Expectation-Maximization and Successive Interference Cancellation Algorithms For Separable Signals

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Abstract

The expectation-maximization (EM) algorithm is well established as a computationally efficient method for separable signal parameter estimation. Here, a new geometric derivation and interpretation of the EM algorithm is given that facilitates the understanding of its convergence properties. Geometric considerations then lead to an alternative separable signal parameter estimator based on successive cancellation. The new Generalized Successive Interference Cancellation (GSIC) algorithm may offer better performance than EM in the presence of large signal power disparities. Finally, application of the GSIC algorithm to CDMA-based radiolocation is discussed, and simulation results are presented.

Key Words

Expectation-maximization, separable signals, radiolocation, CDMA.

Introduction

The problem of parameter estimation for separable signals is encountered in a wide range of array signal processing, sonar, radio, and communications applications. Most such problems can be posed using the following measurement model. Let

$$\mathbf{r} = \sum_{k=1}^{K} \mathbf{s}_k(\theta_k) + \mathbf{n} \tag{1}$$

where $\mathbf{r} \in C^N$ is the measurement vector, and $\mathbf{n} \in C^N$ is circular white Gaussian noise with covariance matrix $\sigma_n^2\mathbf{I}$. For example, in direction-of-arrival estimation, $\mathbf{s}_k(\theta_k) = \mathbf{a}_k(\phi_k)s_k$, where $\mathbf{a}_k(\phi_k)$ is the steering vector for the k-th incoming signal, ϕ_k is its direction of arrival, and s_k is its amplitude [Sch86],[MiF90]. In code-division multiple access (CDMA) systems, θ_k is a mixture of

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linear and nonlinear parameters corresponding to unknown time-of-arrival, user data, and channel fading coefficients. The CDMA model will be considered in detail in the sequel.

The maximum-likelihood solution for the parameters θ_k, under the additive Gaussian noise assumption, corresponds to minimization of the L_2 norm as follows.

$$\theta^{ML} = \arg \min_{\theta} \|\mathbf{r} - \mathbf{s}(\theta)\|^2, \quad \mathbf{s}(\theta) = \sum_{k=1}^{K} \mathbf{s}_k(\theta_k), \tag{2}$$

where $\theta = [\theta_1^T, ..., \theta_K^T]^T$. The ML solution, despite its apparent simplicity, may be intractable when θ_k is a mixture of linear and nonlinear parameters. Even if θ_k is a single nonlinear parameter discretized to Q values, the ML solution has complexity $O(Q^k)$. However, if the minimization in (2) can be carried out independently over the θ_k, then the complexity is obviously only $O(KQ)$. A vast signal processing literature exists on the approximation of the optimum ML solution in (1) by iterative, separable solutions over the individual parameters. For example, in array processing, the MUSIC algorithm [Sch86] uses second-order statistics of $\mathbf{r}$ to find the noise subspace, and then exploits the orthogonality of this subspace to the *individual* vectors $\mathbf{s}_k(\theta_k)$. Alternating maximization [ZiW88] is an iterative algorithm that approximates θ^{ML}, in which the θ_k are sequentially updated, with "undesired" signals projected out. AM is notable, in that the true likelihood is a non-decreasing function of the estimates θ^i thus generated. The EM algorithm [DLR77] was subsequently used for array direction-of-arrival estimation in [MiF90]. As in alternating-maximization, expectation-maximization leads to a non-decreasing likelihood. More recently, EM was applied to the problems of CDMA joint code delay and multipath estimation in [FaA95],[LoC00],[IKT00].

Geometric Interpretation of the EM Algorithm for Separable Signals I

The original development of the EM algorithm and convergence proofs for the separable signal problem are based on statistical arguments [DLR77],[FeW88],[FeH94]. Following [FeW88], construct a set of hidden data which is not available, but permits an approximate separable solution to (1). Specifically, define

$$\mathbf{r}_k = \mathbf{s}_k(\theta_k) + \mathbf{n}_k, \quad \mathbf{r} = \sum_{k=1}^{K} r_k. \tag{3}$$

for k = 1,...K. The noise vectors $\mathbf{n}_k$ are independent circular Gaussian, with covariance matrices $\beta_k \sigma_n^2 \mathbf{I}$, where $\{\beta_k\}$ is a probability vector. The EM algorithm is expressed as follows [FeW88].

$$\theta^{i+1} = \arg \max_{\theta} E\left\{\log p(\{\mathbf{r}_k\} \mid \theta) \mid \mathbf{r}, \theta^i\right\} = \arg \max_{\theta} Q(\theta \mid \theta^i). \tag{4}$$

Evaluation of the log-likelihood and expectations following [DLR77],[FeW88] shows that the maximization (3) is now separable, with

$$\boldsymbol{q}_k^{\,i+1} = \arg \min_{\boldsymbol{q}_k} \left\| \mathbf{s}_k(\boldsymbol{q}_k) - \mathbf{s}_k(\boldsymbol{q}_k^{\,i}) - \boldsymbol{b}_k \left[\mathbf{r} - \mathbf{s}(\boldsymbol{q}^i) \right] \right\|^2 \qquad (5)$$

A straightforward application of Jensen’s inequality reveals that if Q(θ^{i+1}|θ^i)   Q(θ^i|θ^i), then p(**r**|θ^{i+1})   p(**r**|θ^i) [DLR77],[FeW88]. Hence maximization of Q(θ|θ^i) leads to a non-decreasing likelihood function. Furthermore, a second Jensen’s inequality argument shows that the maximum-likelihood solution θ^{ML} is a stable point of EM. Thus, although global convergence cannot be guaranteed, except in the case of linear parameters, the EM algorithm will converge to the ML solution provided initialization is sufficiently “close” to θ^{ML}, as proven in [FeH94].

In order to obtain an intuitive understanding of the EM algorithm for separable signals, an alternative geometric derivation of the recursion (4) is now developed. We seek an algorithm $\theta^{i+1} = A(\theta^i)$ such that

$$\left\| \mathbf{r} - \mathbf{s}(\boldsymbol{q}^{i+1}) \right\| \le \left\| \mathbf{r} - \mathbf{s}(\boldsymbol{q}^{i}) \right\|. \qquad (6)$$

The following Lemma leads to the geometric derivation of EM for separable signals.

Lemma 1: Let ||**r** – **s**(θ)|| represent the L_2 norm of the estimation error for the separable signal problem, where $\mathbf{s}(\boldsymbol{q}) = \sum_{l=1}^{K} \mathbf{s}_l(\boldsymbol{q}_l)$. The norm is upper bounded as follows, for any probability vector {β_1} and any choice of θ^i.

$$\left\| \mathbf{r} - \mathbf{s}(\boldsymbol{q}^{i+1}) \right\| \le \sum_{l=1}^{K} \left\| \boldsymbol{b}_l [\mathbf{r} - \mathbf{s}(\boldsymbol{q}^i)] + \mathbf{s}_l(\boldsymbol{q}_l^i) - \mathbf{s}_l(\boldsymbol{q}_l^{i+1}) \right\|.$$

Proof: Rewrite the L_2 norm (which is a monotonic function of the log likelihood) as

$$\left\| \mathbf{r} - \mathbf{s}(\boldsymbol{q}^{i+1}) \right\| = \left\| \sum_{l=1}^{K} \left(\boldsymbol{b}_l [\mathbf{r} - \mathbf{s}(\boldsymbol{q}^i)] + \mathbf{s}_l(\boldsymbol{q}_l^i) - \mathbf{s}_l(\boldsymbol{q}_l^{i+1}) \right) \right\|, \qquad (7)$$

where $\beta_1 \geq 0$, $\mathbf{1}^T\beta = 1$. Lemma (1) then follows directly from the triangle inequality applied to each summand in (7).

Next, we claim that the following update results in a non-increasing norm for ||**r** – **s**(θ^i)||.

$$\boldsymbol{q}_l^{\,i+1} = \arg \min_{\boldsymbol{q}_l} \left\| \boldsymbol{b}_l [\mathbf{r} - \mathbf{s}(\boldsymbol{q}^i)] + \mathbf{s}_l(\boldsymbol{q}_l^i) - \mathbf{s}_l(\boldsymbol{q}_l) \right\|, l = 1,2,\ldots,K, \qquad (8)$$

which is precisely the EM algorithm for separable signals. To prove this, use the algorithm in (8) and Lemma 1 to show that

$$\left\|\beta_l[\mathbf{r}-\mathbf{s}(\theta^i)]+\mathbf{s}_l(\theta_l^i)-\mathbf{s}_l(\theta_l^{i+1})\right\| \le \left\|\beta_l[\mathbf{r}-\mathbf{s}(\theta^i)]+\mathbf{s}_l(\theta_l^i)-\mathbf{s}_l(\theta_l^i)\right\| = \left\|\beta_l[\mathbf{r}-\mathbf{s}(\theta^i)]\right\|$$

$$\Rightarrow \left\|\mathbf{r}-\mathbf{s}(\theta^{i+1})\right\| \le \sum_{l=1}^{K}\left\|\beta_l[\mathbf{r}-\mathbf{s}(\theta^i)]\right\| = \left\|\mathbf{r}-\mathbf{s}(\theta^i)\right\|. \tag{9}$$

Thus, the EM algorithm has been derived from purely geometric considerations, by minimizing the triangle inequality upper bound on the likelihood in Lemma 1.

As shown in [DLR77], the ML solution is always a stable point of the EM algorithm. A direct geometric proof of this result for the separable signal problem can be likewise obtained. The proof is via contradiction – assume that θ^{ML} is not a stable point, then $Q(\theta|\theta^{ML}) > Q(\theta^{ML}|\theta^{ML})$ for some $\theta \quad \theta^{ML}$. Thus, evaluation of $Q(\theta|\theta^{ML})$ shows that if $Q(\theta|\theta^{ML}) > Q(\theta^{ML}|\theta^{ML})$, equivalently

$$\sum_{k=1}^{K}\frac{1}{\beta_k}\left\|\mathbf{s}_k(\theta_k^{ML})-\mathbf{s}_k(\theta_k)+\beta_k\left[\mathbf{r}-\mathbf{s}(\theta^{ML})\right]\right\|^2 < \sum_{k=1}^{K}\frac{1}{\beta_k}\left\|\mathbf{s}_k(\theta_k^{ML})-\mathbf{s}_k(\theta_k^{ML})+\beta_k\left[\mathbf{r}-\mathbf{s}(\theta^{ML})\right]\right\|^2 = \left\|\mathbf{r}-\mathbf{s}(\theta^{ML})\right\|^2 \tag{10}$$

for some $\theta \quad \theta^{ML}$. Manipulation of the l.h.s. of (10), followed by application of the triangle inequality yields

$$\begin{aligned}\left\|\mathbf{s}_k(\theta_k^{ML})-\mathbf{s}_k(\theta_k)+\beta_k\left[\mathbf{r}-\mathbf{s}(\theta^{ML})\right]\right\| &= \left\|(\mathbf{r}-\mathbf{s}(\theta_{(k)}^{ML},\theta_k))-(\mathbf{r}-\mathbf{s}(\theta^{ML}))+\beta_k\left[\mathbf{r}-\mathbf{s}(\theta^{ML})\right]\right\| \\ &\ge \left\|\mathbf{r}-\mathbf{s}(\theta_{(k)}^{ML},\theta_k)\right\| - (1-\beta_k)\left\|\left[\mathbf{r}-\mathbf{s}(\theta^{ML})\right]\right\| \\ &\ge \beta_k\left\|\left[\mathbf{r}-\mathbf{s}(\theta^{ML})\right]\right\| \ge 0.\end{aligned} \tag{11}$$

where $\mathbf{s}(\theta_{(k)}^{ML},\theta_k)=\sum_{l\neq k}\mathbf{s}_l(\theta_l^{ML})+\mathbf{s}_k(\theta_k)$. Thus,

$$\sum_{k=1}^{K}\frac{1}{\beta_k}\left\|\mathbf{s}_k(\theta_k^{ML})-\mathbf{s}_k(\theta_k)+\beta_k\left[\mathbf{r}-\mathbf{s}(\theta^{ML})\right]\right\|^2 \ge \sum_{k=1}^{K}\beta_k\left\|\mathbf{r}-\mathbf{s}(\theta^{ML})\right\|^2 = \left\|\mathbf{r}-\mathbf{s}(\theta^{ML})\right\|^2 \tag{12}$$

which directly contradicts (10).

Geometric Interpretation of the EM Algorithm for Separable Signals II

The EM algorithm has been derived by minimizing the triangle inequality upper bound (Lemma 1) for the L_2 norm of the estimation error. The minimization in (7) leads to an alternative interpretation. Define the descent increment for signal l at iteration $^{i+1}$ as $\Delta_l^{i+1}=\mathbf{s}_l(\theta_l^{i+1})-\mathbf{s}_l(\theta_l^i)$. The EM algorithm can then be interpreted as minimizing

$$\Delta_l^{i+1} = \arg \min_{\Delta_l} \left\| \Delta_l - \beta_l [\mathbf{r} - \mathbf{s}(\theta^i)] \right\|, \;\; l = 1,2,\ldots,K, \;\; s.t. \;\; \Delta_l = \mathbf{s}_l(\theta_l) - \mathbf{s}_l(\theta_l^i). \quad (13)$$

That is, the EM algorithm tries to find a set of increments, each of which is proportional to the fraction β_l of the error vector $[\mathbf{r} - \mathbf{s}(\theta^i)]$. For example, if the probability vector $\beta = (1/K)\mathbf{1}$, the EM algorithm weighs each signal equally, in trying to match the sum of the increments $\Delta_l^1 = \mathbf{s}_l(\theta_l^1)$ to the error vector. The procedure is shown graphically in Figure 1 for the case K = 3.

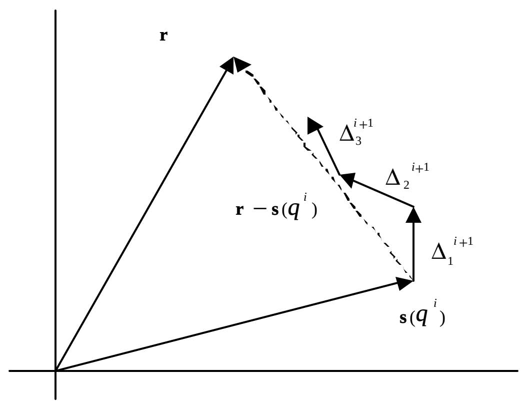

Figure 1: Matching Search Increments to the Error Vector – EM Algorithm

The choice of β is arbitrary. For example, if $\beta_1 = 1$, and $\mathbf{s}(\theta^0) = \mathbf{0}$ is a feasible initialization, then the EM algorithm will attempt to find the single vector $\mathbf{s}_1(\theta_1)$ that minimizes $\|\mathbf{r} - \mathbf{s}_1(\theta_1)\|$. In this case, the algorithm terminates in one step, and $\theta_1^1 = \arg\min_\theta \|\mathbf{r} - \mathbf{s}_1(\theta_1)\|$ is a stable point. Obviously, this choice of initialization and β is undesirable, unless it is known that only signal $\mathbf{s}_1(\theta_1)$ predominates. For the choice $\beta_l = 1/K$, it is clear that the EM algorithm takes a conservative approach to "minimizing the gap" between $\mathbf{r}$ and $\mathbf{s}(\theta^i)$ on iteration $^{i+1}$, by trying to use all signals equally to make up the total remaining error vector.

Generalized Successive Interference Cancellation Algorithm

When all signals have equal energy, and approximately the same degrees of freedom, then the EM strategy as shown in Figure 1 is a sensible way to minimize the error norm, while hopefully avoiding local minima. In fact, when the signals are linear in θ, that is $\mathbf{s}_k(\theta_k) = \mathbf{S}\theta_k$, then the resulting EM algorithm always converges to the minimum-norm solution [FeH94]. However, in the nonlinear case, there is again no guarantee of global convergence, unless initialization is suitably close to θ^{ML}. In many communications applications, a substantial disparity in signal energy may exist, as in CDMA systems without power control. For example, in the radiolocation application discussed in the next section, a master node transmits request-to-send signals to multiple reference nodes (RTS), and round-trip travel time is used to determine range and position. Since the reference nodes are likely to be at varying distances from the master, and power control is infeasible due to the short packet length, large signal power variations are quite likely and should be exploited. It would make sense in the EM algorithm to then allocate the largest value of β_l to the signal $\mathbf{s}_l(\theta_l)$ with the largest power, to obtain the most rapid decrease in the error norm. However, the signal energy is rarely known a-priori, hence we propose an alternative GSIC algorithm for sequentially minimizing the error, in a manner which is efficient for large signal power disparities.

The GSIC algorithm is stated as follows for the first step. Let p_i, i = 1,2,...K represent a permutation of the indices {1,2,...K}. Then

$$p_1 = \arg \min_k \left\{ \min_{\theta_k} \left\| \mathbf{r} - \mathbf{s}_k(\theta_k) \right\|^2 \right\}, \quad \hat{\theta}_{p_1} = \arg \min_{\theta_{p_1}} \left\| \mathbf{r} - \mathbf{s}_{p_1}(\theta_{p_1}) \right\|^2 \tag{14}$$

That is, the first step of GSIC finds the vector $\mathbf{s}_k(\theta_k)$ which lies closest to $\mathbf{r}$, over all choices of θ_k. Again, this step would be identical to a "clairvoyant" EM algorithm, which assigns $\beta_k = 1$ to the vector which achieves the minimum error norm over all θ_k. The GSIC algorithm is defined in Table 1 for successive steps.

For k = 1,2,....K
Form the cancelled signal $$\mathbf{r}^k = \mathbf{r} - \sum_{l=1}^{k-1} \mathbf{s}_{p_l}(\hat{\theta}_{p_l})$$
Find p_k $$p_k = \arg \min_{l \neq p_1, p_2, \ldots, p_{k-1}} \left\{ \min_{\theta_l} \left\lVert \mathbf{r}^k - \mathbf{s}_l(\theta_l) \right\rVert^2 \right\}$$
Save p_k-th parameter estimate (computed from previous step.) $$\hat{\theta}_{p_k} = \arg \min_{\theta_{p_k}} \left\lVert \mathbf{r}^k - \mathbf{s}_{p_k}(\theta_{p_k}) \right\rVert^2$$
Next k

Table 1 – GSIC Algorithm for Separable Signals.

The GSIC algorithm is an "aggressive" or "greedy" approach to minimizing the error norm compared with EM for $\beta_k = 1/K$. At every step, GSIC tries to find a single signal and its parameter estimate that leads to the largest reduction in the error vector L_2 norm. Intuitively, GSIC should perform well when one signal predominates over all others.

The following proposition states that GSIC results in a non-decreasing likelihood, so long as as each signal vector satisfies $\mathbf{s}_k(\theta_\kappa^0) = \mathbf{0}$ for some θ_κ^0. Note that this is a mild constraint – in array processing and communications, θ_κ generally includes a linear parameter (e.g. signal amplitude, multipath) which is unconstrained, hence $\mathbf{s}_k(\theta_\kappa^0) = \mathbf{0}$ is readily satisfied.

Proposition: Assume that a point θ^0 exists such that $\mathbf{s}_k(\theta_\kappa^0) = \mathbf{0}$ for k = 1,...,K. The sequence of estimates in the GSIC algorithm (Table 1) results in a non-increasing error norm, such that

$$\left\| \mathbf{r} - \mathbf{s}(\hat{\theta}^k) \right\|^2 \leq \left\| \mathbf{r} - \mathbf{s}(\hat{\theta}^{k-1}) \right\|^2$$

where the estimated signal at iteration k is

$$\mathbf{s}(q^k) = \sum_{l=1}^{k} \mathbf{s}_{p_l}(\hat{q}_{p_l})$$

Proof: From the construction of the GSIC algorithm, we have

$$\left\|\mathbf{r} - \hat{\mathbf{s}}(q^k)\right\|^2 = \left\|\mathbf{r}^k - \hat{\mathbf{s}}_{p_k}(q_{p_k})\right\|^2 \le \left\|\mathbf{r}^k\right\|^2 = \left\|\mathbf{r} - \hat{\mathbf{s}}(q^{k-1})\right\|^2$$

Again, note that the above equation hinges on $\mathbf{s}_k(\theta_k^0) = \mathbf{0}$ being a valid signal vector.

Application to CDMA-Based Radiolocation

The application of GSIC to CDMA-based radiolocation is now considered. Following [IKT00], a a system of master and reference nodes is assumed, in which the references have absolute or relative positioning information. The master transmits a RTS packet, and each reference, upon reception of the RTS, immediately transmits an ACK packet. All packets are direct-sequence CDMA waveforms. Hence, the received continuous-time signal in flat fading at the master node is

$$r(t) = \sum_{k=1}^{K} \sum_{m=0}^{M} a_k(m) s_k(t - mT - T_k) + n(t), \qquad (15)$$

for 0 t < MT. The $a_k(m)$ are complex variables representing the product of flat fading and data (navigation message,) and n(t) is additive white Gaussian noise. Each DS waveform $s_k(t)$ has duration 0 t < T s., and T_k is the unknown time-of-arrival. Following [IKT00], vectors **r**(n) of Nyquist samples are formed, defined by

$$\mathbf{r}(n) = [r(((n+1)N_s - 1)T_s), r(((n+1)N_s - 2)T_s), \ldots, r(nN_s T_s)]^T$$

where T_s is the Nyquist sampling interval, and $N_s = T/T_s$ is the number of samples per symbol interval. It is readily shown that

$$\mathbf{r}(n) = \sum_{k=1}^{K} \sum_{l=0}^{1} a_k(n-l) \mathbf{s}_k(T_k - lT) + \mathbf{n}(n) \qquad (16)$$
$$\mathbf{s}_k(\boldsymbol{t}) = [s_k(((n+1)N_s - 1)T_s - \boldsymbol{t}), s_k(((n+1)N_s - 2)T_s - \boldsymbol{t}), \ldots, s_k(nN_s T_s - \boldsymbol{t})]^T$$

Finally, a received signal model of the form of (1) is obtained by stacking the **r**(n) into a vector of M+1 symbols.

$$\mathbf{r} = \sum_{k=1}^{K} \mathbf{S}_k(T_k)\mathbf{a}_k + \mathbf{n}, \qquad \mathbf{a}_k = [a_k(M), a_k(M-1), \ldots, a_k(0)]^T, \qquad (17)$$

where the block-Toeplitz signal matrix $\mathbf{S}_k(T_k)$ is defined by

$$
\mathbf{S}_k(T_k) = \begin{bmatrix} \mathbf{s}_k(T_k) & \mathbf{s}_k(T_k - T) & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{s}_k(T_k) & \mathbf{s}_k(T_k - T) & & \\ & & & \ddots & \mathbf{s}_k(T_k - T) \\ \mathbf{0} & \cdots & \cdots & \mathbf{0} & \mathbf{s}_k(T_k) \end{bmatrix}. \tag{18}
$$

Thus, $\mathbf{r}$ is in the form of the separable signal model, with $\theta_k = [T_k, \mathbf{a}_k^T]^T$, and $\mathbf{s}_k(\theta) = \mathbf{S}_k(T_k)\mathbf{a}_k$. The generic GSIC procedure can now be applied, resulting in the algorithm of Table 2.

For k = 1,2,....K
Form the cancelled signal $\mathbf{r}^k = \mathbf{r} - \sum_{l=1}^{k-1} \mathbf{S}_{p_l}(\hat{T}_{p_l})\hat{\mathbf{a}}_{p_l}$
Compute the next signal index p_k $p_k = \arg \max_l \left\{ \max_{T_l} \mathbf{r}^{kH}\mathbf{S}_l(T_l)\left[\mathbf{S}_l(T_l)^H \mathbf{S}_k(T_l)\right]^{-1} \mathbf{S}_l(T_l)^H \mathbf{r}^k \right\}$ $\approx \arg \max_l \left\{ \max_{T_l} \sum_{n=0}^{M} \lVert \mathbf{s}_l(T_l)^H \mathbf{r}^k(n) + \mathbf{s}_l(T_l - T)^H \mathbf{r}^k(n+1) \rVert^2 \right\}$
Compute the delay estimate for signal p_k $\hat{T}_{p_k} = \arg \max_{T_{p_k}} \mathbf{r}^{kH}\mathbf{S}_{p_k}(T_{p_k})\left[\mathbf{S}_{p_k}(T_{p_k})^H \mathbf{S}_{p_k}(T_{p_k})\right]^{-1} \mathbf{S}_{p_k}(T_{p_k})^H \mathbf{r}^k$ $\approx \arg \max_{T_{p_k}} \sum_{n=0}^{M} \lVert \mathbf{s}_{p_k}(T_{p_k})^H \mathbf{r}^k(n) + \mathbf{s}_{p_k}(T_{p_k} - T)^H \mathbf{r}^k(n+1) \rVert^2$
Compute the amplitude estimates for signal p_k $\hat{\mathbf{a}}_{p_k} = \left[\mathbf{S}_{p_k}(\hat{T}_{p_k})^H \mathbf{S}_{p_k}(\hat{T}_{p_k})\right]^{-1} \mathbf{S}_{p_k}(\hat{T}_{p_k})^H \mathbf{r}^k \Rightarrow$ $\hat{\mathbf{a}}_{p_k}(n) \approx \frac{1}{\lVert \mathbf{s}_{p_k}(\hat{T}_{p_k}) \rVert^2 + \lVert \mathbf{s}_{p_k}(\hat{T}_{p_k} - T) \rVert^2} \left[\mathbf{s}_{p_k}(\hat{T}_{p_k})^H \mathbf{r}^k(n) + \mathbf{s}_{p_k}(\hat{T}_{p_k} - T)^H \mathbf{r}^k(n+1)\right]$
Next k

Table 2 – GSIC Algorithm CDMA Delay Estimation.

Results

The GSIC algorithm was simulated for the radiolocation problem under the following conditions. The waveforms $s_k(t)$ were 31 chip Gold sequences, with nominal SNR of $E_b/N_0 = 10$ dB. The delays

$\{T_k\}$ were chosen randomly in [0,T) for each user and on each simulation run. For user k, ρ_k is the power relative to 10 dB E_b/N_0. Hence, if $\rho_k = 10$, user k has an effective SNR of 20 dB. An ensemble average timing error is defined by $e_k = \frac{1}{R}\sum_{r=1}^{R}\left|\hat{T}_k^{\,r} - T_k\right|$. The variable α_k is the percentage of runs for which the instantaneous timing error was less than one chip. Table 3 shows results for K = 5 users averaged over R = 50 runs. The matched filter results correspond to the algorithm of Table 2, with $\mathbf{r}^k = \mathbf{r}$ for each step (no successive cancellation.) Clearly, the GSIC algorithm far outperforms a conventional matched filter for delay estimation. Although the two stronger users are acquired with 100% probability by the matched filter, acquisition probability drops to 25% or less for the weaker users. In contrast, acquisition probability is a minimum of 98% for GSIC.

ρ_k	1	1	1	10	10
e_k	.066	.062	.68	.68	.066
α_k	1	1	.98	.98	1

GSIC Results

ρ_k	1	10	1	10	1
e_k	7.4	.066	8.85	.062	7.05
α_k	.4	1	.26	1	.24

Matched Filter Results

Table 3. GSIC simulation results for five users

Table 4 shows corresponding results for K = 10 users, again using E_b/N_0 = 10 dB, and 31 chip Gold sequences. Averaging was performed over R = 25 simulation runs.

ρ_k	10	1	10	1	10
e_k	.069	2.18	3.61	.06	.07
α_k	1	.88	1	1	1

ρ_k	1	10	1	10	1
e_k	.499	1.30	1.59	.065	3.55
α_k	.96	.96	.92	1	.8

Table 4. GSIC simulation results for ten users.

At first, the estimation errors for the K = 10 user case appear large. However, note that the weaker users nevertheless had acquisition probabilities of 80% or greater, and the stronger users were acquired with nearly 100% probability.

Conclusions

The EM algorithm for separable signals was rederived based on geometric considerations. It was shown that EM corresponds to minimization of a triangle inequality upper bound on the estimation error. Based on the error vector minimization interpretation, an alternative GSIC algorithm was developed which may offer better performance when large power differences exist between users. Preliminary simulation results for GSIC for CDMA delay estimation demonstrate far superior performance to matched filter acquisition. Future results will include comparisons between EM, true maximum likelihood, and GSIC, as well as an extension to frequency-selective channels.

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