BIT ERROR RATES IN THE PRESENCE OF UNTRACKED TIME BASE FLUCTUATION

A. O. ROCHE
General Dynamics

P. MALLORY
Dynatronics

Summary This paper presents a simple four-step procedure for estimating the error probability of an NRZ PCM Synchronizer and Detector operating on an NRZ Bit Stream in the presence of a fluctuating data frequency source. The four steps are as follows. First, the bit error probability is calculated for Gaussian time base fluctuation as a function of the energy per bit to noise power density ratio. The second step is to model the synchronizer as an ordinary linear servo for small phase errors and a closed loop bandwidth, small compared to the bit rate, so that effect of the randomness of the data is averaged out. With the linear model, the time base error in tracking the input signal is calculated also utilizing this approximation as if there were no additive noise. The third step is to calculate the mean squared time base error due to the additive Gaussian noise alone. The fourth step is to combine the errors found in steps two and three as if they were independent and use the graphs found in Step 1 to determine the error rates. It is assumed that the total untracked time base fluctuation is Gauss ian.

The calculated error probabilities are compared with measured data. There appears to be good correspondence between the calculated and measured error probability.

Experimental Set-Up The bit synchronizer and its accompanying input signal are shown in Figure 1[1]. The bit rate of the input signal simulator is determined by a VCO which is noise modulated by lowpass filtered,white, Gaussian noise to generate the time base fluctuation. The rms value, σ_f, of the frequency modulation of the bit clock is measured by triggering an' oscilloscope on the leading edge of the bit clock and observing the excursion of the following zero crossing when frequency modulating with a sine wave. The rms value of the sine wave is measured using a true rms meter. The corresponding rms frequency modulation is the peak-to-peak value divided by $2\sqrt{2}$. A noise generator is substituted for the sine wave generator and its rms value measured by the same meter. A random bit stream is generated by sampling wide band noise.

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[1] The bit synchronizer used is Dynatronics Model BSC-307.

Wideband, Gaussian noise is added to the signal. The bandwidth of the signal plus noise is ten times the bit rate, f_b. The bit rate for all data reported here is 100Kb. The NRZ signal plus noise is then put into a phase detector which is designed to give an approximate maximum likelihood estimate of the bit phase when used in conjunction with a phase lock loop. In the phase-lock loop, the phase error signals are filtered in the compensation network and used to control the local clock which is a voltage controlled oscillator (VCO) with a linear control characteristic. The control loop bandwidth is determined by the gain factors of the compensation network and the VCO. Phase lead compensation, to provide stability with the appropriate damping factor, is introduced by adding the phase error signal directly to the VCO with a proper gain factor in parallel with the integral of the phase error which is generated by an up down counter. Under the assumed conditions, where the control loop undamped natural frequency is between 0.1% and 1% of bit rate, and for bit error rates of practical interest, the quantization error of the instrumentation can be neglected. The detected bit stream is compared with the input bit stream and the errors counted. Sufficiently large samples were taken to provide good bit error probability estimates.

Error Probability Calculations The bit detection is done by a “dumping integrator” which integrates for t_o seconds where t_o is the bit duration. In order to calculate the error rate in the presence of time base perturbations, it is assumed that the perturbations are of low frequency compared to $1/t_o$, so that the bit duration is almost constant and the time base error can be modeled as creating a time displacement between the beginning of a bit and the beginning of the integration period, while at the same time, the bit period and the integration period stay fixed at to seconds. With this assumption, the bit error rate for NRZ PCM is given approximately by the equation given along with its plot in Figure 2.

Time Base Fluctuation Tracking Errors The servo system shown in figure 1 is approximated as being a second order servo with an undamped natural frequency of f_n cps and a damping factor of .7. It can be shown that such a servo has an error transfer function which can be approximated as shown in figure 3. The phase perturbation of the data stream is produced by frequency modulating the bit clock with white noise which has been passed through a four pole butterworth which cuts off at f_q cps. This gives a flat frequency modulation spectrum, as referred to baseBand, of one sided density J out to f_q. The one sided power spectrum, G(f), of the resulting phase perturbation can be obtained by dividing the frequency modulation spectrum by $(2 \pi f)$ and is shown in figure 4.

The mean square value of the resulting phase perturbation is then

$$\sigma_e^2 = \frac{J}{(2\pi)^2 f_n^4} \int_0^{f_n} f^2 df + \frac{J}{(2\pi)^2} \int_{f_n}^{f_q} \frac{df}{f^2} + \frac{J f_q^8}{(2\pi)^2} \int_{f_q}^{\infty} \frac{df}{f^{10}}$$

$$= \frac{J}{(2\pi)^2} \left(\frac{4}{3f_n} - \frac{8}{9f_q}\right) \text{ cycles}^2. \quad (1)$$

Since the noise bandwidth of the four pole filter is very close to f_q, J can be obtained from the relation

$$\sigma_f^2 = J f_q \; , \quad (2)$$

where σ_f^2 is the mean square value of the frequency modulation. Actually, the spectrum of the noise source used cuts off below about 10 cps, but it has been assumed in the above, as an approximation, that the spectrum extends to zero frequency.

Total System Time Base Error The system is assumed to have two sources of time base error. The first is the error in tracking the time base fluctuation of the input signal and the second is the error caused by the presence of additive Gaussian noise with the sigpal. The mean squared values of these two errors are delineated by σ_e^2 and σ_n^2 respectively. It will be assumed that these two errors are independent and therefore the total mean squared error is given by

$$\sigma^2 = \sigma_n^2 + \sigma_e^2 . \text{ (cycle)}^2 . \quad (3)$$

We will use the result obtained earlier in the equation (39) of “A Maximum Likelihood Bit Synchronizer”, given in Session I of this conference that

$$\sigma_n^2 = \frac{1.5 f_n N_o \Delta}{t_o S} \text{ (cycle)}^2 , \quad (4)$$

where N_o is the two-sided noise power density, t_o is the bit duration, S is the signal power and Δ is the time displacement between the sampling times of the two matched filters in the maximum likelihood synchronizer.

Discussion Figure 5 shows test results on bit error probability versus (S/N) in a bandwidth equal to half the bit rate. This corresponds to E/N_o where E = bit energy and N_o is the two sided noise power spectral density. For this figure, the bit clock was not frequency modulated. A random bit stream was used. The curve drawn is the theoretical curve for NRZ plus wide band Gaussian noise using a perfectly synchronized integrate and dump detector. Note that both for hard wire and for narrow band (f_n=100) synchronization, very good agreement with theory is obtained. With the narrow band, it is seen from Table I that $\sigma_n < 0.01$ so very little performance is lost as indicated. For the wide band, f_n =1000 Hz the number of errors is increased. As seen from Table I, for zero frequency modulation of the clock, the agreement with linear theory is quite good.

Figure 6 shows test results with noise and bit clock frequency modulation. As seen from Table I the agreement with theory is good as long as there is no slippage. It is interesting to note that when the calculated time base error due to bit clock modulation is larger than the calculated error due to the additive noise, the agreement with theory is better than vice vers a. This may indicate that the system is more sensitive to the assumptions and approximations in the additive noise theory leading to equation (4). It may also be seen that the system seems to be more sensitive to noise induced time base error, relative to PLL slippage, than to clock induced time base error possibly for the same reason. The slippage seems to start when the phase errors get to 0.05 bit periods rms. This corresponds to about 0.3 radiams rms.

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TABLE 1 COMPARISON OF THEORY AND TEST DATA

f_n	$\frac{S}{N}$	% Frequency Modulation RMS	σ_n^2	σ_e^2	$\sigma_n^2+\sigma_e^2$	σ	Pe_{cal}	Pe_{mea}	
100	4	0.175	0.0001	0.0051	0.0052	0.072	$3.3x10^{-2}$	$3.5x10^{-2}$	
	8	0.175	0.00005	0.0051	0.0051	0.072	$5.7x10^{-3}$	$5.4x10^{-3}$	
	16	0.175	0.000025	0.0051	0.0051	0.072	$2.8x10^{-4}$	$2.5x10^{-3}$	
	4	0.25	0.0001	0.010	0.0101	0.010	$3.6x10^{-2}$	$4.5x10^{-2}$	slippage
	8	0.25	0.00005	0.010	0.0100	0.010	$7.6x10^{-3}$	$1.3x10^{-2}$	slippage
	16	0.25	0.000025	0.010	0.0100	0.010	$9.0x10^{-4}$	$3.0x10^{-3}$	slippage
1000	4	0.35	0.001	0.002	0.0030	0.055	$3.0x10^{-2}$	$5.0x10^{-2}$	slippage
	8	0.35	0.0005	0.002	0.0025	0.050	$4.0x10^{-3}$	$5.4x10^{-3}$	
	16	0.35	0.00025	0.0002	0.0022	0.047	$1.2x10^{-4}$	$1.1x10^{-4}$	
	4	0.49	0.001	0.004	0.0050	0.070	$3.0x10^{-2}$	$5.0x10^{-2}$	slippage
	8	0.49	0.0005	0.004	0.0045	0.067	$5.0x10^{-3}$	$5.6x10^{-3}$	slippage
	16	0.49	0.00025	0.004	0.00425	0.065	$2.0x10^{-4}$	$1.5x10^{-4}$	
	4	0.70	0.001	0.008	0.0090	0.095	$3.5x10^{-2}$	$5.0x10^{-2}$	slippage
	8	0.70	0.0005	0.008	0.0085	0.091	$7.0x10^{-3}$	$7.0x10^{-3}$	
	16	0.70	0.00025	0.008	0.0082	0.090	$8.0x10^{-4}$	$3.2x10^{-4}$	
	4	0	0.001	0	0.0010	0.032	$2.6x10^{-2}$	$3.7x10^{-2}$	
	8	0	0.0005	0	0.0005	0.022	$3.0x10^{-3}$	$4.5x10^{-3}$	
	16	0	0.00025	0	0.00025	0.016	$7.5x10^{-5}$	$9.0x10^{-5}$	

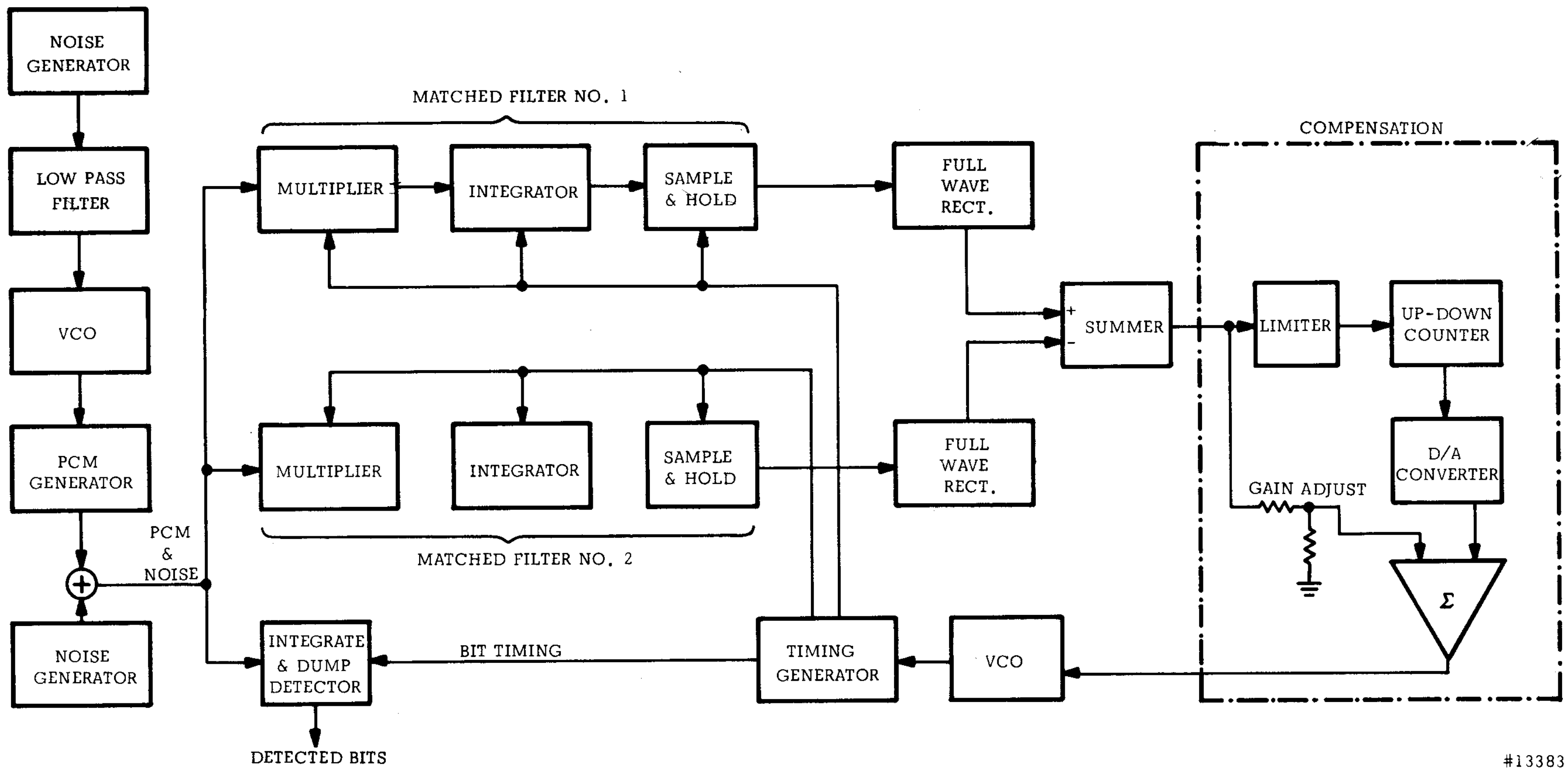
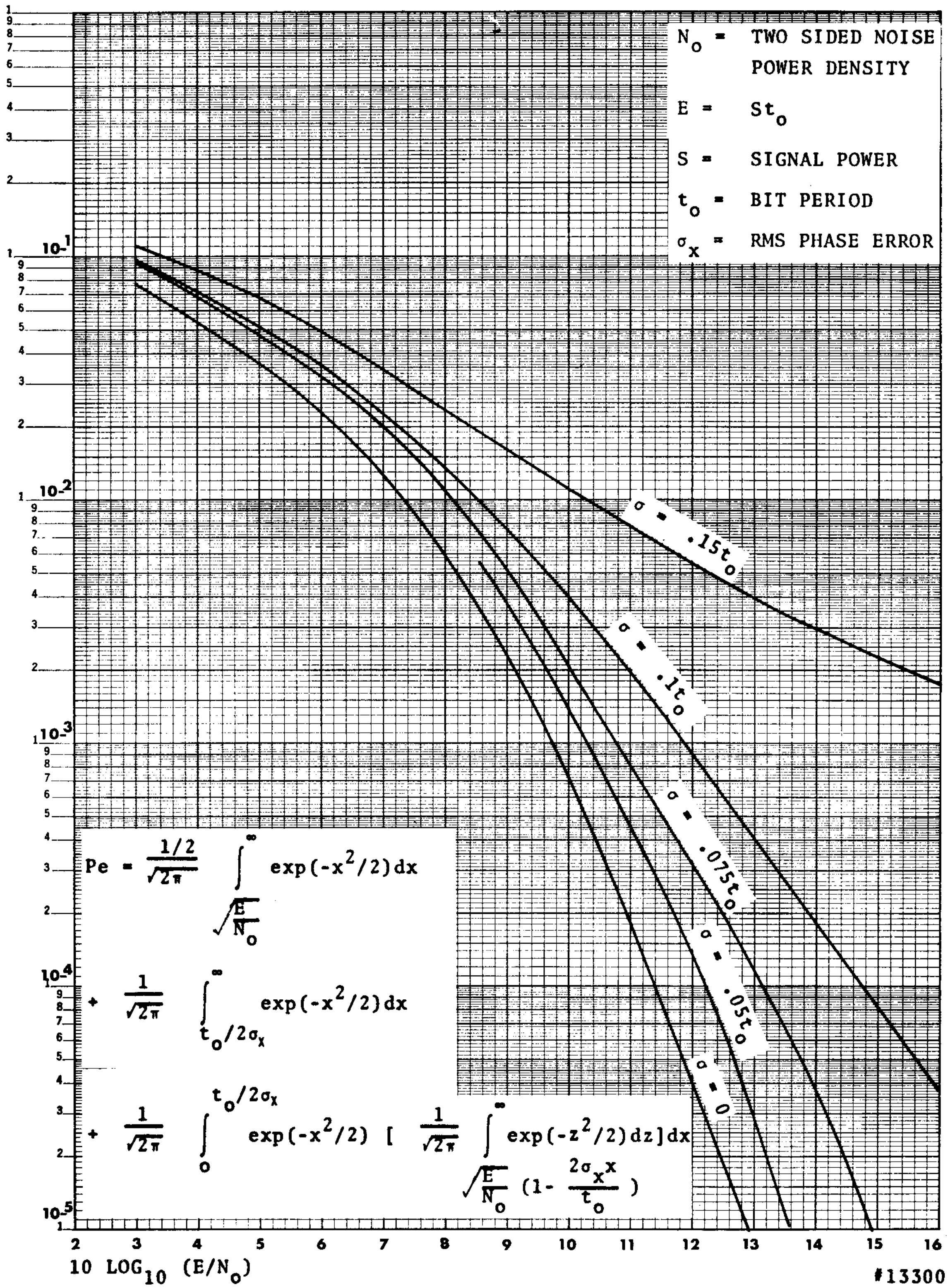


Figure 1. System Block Diagram



$$Pe = \frac{1/2}{\sqrt{2\pi}} \int_{\sqrt{E/N_o}}^{\infty} \exp(-x^2/2)\,dx + \frac{1}{\sqrt{2\pi}} \int_{t_o/2\sigma_x}^{\infty} \exp(-x^2/2)\,dx + \frac{1}{\sqrt{2\pi}} \int_{0}^{t_o/2\sigma_x} \exp(-x^2/2) \left[\frac{1}{\sqrt{2\pi}} \int_{\sqrt{E/N_o}\left(1-\frac{2\sigma_x x}{t_o}\right)}^{\infty} \exp(-z^2/2)\,dz \right] dx$$

Figure 2. Bit Error Rate As A Function Of Untracked Gaussian Time Base Error

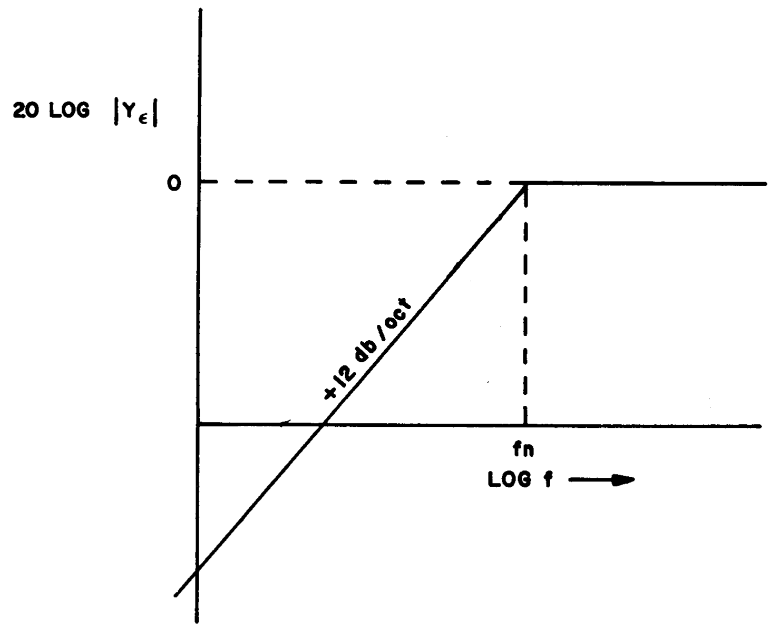


Figure 3. Approximate Phase Error Transfer Function

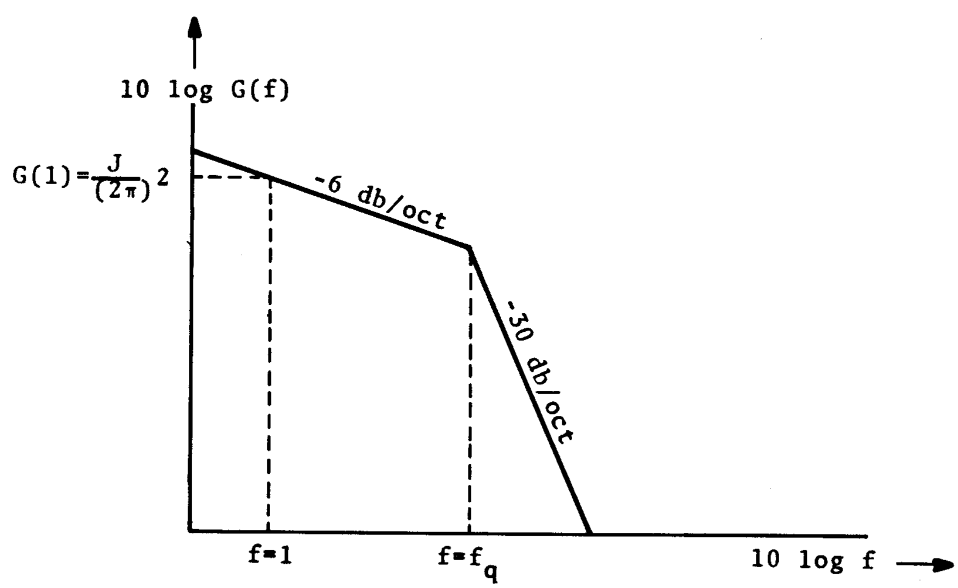


Figure 4. Time Base Spectrum

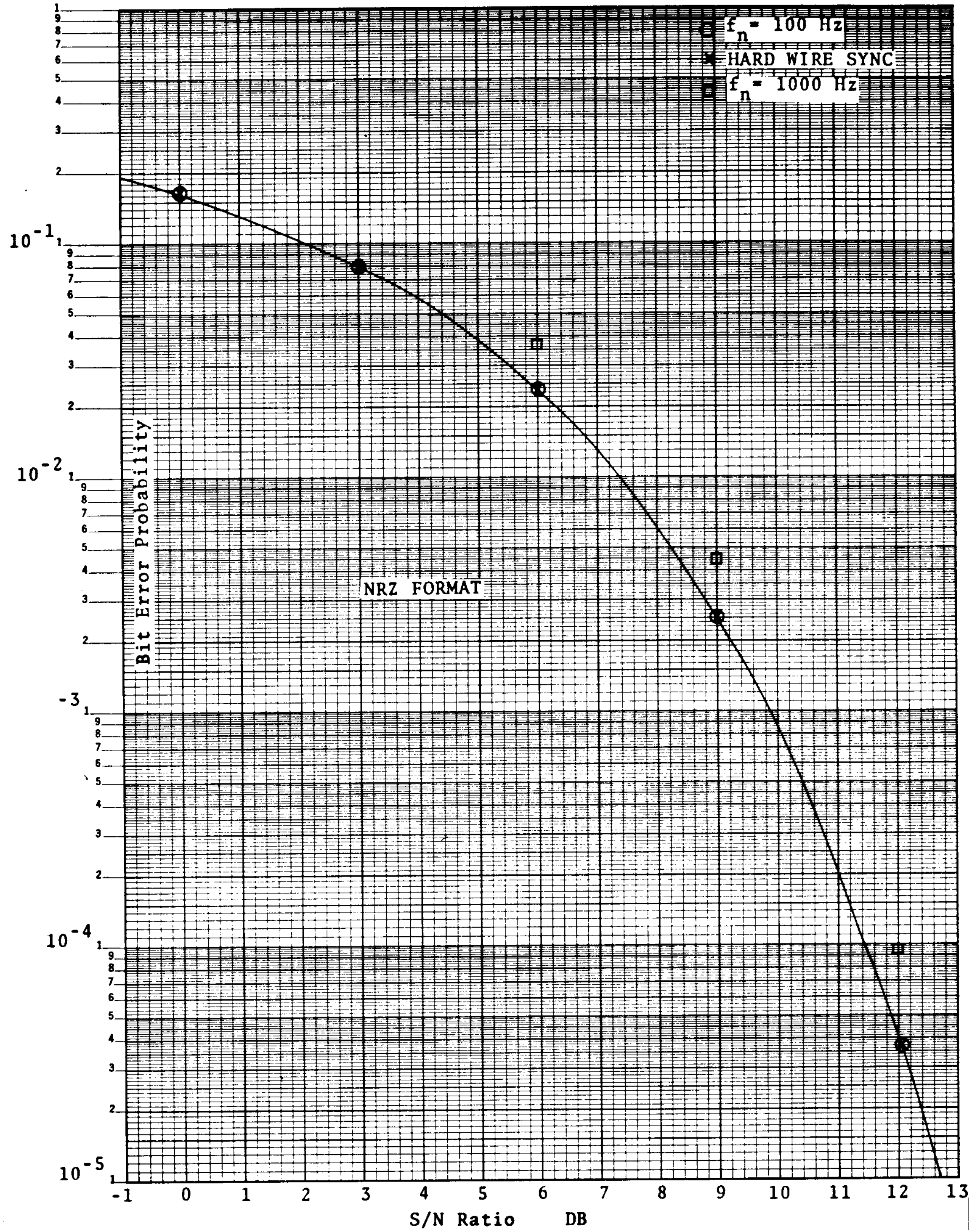


Figure 5. Bit Error Probability Data With No bit Clock Modulation

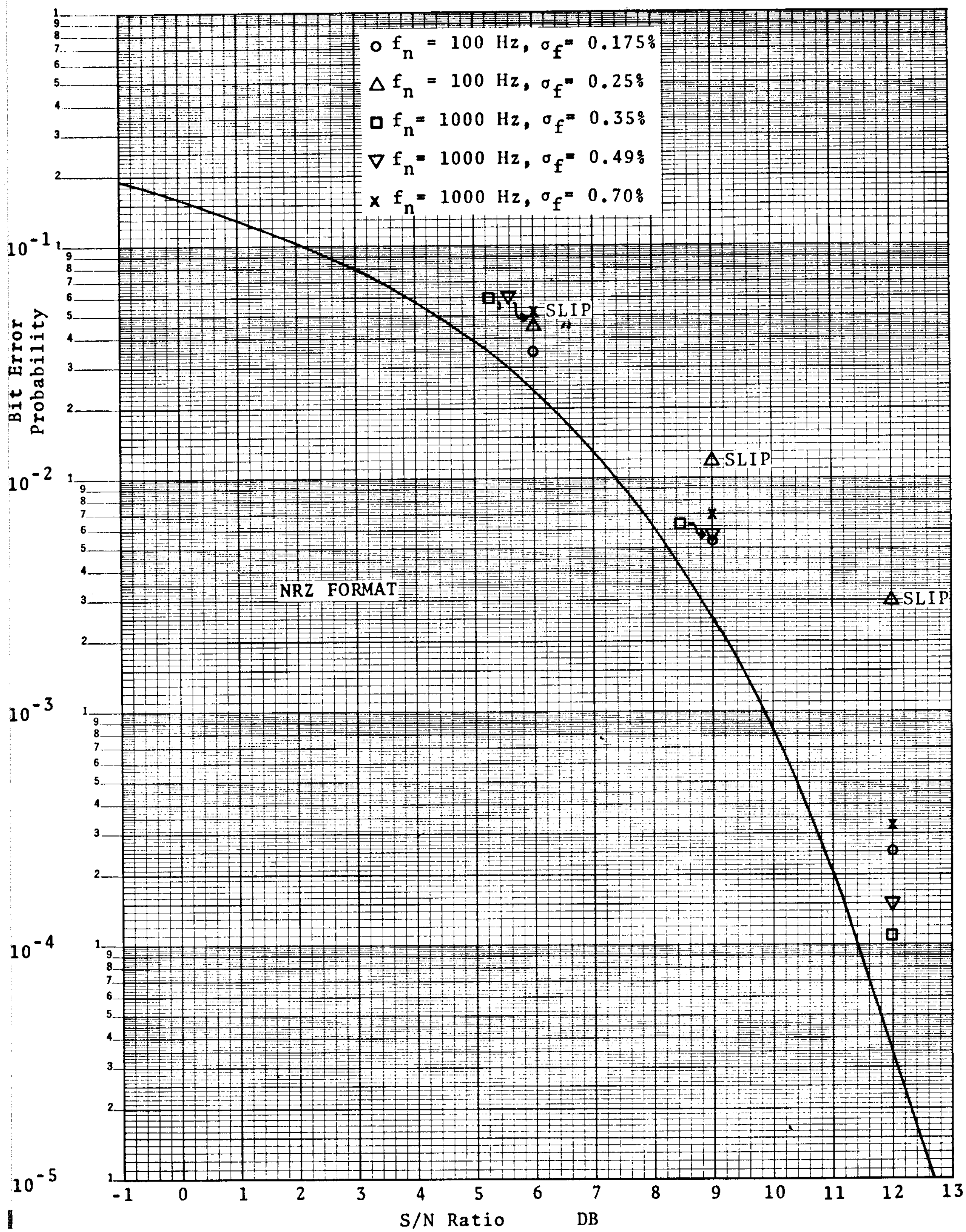


Figure 6. Bit Error Probability Data With Bit Clock Modulation