‘This is because the mean squared portion due to noise blows up as fast as the portion due to signal in equation (26).

Now consider the jitter in the large signal instrumentation Using the likelihood function of equation (13) and assuming the signal to noise ratio large the likelihood function is approximated by

$$\ln\Lambda(y(t),\alpha) = \sum_{j=-K}^{o} \left| \frac{\sqrt{S}}{N_o} \int_{-\infty}^{\infty} \left(\sum_{i=-\infty}^{\infty} \sqrt{S}\, a_i\, g(t-\alpha_o-it_o) \right) g(t-\alpha-jt_o)dt \right| + \sum_{j=-K}^{o} \frac{\sqrt{S}}{N_o} a_j \int_{-\infty}^{\infty} n(t)\, g(t-\alpha-jt_o)dt, \tag{33}$$

where the first term is due to signal and the second is due to noise. Now if 33 is substituted into equation 14 the result (after many manipulations) is the approximation

$$\ln\Lambda(y(t),\alpha+\Delta/2) - \ln\Lambda(y(t),\alpha-\Delta/2) = \sum_{j=-K}^{o} \left| a_j - a_{j+1} \right| \frac{2S}{N_o} \delta_\alpha + \sum_{j=-K}^{o} \frac{\sqrt{S}}{N_o} a_j \int_{-\infty}^{\infty} n(t) \left[g(t-\alpha+\Delta/2-jt) - g(t-\alpha-\Delta/2-jt_o) \right] dt \tag{34}$$

where the fringe effects for j=0 and j=K have been neglected and g(t) has the form

$$g(t) = \begin{cases} 1, & 0 \le t \le t_o \\ 0, & \text{elsewhere} \end{cases} \tag{35}$$

and δ_α is the error between α_o and α.

The first term in equation (34) is due to signal while the second term is due to noise. The mean squared signal term can be shown to be very nearly equal to

$$\sigma_s^2 = \frac{S^2}{N_o^2} 4K^2 <(\delta_\alpha)^2> \tag{36}$$

while the mean squared noise term is

$$\sigma_n^2 = \frac{2S\Delta}{N_o} \quad . \tag{37}$$

If it is assumed that when the synchronizer is "locked up" then $\sigma_n^2 = \sigma_s^2$, the result is

$$<(\delta_\alpha)^2> = \frac{N_o \Delta}{2SK} \quad . \tag{38}$$

Taken at face value this equation indicates that the mean squared jitter due to noise is proportional to the noise, inversely proportional to the sync servo bandwidth B, and proportional to the A used in the approximation to the derivitive. If the servo bandwidth B is in cps approximated as $1/2Kt_o$, then

the equation can be rewritten

$$<(\delta_\alpha)^2> = \frac{N_o \Delta B t_o}{S} \quad . \tag{39}$$

If, further, B is expressed in terms of percent of bit rate % then the result can be written

$$<(\delta_\alpha)^2> = \frac{N_o}{S} \; \frac{\%}{100} \Delta \quad . \tag{40}$$

This formula is plotted in Figure 2 along with the measured test results.

5.0 Comparison With Other Systems

We will conclude this paper by comparing this technique with two other techniques which are commonly used. The first technique is the one which is used in the old Dynatronics BSC 7 series Bit Synchronizers. This synchronizer works by limiting, differentiating, and rectifying, in order to produce a sharp spike at each bit transition. These spikes are then compared with the crossover points of a reference waveform with a standard phasemeter technique. The phase meter signal is fed back through a servo loop compensation. A schematic of the BSC 7 is given in Figure 3. “Peak-to-peak” jitter due to noise for various bandwidths and signal-to-noise ratios is shown for both the maximum likelihood synchronizer and the BSC 7 in Figure 4. It should be noted that due to the subjective nature of the peak-to-peak readings, the values of the jitter due to noise could be off by as much as 2 to 1, however, the relative performance should stay the same. With this in mind it appears that the new synchronizer is better than the BSC 7 in noise performance.

As a final comparison, consider a synchronizer which derives an error signal by integrating across a bit transition. It is is assumed that the interval of intergation has length t'. The noise accumulated by the integrator in a particular interval is

$$n_o(t) = \int_{t-t'}^{t} n(T)d(T) \quad . \tag{41}$$

Assuming white noise with two-sided power density N_o, the mean squared noise output of this integration can be shown to be

$$\sigma_n^2 = N_o t' \quad . \tag{42}$$

If the signal has amplitude √S and the transition occurs off the center of the integration period by an amount δ_I, the signal output at each transition will have a magnitude given by

$$2\sqrt{S} \; |\delta_I| \quad . \tag{43}$$

If a sequence of K bits are observed, an average of K/2 transitions will take place and the mean squared output is for large K’s very nearly

$$\sigma_s^2 = K^2 \; S < \delta_I^2 > \tag{44}$$

where it has also been assumed that δ_I is independent of the number of transitions. Now assume that $\sigma_s^2 = K \; \sigma_n^2$ to get the result

$$< \delta_I^2 > = N_o t'/KS \quad . \tag{45}$$

comparison with equation (38) reveals that

$$\frac{< \delta_I^2 >}{< \delta_\alpha^2 >} = \frac{2T'}{\Delta} \quad . \tag{46}$$

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REFERENCE

1) Murphy et al "ULTRA LONG-RANGE TELEMETRY STUDY" ALTDR 64-87 APRIL 1964.

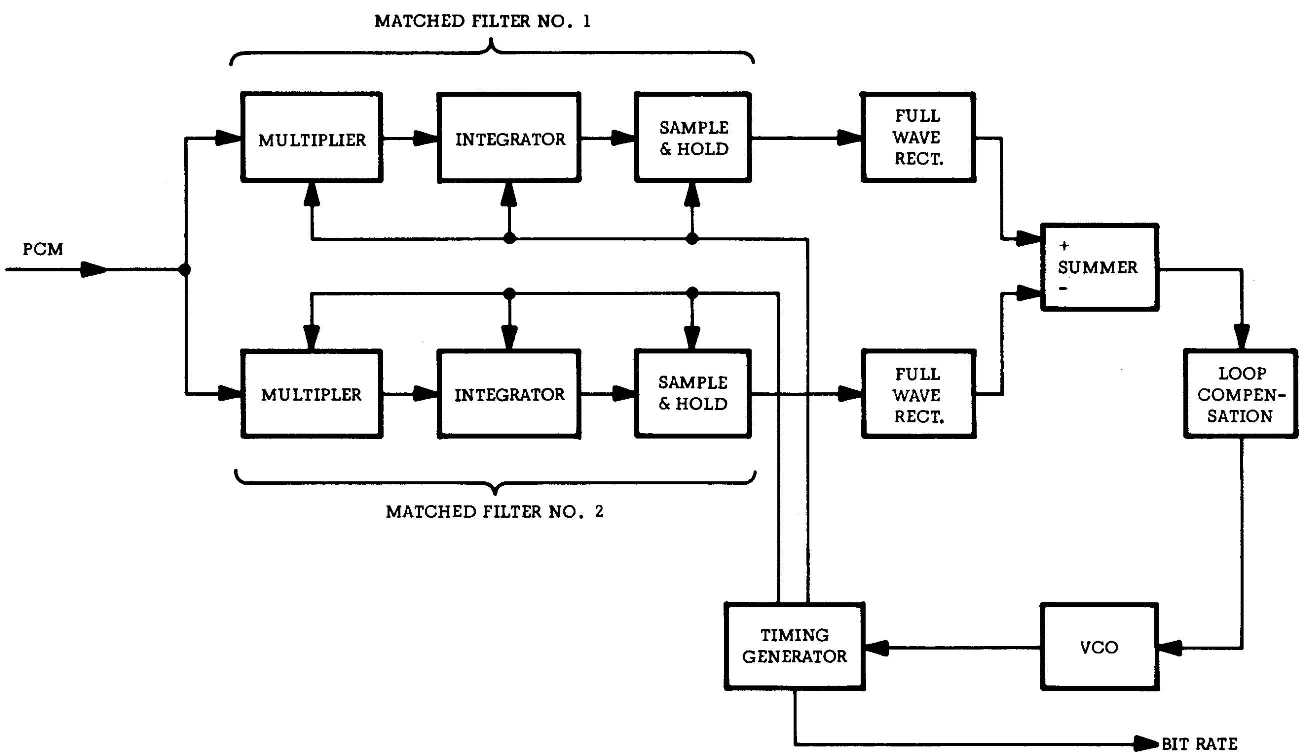

Figure 1. Maximum Likelihood Synchronizer

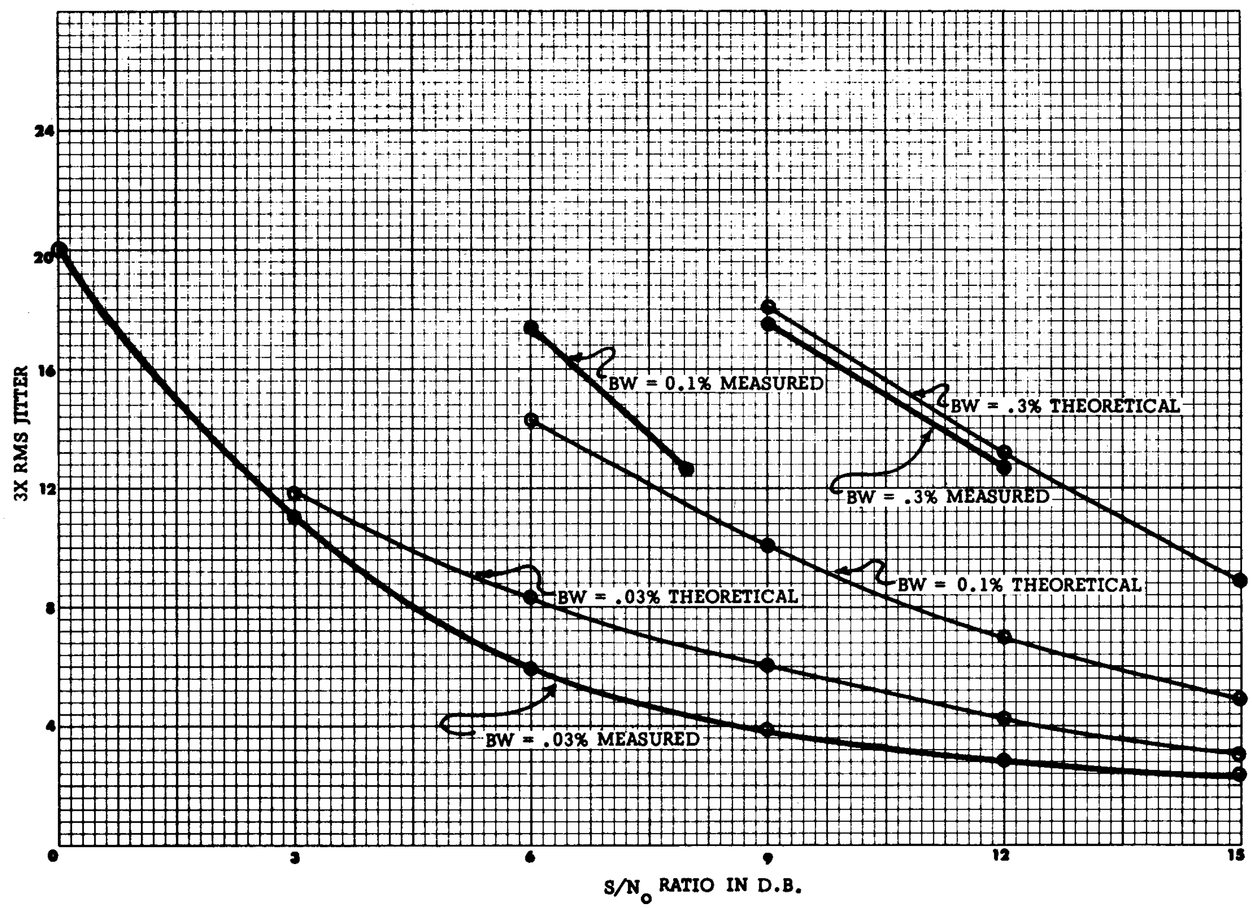

Figure 2. RMS Jitter Due to Wideband Noise For Maximum Likelihood Synchronizer With Random Data

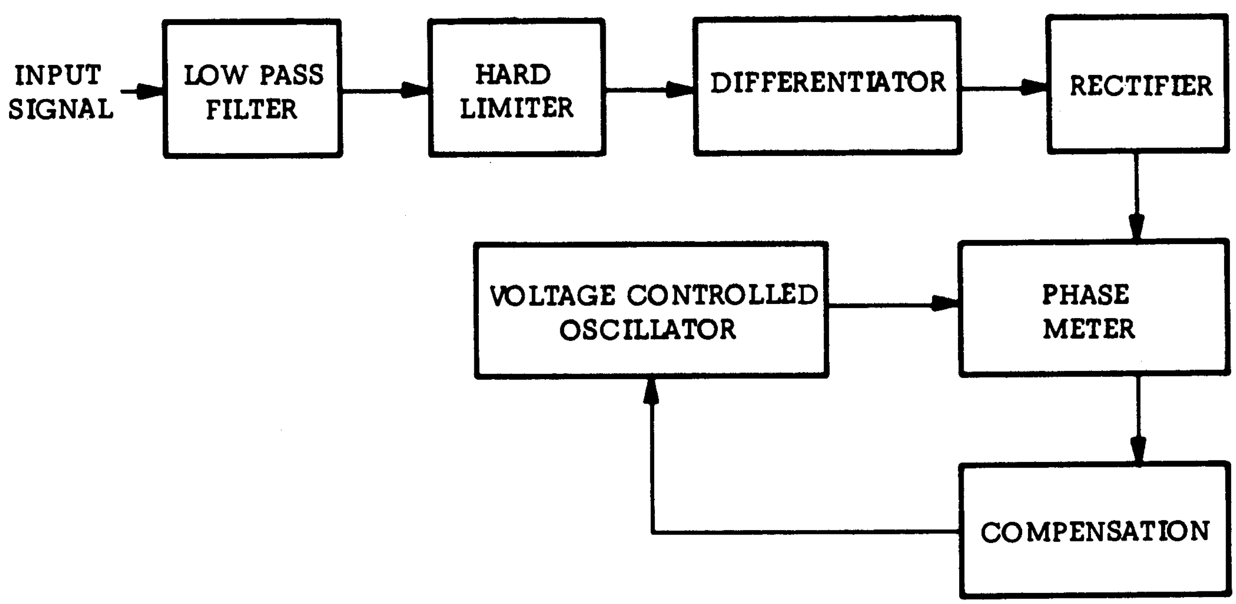

Figure 3. Schematic of BSC-7 Synchronizer

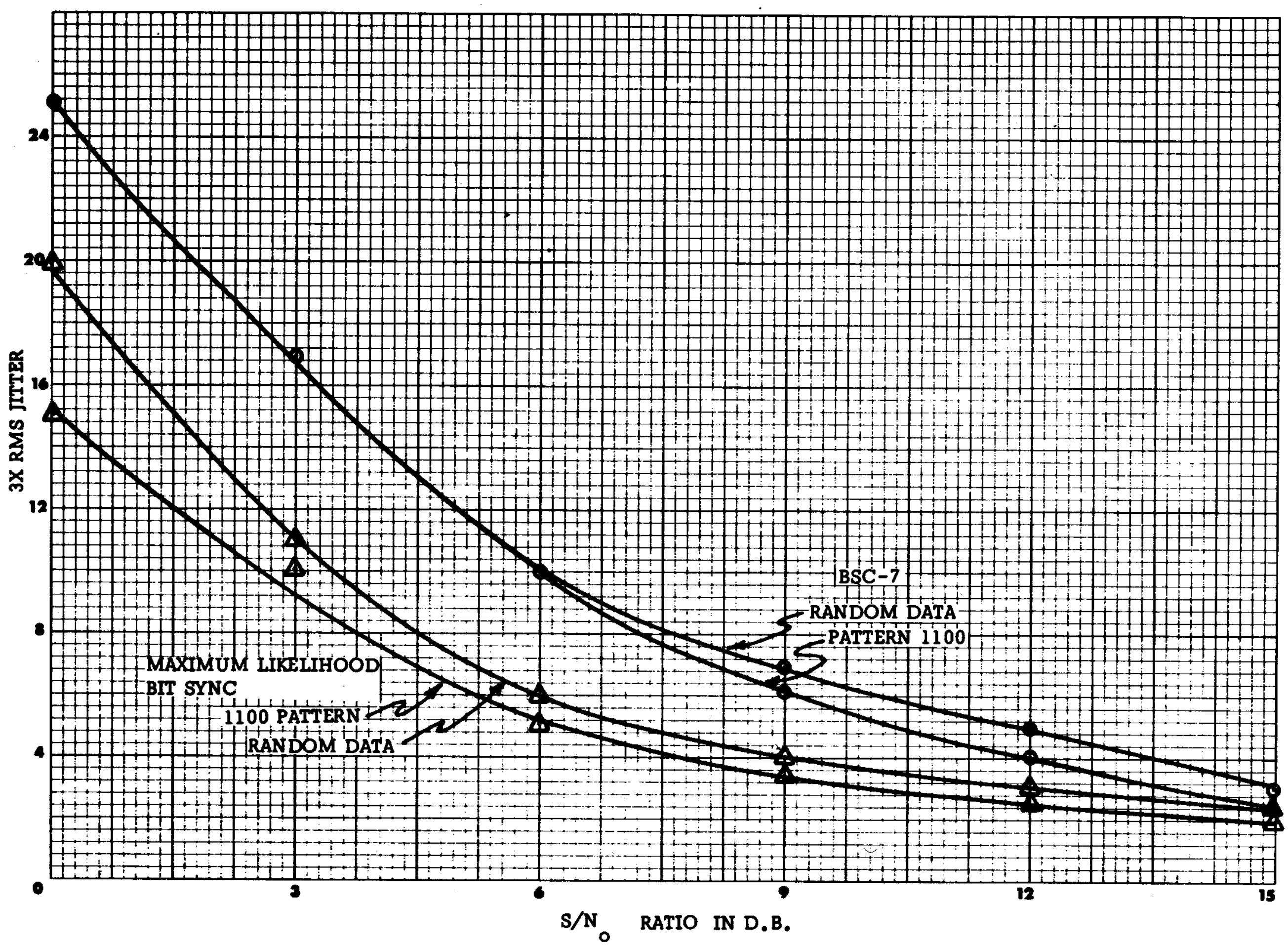

Figure 4. Comparison of Maximum Likelihood Synchronizer With BSC-7 on .3% Bandwidth