RICIAN INTERSYMBOL INTERFERENCE IN FREQUENCY SHIFT KEYING

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Summary   Many communications links involve a reflected signal which is Rician in nature. In troposcatter systems, this reflection constitutes the entire received signal while in communications between satellites or aircraft, such a reflection from the surface corrupts the direct signal. Other such situations involve low elevation tracking or intentional coherent jamming.

This paper derives the bit error probability for a non-coherent binary FSK link in this environment for any order of diversity and any ratio of specular to diffuse reflection assuming orthogonal signalling frequencies, matched filter detection and perfect bit synchronization. The interfering signal may represent the same datum as the direct signal (Mark-Mark interference) or, for delays longer than a bit period, the interference may appear in the opposite receiver channel from the direct signal (Mark-Space interference). Results are stated in terms of the direct signal energy to noise density ratio and factors determined by the geometry of the situation; the ratio of direct signal to interfering power, the ratio of specular to diffuse reflected power and the relative carrier phases of the direct signal and the specular reflection in the Mark-Mark case. These geometric parameters are most conveniently treated separately from the modulation and detection problem.

Introduction   Previous studies of the problem of communication through fading channels can be summarized as follows: Pierce[1] evaluated the probability of error for troposcatter links by integrating the error probability for a given signal strength weighted by the signal strength probability density. Glenn and Lieberman[2] determined the percentage of time that the received signal strength is above the threshold required for "satisfactory operation". Turin[3] determined the error probability averaged over a fade cycle for a Rician received signal, and Lindsey[4] added the effects of diversity to Turin's results. All of these works assume slow fading such that the received signal amplitude is essentially constant for the duration of a bit. This assumption is retained in this paper. The previous works also assume that the interfering signal represents the same data bit as the direct signal so that for FSK, both signals excite the same matched filter (Mark-Mark interference). However, in many cases of interest, the differential time delay over the two paths is longer than a bit period. In this situation, both matched filters of the receiver

may be excited, one by the direct signal and one by the interference (Mark-Space interference). The analysis in this paper therefore treats both Mark-Mark and Mark-Space interference. This analysis also considers the specular interference component on an instantaneous basis with phase independent of the direct signal.

System Model   Figure 1 is a model of the channel and illustrates the channel symbols. Note that the indirect path, J, is "derived" from the direct path, D, and that the indirect path may have any degree of Ricianness from pure specular to pure Rayleigh. As the differential time delay between the two paths varies, the receiver may see the interfering symbol as the same symbol received over the direct path (Mark-Mark case), or the receiver may see a <u>different</u> interfering symbol than that received over the direct path (Mark-Space case).

Figure 2 is the receiver model, and illustrates the receiver symbols. The "type" of diversity may be any form. (frequency, time, space; polarization), however, the analysis is limited to square law diversity combiners.

Mark-Space Error Probability   The Mark-Space error probability is derived by assuming that a Mark is received via the direct path and a Space via the interfering path and integrating the general expression:

$$P_e = \int_{X=0}^{\infty} p(X) \int_{Y=X}^{\infty} \int_{A=0}^{\infty} p(Y|A) dA \, dY \, dX \qquad (1)$$

where $X = \sum_{m=1}^{M} x_m^2$, $Y = \sum_{m=1}^{M} y_m^2$ as shown in Figure 2, and A represents a composite interfering signal.

In this situation, all of the Mark and Space channels have inputs consisting of signal components and thermal noise. The Mark channels have "direct" signal components and thermal noise, therefore

$$p(x_m) = x_m \exp\left(-\frac{x_m^2 + 2R_m}{2}\right) I_o\left(x_m \sqrt{2R_m}\right), \qquad (2)$$

where $I_o(x)$ is the modified Bessel function, and R_m is the direct path signal energy to noise density ratio, E_m/N_o, in the mth diversity channel. The Space channels have "indrect" signal components and thermal noise, therefore

$$P(y_m|a_m) = y_m \exp\left(-\frac{y_m^2 + \frac{a_m^2}{\alpha_{sm}^2} \, 2R_m}{2}\right) I_o\left(y_m \frac{a_m}{\alpha_{sm}} \sqrt{2R_m}\right), \qquad (3)$$

where a_m^2/α_{sm}^2 is the ratio of interference to direct signal power. The probability density of a_m is

$$p(a_m) = \frac{a_m}{\sigma_j^2} \exp\left(-\frac{a_m^2 + \alpha_{jm}^2}{2\sigma_j^2}\right) I_0\left(\frac{a_m \alpha_{jm}}{\sigma_j^2}\right),$$

(4)

where α_{jm} is the amplitude of the specular interference component and $2\sigma_j$ is the rms amplitude of the diffuse interference component. The fading on the different diversity channels is assumed to be statistically independent, and the diffuse interference component is assumed to have the same average power for all diversity channels. The probability density of y_m may be found from

$$p(y_m) = \int_0^\infty p(y_m|a_m)\, p(a_m)\, da_m,$$

(5)

and by applying Weber's second exponential integral, [5]

$$p(y_m) = \frac{y_m}{\sigma_j^2}\, \exp\left[-\frac{y_m^2 + \dfrac{\alpha_{jm}^2}{\alpha_{sm}^2}2R_m}{2\left(1 + \dfrac{\sigma_j^2}{\alpha_{sm}^2}2R_m\right)}\right] I_0\left(y_m \frac{\alpha_{jm}}{\alpha_{sm}} \frac{\sqrt{2R_m}}{1 + \dfrac{\sigma_j^2}{\alpha_{sm}^2}2R_m}\right)$$

(6)

The parameters $\dfrac{\alpha_{jm}^2}{\alpha_{sm}^2}R_m$ and $\dfrac{\sigma_j^2}{\alpha_{sm}^2}R_m$ may be recognized as the "signal" to thermal noise ratios for the specular and diffuse interference components. Following Lindsey's Characteristic Function approach to transform $p(y_m)$ into $p(Y)$, making the substitutions

$$u^2 = X, \quad v^2 = \frac{Y}{1 + \dfrac{\sigma_j^2 T}{N_o}}$$

(7)

where N_o is the single sided noise power spectral density in watts per Hertz, and,

$$a_{ms}^2 = \sum_{m=1}^{M}{}' 2R_m$$

$$b_{ms}^2 = \frac{\dfrac{T}{N_o}\displaystyle\sum_{m=1}^{M}\alpha_{jm}^2}{1 + \dfrac{\sigma_j^2 T}{N_o}}$$

(8)

$$r_{ms}^2 = \frac{1}{1 + \dfrac{\sigma_j^2 T}{2N_o}}$$

with $\quad R_m = \alpha_{sm}^2 T/2N_o$

(T being the bit period), then after some manipulation

$$p(X)dX = a_{ms}^{1-M}\, u^M\, \exp\left(-\frac{u^2 + a_{ms}^2}{2}\right)\, I_{M-1}(u\, a_{ms})\, du$$

(9)

and

$$p(Y)dY = b_{ms}^{1-M}\, v^M\, \exp\left(-\frac{v^2 + b_{ms}^2}{2}\right)\, I_{M-1}(v\, b_{ms})dv.$$

(10)

$$P_{ems} = 1 - (a_{ms} b_{ms})^{1-M} \int_{u=0}^{\infty} u^M \exp\left(-\frac{u^2 + a_{ms}^2}{2}\right)$$

$$I_{M-1}(u\, a_{ms}) \int_{V=0}^{r_{ms}u} v^M \exp\left(-\frac{v^2 + b_{ms}^2}{2}\right)$$

(11)

$$I_{M-1}(v\, b_{ms})dv\, du.$$

This integral has been solved in Price.[6] Thus,

$$P_{ems} = 1 - P_{M-1}(a_{ms}, b_{ms}, r_{ms})$$

(12)

in terms of $P_{M-1}(a,b,r)$ given by Price.

Mark-Mark Interference   In the Mark-Mark situation, the Space channels contain only thermal noise, thus

$$p(y_m) = y_m \exp\left(-\frac{y_m^2}{2}\right) \tag{13}$$

so, $$p(Y)dY = \lim_{b_{mm} \to 0} b_{mm}^{1-M} v^M \exp\left(-\frac{v^2 + b_{mm}^2}{2}\right) I_{M-1} (v\, b_{mm}) dv \tag{14}$$

where $$v^2 = Y. \tag{15}$$

The Mark channel input contains the direct signal, thermal noise, and the interfering Rician signal, with resultant amplitude

$$\alpha_m^2 = \alpha_{sm}^2 + \alpha_{jm}^2 + 2\alpha_{sm}\,\alpha_{jm}\,\cos\psi_m \tag{16}$$

where ψ_m is the relative carrier phase between the direct signal and the specular portion of the Rician signal.

By analogy with the Mark-Space case

$$p(X)dX = a_{mm}^{1-M} u^M \exp\left(-\frac{u^2 + a_{mm}^2}{2}\right) I_{M-1} (u\, a_{mm}) du \tag{17}$$

where $$u^2 = \frac{X}{1 + \dfrac{\sigma_j^2 T}{N_o}}$$

and $$a_{mm}^2 = \frac{\dfrac{\alpha^2 T}{N_o}}{1 + \dfrac{\sigma_j^2 T}{N_o}} \tag{18}$$

with $$\alpha^2 = \sum_{m-1}^{M} \alpha_m^2 .$$

The probability of error is then,

$$P_{emm} = 1 - \lim_{\substack{b \to 0 \\ mm}} P_{M-1}\ (a_{mm},\ b_{mm},\ r_{mm}) \tag{19}$$

or
$$P_{emm} = 1 - P_{M-1}\ (a_{mm},\ 0,\ r_{mm}) \tag{20}$$

where
$$r_{mm} = 1/r_{ms} \tag{21}$$

Results   The probability of error for either case is then, using Price's expression for P_{m-1} (a. b. r),

$$P_e = 1 - Q\left(\frac{ar}{\sqrt{r^2 + 1}},\ \frac{b}{\sqrt{r^2 + 1}}\right) + \exp\left[-\frac{(ar)^2 + b^2}{2(r^2 + 1)}\right]\left\{\frac{1}{r^2 + 1}\right.$$

$$I_o\left(\frac{abr}{r^2 + 1}\right)$$

$$-\frac{r^2 - 1}{2(r^2 + 1)}\ \sum_{m=0}^{M-1}\ [(\frac{br}{a})^m + (\frac{a}{br})^m]\ I_m\left(\frac{abr}{r^2 + 1}\right)\ \sum_{j=m+\delta_{mo}}^{M-1}$$

$$\binom{2j-\delta_{mo}}{j+m}\ \frac{r}{r^2 + 1}\ 2j$$

$$-\sum_{m=1}^{M-1}\ m[(\frac{br}{a})^m - (\frac{a}{br})^m]\ I_m\left(\frac{abr}{r^2 + 1}\right)\ \sum_{j=m}^{M-1}\ \frac{1}{2j}\ \binom{2j}{j+m}(\frac{r}{r^2 + 1})2j\right\} \tag{22}$$

where the Marcum Q function is

$$Q(\alpha,\beta) = \int_{\beta}^{\infty}\ z\ \exp\left(-\frac{z^2 + \alpha^2}{2}\right)\ I_o\ (\alpha z)dz \tag{23}$$

the binomial coefficient is

$$\binom{n}{k} = n!/k!\ (n-k)! \tag{24}$$

and the Kroniker delta function is

$$\delta_{mo} = \begin{cases} 1;\ m=0 \\ 0;\ m\neq 0 \end{cases} \tag{25}$$

Examples   Graphs of error probability are given in Figures 3, 4, 5, and 6, as functions of the "geometry" factor

$$D/J = \frac{\alpha_s^2}{\alpha_j^2 + 2\sigma_j^2} \tag{26}$$

i.e., the relative strength of the direct and indirect paths, the direct path energy to noise density

$$R = E/N_o \tag{27}$$

the degree of diversity, M, and the degree of specularity of the indirect path

$$\zeta = \frac{\alpha_j^2}{2\sigma_j^2} \tag{28}$$

It is interesting to see from Figure 3 that the D/J must be <u>very</u> high (greater than 25 dB), for the interference to be termed "negligible". At moderate D/J levels, say 5 to 10 dB, the uncertainty of the quality of the link can be <u>two orders</u> of magnitude From Figure 4 it is clear that the increasing of link power (or sensitivity) alone is an expensive technique to combat multipath. For example, a link operating at error rate 2 x 10^{-4} in a multipath free environment would degrade over an order of magnitude if an interference signal of one tenth the power were occuring. Diversity can be useful in most multipath environments as seen from Figure 5, however, at very low D/J ratios the use of diversity can degrade the link performance. Another surprising result, shown in Figure 6, is the significance of the degree of specularity. It appears that the degree of specularity may outweigh the importance of the total strength of the interfering signal in many multipath situations.

Conclusion   This paper has extended the analysis of previous FSK multipath work to include the generic case of a Rician interfering signal. The situations where the interfering bit is the same as the direct bit, or different than the direct bit have also been shown. This is of particular significance when the differential path length between the two transmission paths is of the order of a bit period, or greater. The results have shown that the interfering signal must be "much" less than the direct signal before the interference can be considered negligible. The results have also shown that the degree of interference is strongly influenced by the amount of specularity of the indirect path.

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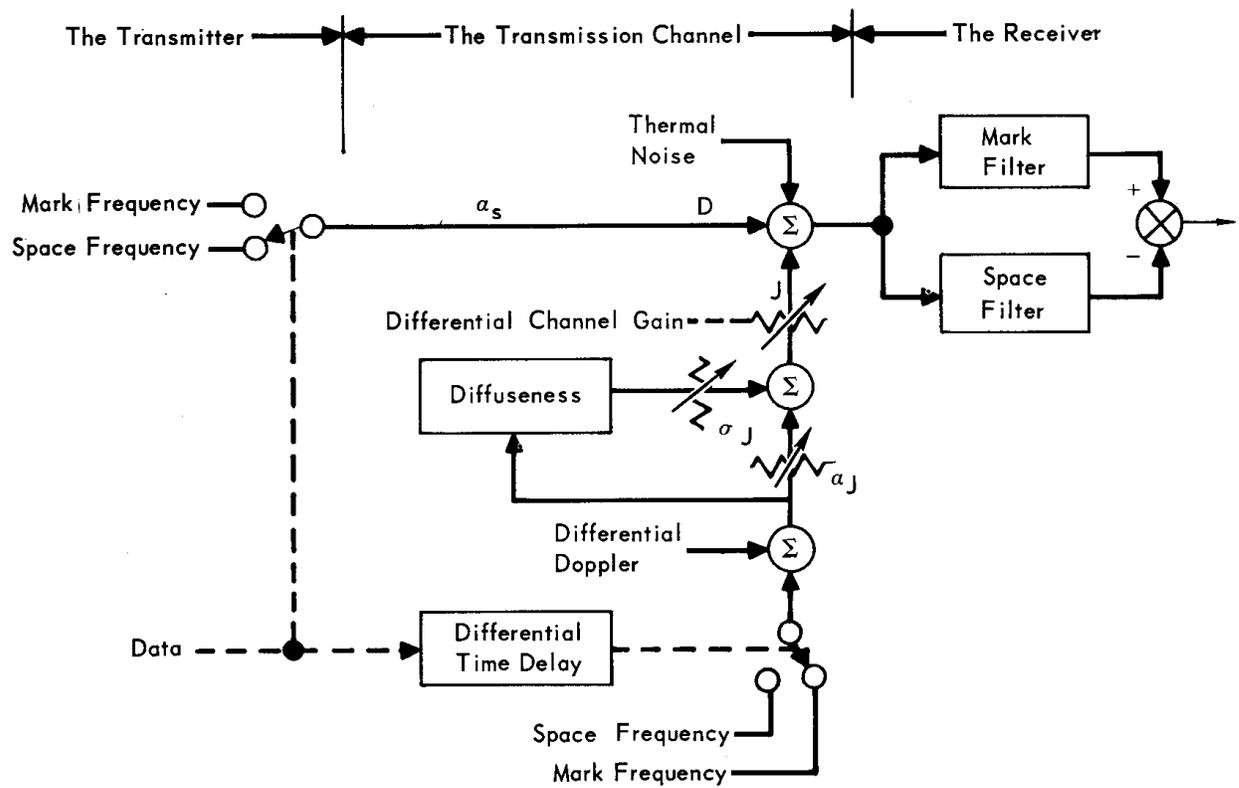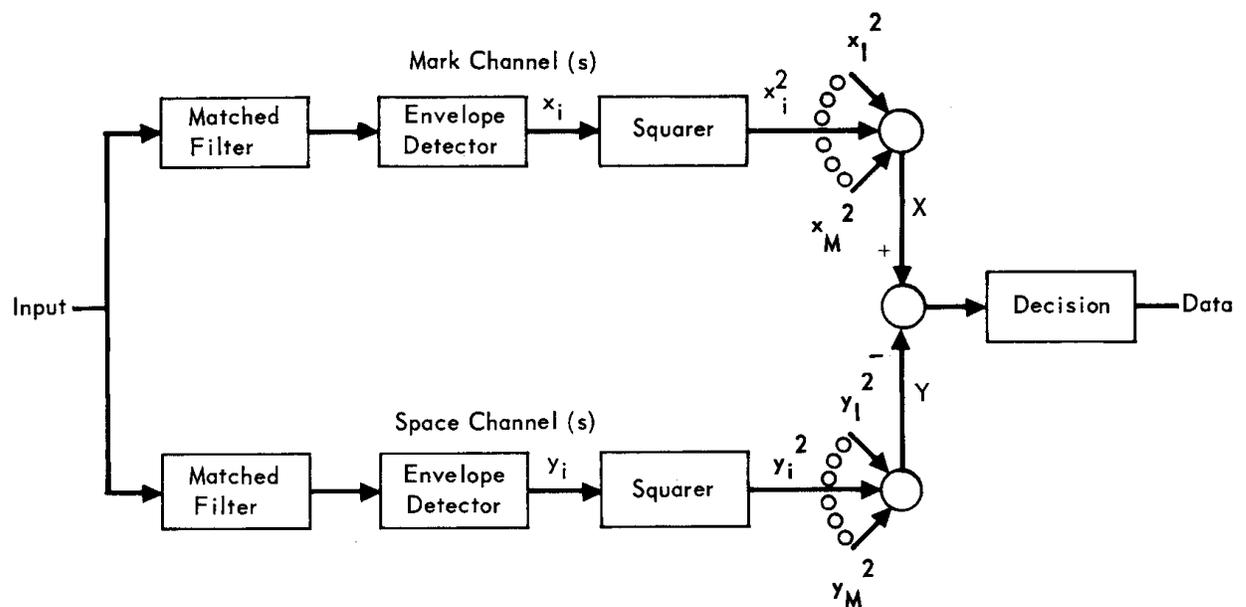
Fig. 1 - The Channel Model
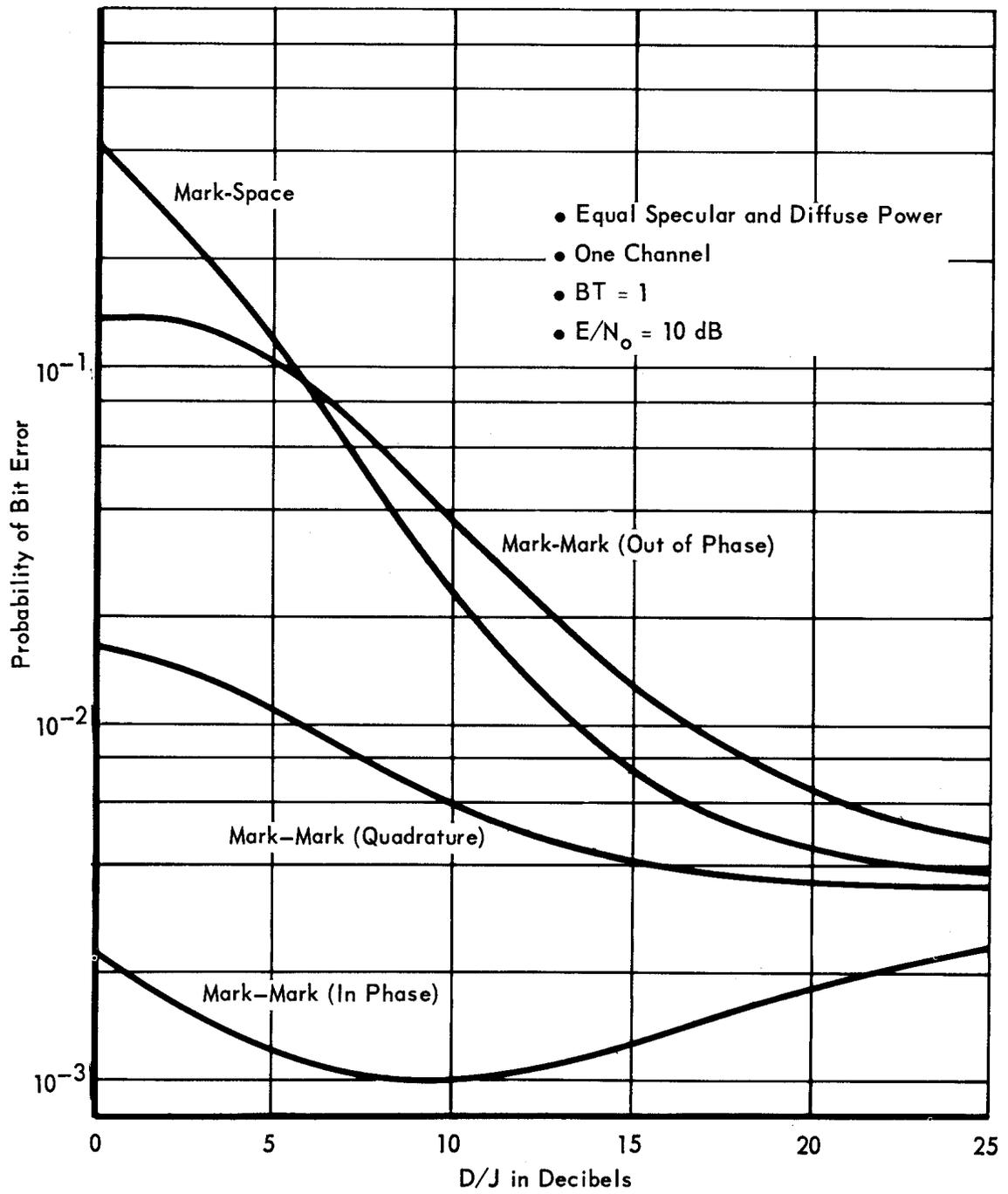

Fig. 2 - The Receiver Model
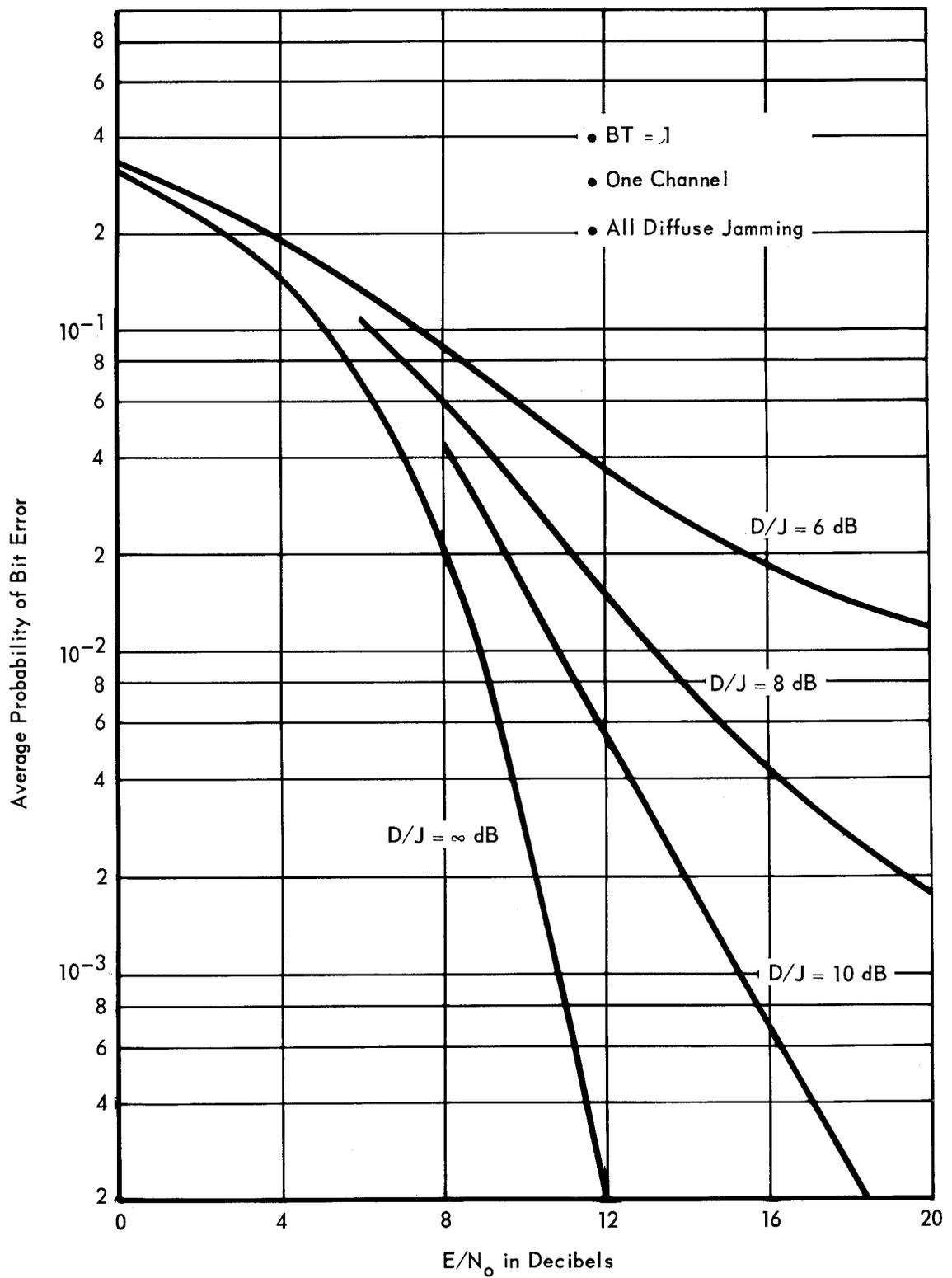
Fig. 3 - The Effect of the D/J Ratio
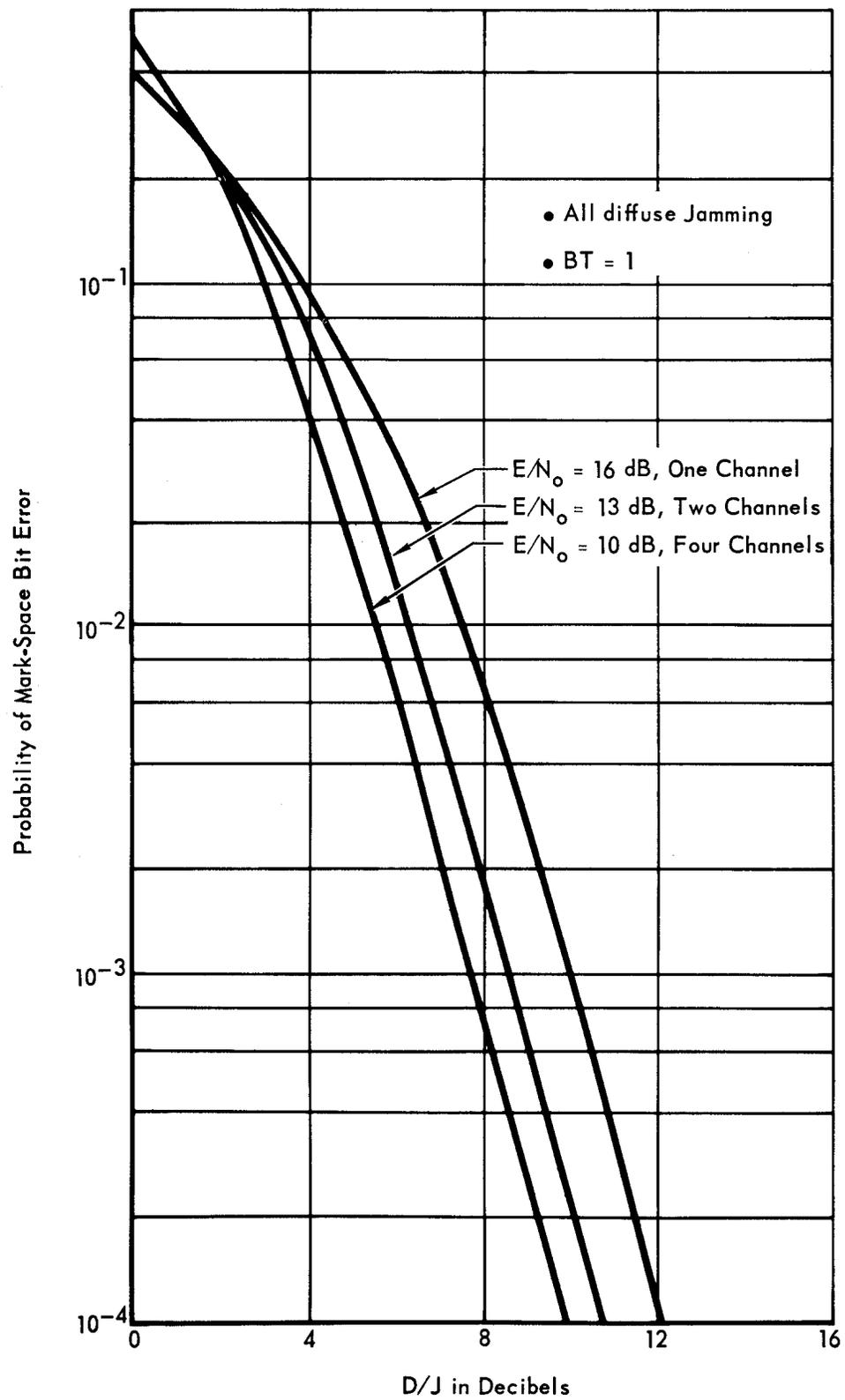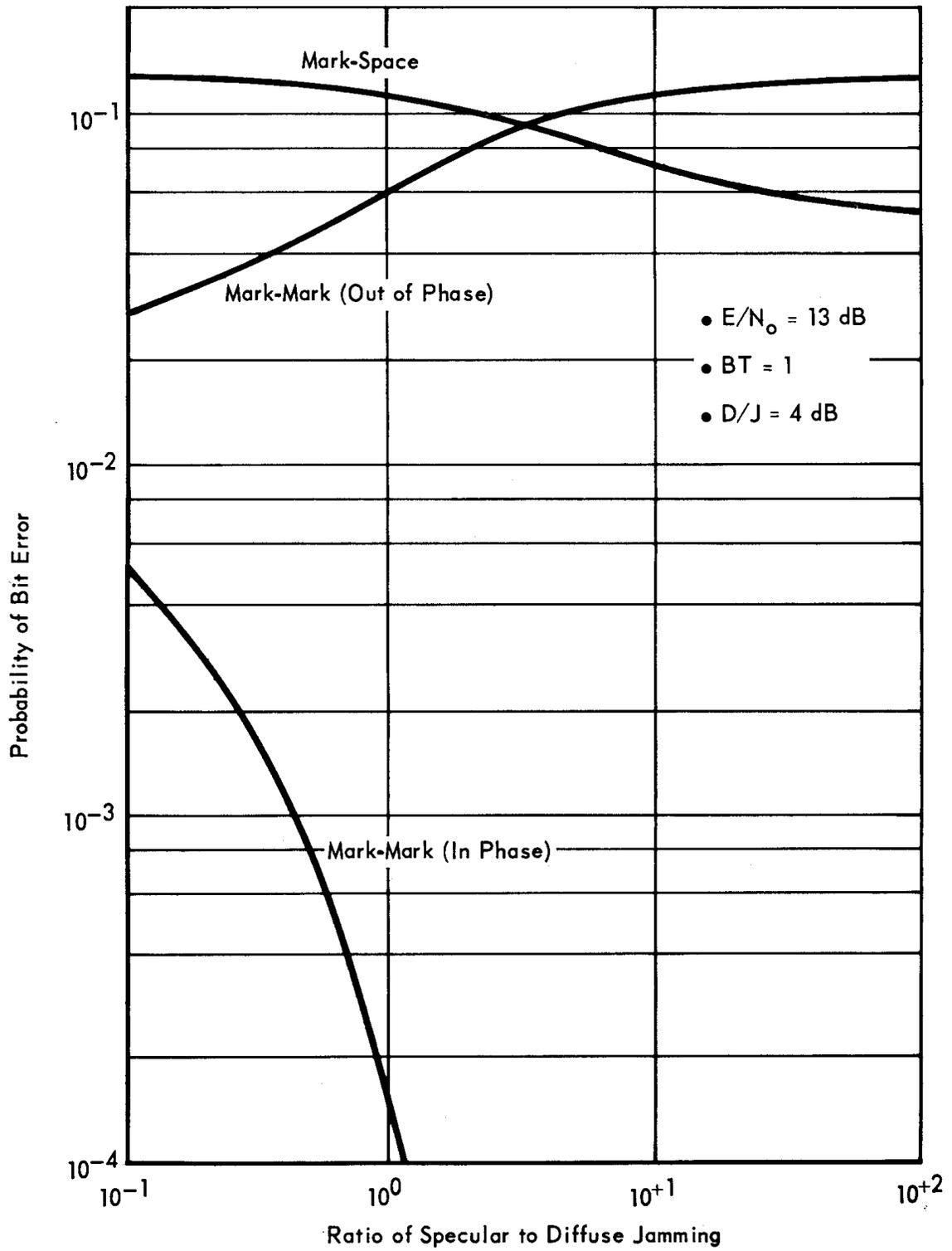
Fig. 4 - The Effect of Transmitter Power

Fig. 5 - The Effect of Diversity

Fig. 6 - The Effect of Specularity