ERRORS RESULTING FROM CHANNEL FILTERS AND ADJACENT CHANNEL CROSSTALK IN DSB/SC TELEMETRY SYSTEMS

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Summary The waveform distortion resulting from adjacent channel crosstalk and from amplitude and phase nonlinearity in channel filters limits the minimum channel spacing, and hence the bandwidth utilization efficiency of a double sideband/suppressed carrier (DSB/SC) telemetry link. The paper presents results of an analysis defining the minimum achievable mean-square error when Butterworth filters are used in the DSB demodulator/demultiplexer. With data inputs consisting of band-limited random signals, solutions are given for various combinations of data order, filter order, channel spacing, and filter cut-off. The trade-off between waveform distortion and channel spacing is illustrated, and optimum locations for the filter cut-off are defined. The irremovable error based on Weiner optimum filter theory is presented as an interesting basis for comparison.

Introduction The search for efficient transmission techniques for wideband data such as shock, vibration, and acoustical measurements has directed attention to frequency division arrangements of AM suppressed carrier subcarriers. Single sideband/suppressed carrier (SSB/SC) techniques have been widely used in several large space vehicle programs since 1962.[1] More recently, use of the DSB/SC technique has been proposed for applications where the practical limitations of SSB/SC offset its advantage in bandwidth efficiency.

Two essential attributes of a transmission link used for wideband data are radio frequency (RF) signal-to-noise (SIN) performance and RF bandwidth efficiency. With FM carrier modulation, both are very sensitive to the spacing of channels in the frequency baseband, and each is improved by packing the channels more closely together.

Although waveform fidelity is not a stringent requirement for many wideband data applications, it is an important attribute for measurements such as shock. Good waveform fidelity involves attention not only to noise originating from RF sources, intermodulation, and crosstalk, but also to distortion resulting from amplitude and phase nonlinearities in channel filters. The channel filters associated with the demodulator/demultiplexer should eliminate as much of the adjacent channel signals as possible, while committing minimum distortion on the channel signal of interest. This involves careful selection of filter order and cut-off frequency.

In principle, errors resulting from crosstalk and channel filters can be made negligible by increasing channel spacing; but this alternative leads to degradation in RF bandwidth utilization and S/N performance. Therefore the system user must seek a suitable compromise between these sources of error and the RF link errors to achieve optimum performance. This paper presents an analysis of the waveform distortion arising from adjacent channel crosstalk and channel filter errors in a DSB/SC link where the channel filters are the familiar Butterworth type. The results are presented in a form which defines the optimum selection of filter parameters for a specified channel spacing and identifies the trade-off relation between signal distortion and channel spacing.

System Description The simplified block diagram in Figure i shows the data now path and identifies the individual elements of a DSB/SC transmission link. The data channel of a typical ground system includes the channel demodulator and, depending on the method of implementation, either one or two channel filters. If sinusoidal reference carriers and ideal product demodulators are used, channel separation can be accomplished with only a lowpass filter following the demodulator output. In practice, however, a bandpass filter is sometimes needed at the demodulator input for proper operation. In either case, for the purpose of analysis, channel filtering can be represented by an equivalent lowpass filter following the demodulator output.

Since the magnitude of errors resulting from channel filters depends on characteristics of the input signal, it is difficult to correlate results, using a simple test signal such as a single sinusoid, to performance with actual data signals. Therefore, a band-limited random signal with uniform spectral density was selected to simulate the input data. As shown in Figure 2(a) the single-sided spectrum of the signal is defined as flat from dc to f_o and attenuated at a rate of 6m db/octave above f_o. All channel signals are assumed to be identical in spectral shape, to occupy equal bandwidths, and to be equally spaced in the baseband.

The signal at the output of the telemetry channel, shown in Figure 2(b) , is assumed to be a perfect DSB/SC image of the input signal s(t) together with equally spaced images of adjacent channels. Hence the analysis does not consider distortion caused by channel filters at the transmitter or errors arising from RF and intermodulation noise. The demodulation reference carrier phase is assumed identical with that of the carrier at the modulator, so that the two sidebands add directly. Thus the product modulation and demodulation are reversing operations on the signal s(t).

The signal $s_i(t)$, which results from the second product operation in Figure 1, consists of s(t) plus superimposed images of the two adjacent channels centered about a frequency equal to the channel spacing. The one-sided spectrum of $s_i(t)$ is shown in Figure 2(c) . Note that the adjacent channel signals have a spectral amplitude less than the signal of interest by a factor of $\sqrt{2}$. This results because the sidebands of the DSB subcarriers add directly while the two adjacent channels (assumed noncoherent) add rms-wise.

Error Calculations A meaningful evaluation of a systems performance requires that a criterion be established for judging its merits or comparing it with other systems. Although a criterion based on S/N ratio or dynamic range presents a useful measure of performance for certain types of analyses, it is not suitable for the measurement of waveform distortion because it neglects errors caused by nonideal amplitude and phase characteristics of the channel filters.

A realistic and useful representation for these errors is the mean of the squared difference between the output signal $s_o(t)$ and $s(t - \alpha)$, which is the input signal delayed by the constant time a that minimizes the mean-square difference. This so-called mean-square error criterion 3 provides results that may be readily summed with mean-square errors from other sources to produce overall system error.

A mathematical model for the analysis is shown in Figure 3. The input signal s(t) is summed with n(t), the translated and superimposed images of the two adjacent channels, to form $s_i(t)$ with spectrum indicated in Figure 2(c). This signal is passed through a filter with characteristics $H(\omega)$ which is the equivalent of the demodulator output lowpass filter in series with the input bandpass filter (if used) translated to zero frequency. The result is then compared with $s(t - \alpha)$, the delayed replica of the input signal; the difference represents the instantaneous system error.

In accordance with the model depicted in Figure 3, signal distortion introduced into the data channel is classified as either adjacent channel crosstalk error or channel filter error. Crosstalk error occurs because portions of the translated adjacent channel spectra are passed by the channel filter. Channel filter error results from distortion of the data signal caused by nonideal amplitude and phase characteristics of the channel filter.

To obtain an expression for crosstalk error, we multiply the power density spectrum of n(t) by $|H(\omega)|^2$ and integrate over all frequencies. The phase of the interfering components may be neglected since they are uncorrelated with the desired signal.

Expressed in closed form with H(ω) corresponding to the familiar Butterworth filter function, this gives for the mean-square error resulting from adjacent channel crosstalk,

$$\sigma_{ct}^2 = \left[\frac{2m-1}{4m}\right] \int_{\beta_s - 1}^{\beta_s + 1} \frac{dz}{[z/(f_{co}/f_o)]^{2n} + 1} + \left[\frac{2m-1}{4m\beta_s}\right] \int_{-\beta_s - 1}^{\beta_s - 1} \frac{[(z+1)/\beta_s]^{2m-2}}{[\beta_s z/(f_{co}/f_o)(z+1)]^{2n} + 1} dz \,, \tag{1}$$

where

z = f/f_{co}
f_o = data cut-off frequency
f_{co} = filter cut-off frequency
m = data order
n = filter order
β_s = channel separation factor

The amplitude response characteristic of the filter is down 3 db at the cut-off frequency f_{co} and is attenuated at a rate of 6n db/octave outside the passband. The channel separation factor β_s is equal to the channel spacing divided by the data cut-off frequency.

To determine the channel filter error, the random input signal s(t) is represented as an infinite sum of sinusoids spaced equally in frequency and having relative magnitudes corresponding to the envelope of the voltage spectrum.[4] This produces the following expression for mean-square error caused by channel filters:

$$\sigma_f^2 = \frac{(2m-1)(f_{co}/f_o)}{2m} \int_0^{f_o/f_{co}} \{1 - 2A(x)\cos[\phi(x) + \alpha_o x) + A^2(x)\} dx + \frac{(2m-1)(f_{co}/f_o)^{-2m+1}}{2m} \int_{f_o/f_{co}}^{\infty} \frac{1}{x^{2m}} \{1 - 2A(x)\cos[\phi(x) + \alpha_o x] + A^2(x)\} dx \tag{2}$$

where

x = f/f_{co}
$A(x)$ = amplitude characteristic of the channel filter
$\phi(x)$ = phase characteristic of the channel filter
α_o = time delay factor

Equations (1) and (2) as shown are normalized to the mean-square value of the input signal s(t). Since the two error waveforms are uncorrelated, their mean-square values may be surnmed to give the total mean-square error,

$$\sigma_e^2 = \sigma_f^2 + \sigma_{ct}^2 \quad . \qquad (3)$$

Using a digital computer, solutions to Equations (1) and (2) were obtained for various combinations of the variables: f_{co}/f_o, m, n, and β_s. Using reiteration procedures, a value was selected for the time delay factor α_o to minimize the mean-square error, thereby assuring a true representation of waveform distortion. [5]

Results Figure 4 shows how crosstalk error, filter error, and total waveform distortion varies with filter cut-off frequency for a specific choice of data order, filter order, and channel spacing. Note that crosstalk error σ_{ct} rises steadily as the filter cut-off frequency is increased while filter error σ_f has the reverse relationship. The curve for σ_e, the total waveform distortion obtained by rms summation of σ_{ct} and σ_f, clearly identifies an optimum ratio between the filter and data cut-off frequencies. For the parameters depicted in Figure 4 (m = 3, n = 6, β_s = 4) , we see that a filter with cut-off frequency slightly less than 2 times the data cut-off frequency provides minimum distortion.

The values for rms error shown on the graphs are normalized to rms values of the input signal. To obtain rms error normalized to peak-to-peak signal which is sometimes preferred, one should divide the ordinate values by two times the crest factor (peak/rms ratio) of the input signal. For example, if we assume a crest factor of 3 as is often used for vibration data, the ordinate values would be divided by 6.

Figures 5, 6, and 7 depict families of curves similar to Figure 4 where each curve represents a specific value for the channel separation factor β_s. From these curves the minimum achievable distortion for each selection of β_s and the filter cut-off frequency necessary to achieve this minimum is readily identified. One should notice that the selection of filter cut-off frequency becomes more critical as channel spacing is decreased. The three figures illustrate results for data order m equal to 2, 3, and 6, respectively. In each case the filter order n = 6 was used, a selection which may be justified from the next set of curves.

Figures 8, 9, and 10 illustrate the trade-off between error, channel spacing, and filter order. Here each point is plotted using the ratio of f_{co}/f_o that yields minimum distortion, thus providing a convenient form for determining β_s for a specified rms error. Although error decreases with higher filter order, it is apparent that negligible improvement results for n greater than 6.

Irremovable Error The dashed curves shown in Figures 8, 9, and 10 represent the waveform distortion resulting from a system implemented with an optimum infinite lag filter. If we apply Parseval's theorem to the expression for the minimum mean-square error of an optimum system and use the property of zero covariance between channels, the irremovable error is given by[6]

$$\sigma^2_{opt} = \frac{1}{\sigma^2_d} \int_{-\infty}^{+\infty} \frac{\Phi_{mm}(\omega) \cdot \Phi_{nn}(\omega)}{\Phi_{mm}(\omega) + \Phi_{nn}(\omega)} \, d\omega \ , \qquad (4)$$

where

$\Phi_{mm}(\omega)$ = power density spectrum of the input data signal
$\Phi_{nn}(\omega)$ = power density spectrum of the adjacent channel signals
σ_d = rms value of the input data signal

The irremovable error represents a lower bound on system distortion and thus presents an interesting basis for comparison.

Experimental Results Channel filter distortion resulting from the nonlinear amplitude and phase characteristics of a sixth-order Butterworth filter was measured using shaped random noise to simulate third and sixth-order data signals. As shown in Figure 10, the measured values agree very closely with the corresponding calculated values of σ_f.

Conclusions Optimum performance from a DSB/SC transmission link, where the criterion is minimum mean-square error, requires careful selection of channel spacing/data bandwidth ratio and channel filter cut-off frequency. A logical design choice for most applications should be to make the filter and crosstalk error approximately equal to the expected error originating from RF and intermodulation noise. Although filters other than the Butterworth design used in this analysis may give better results, the relationship shown in Figures 8, 9, and 10 between the curve for n = 6 and the dashed curves representing the performance of an optimum infinite lag filter indicates that one should not expect phenomenal improvement from other filter designs.

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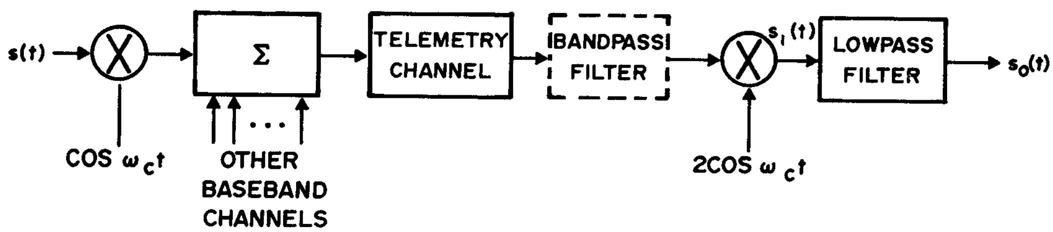

Figure 1 - DSB/SC Transmission Link

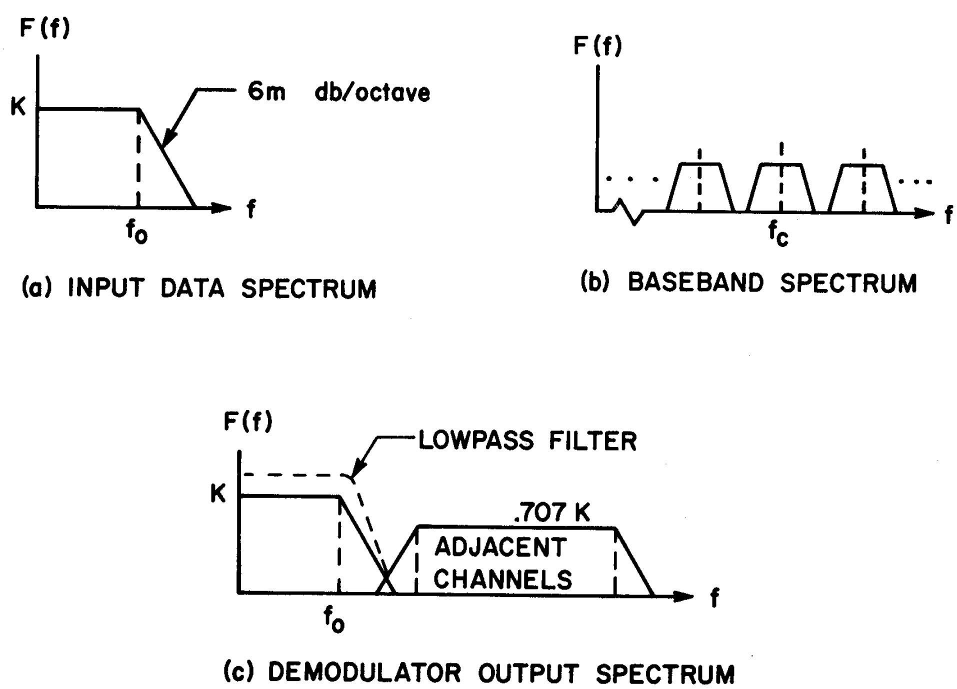

Figure 2 - Signal Characteristics

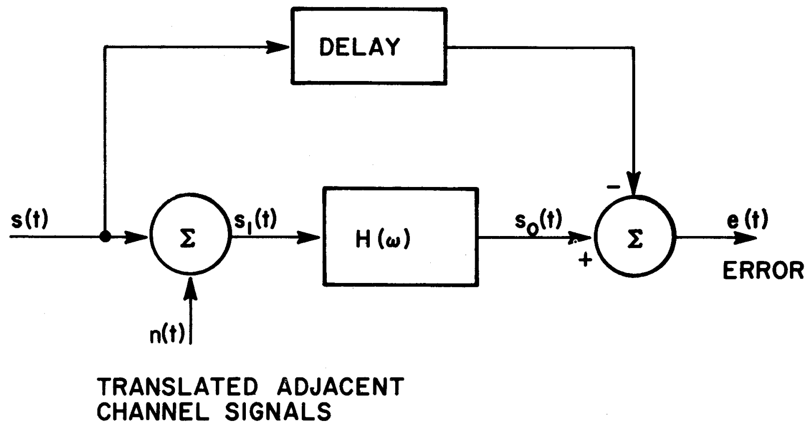

Figure 3 - System Model

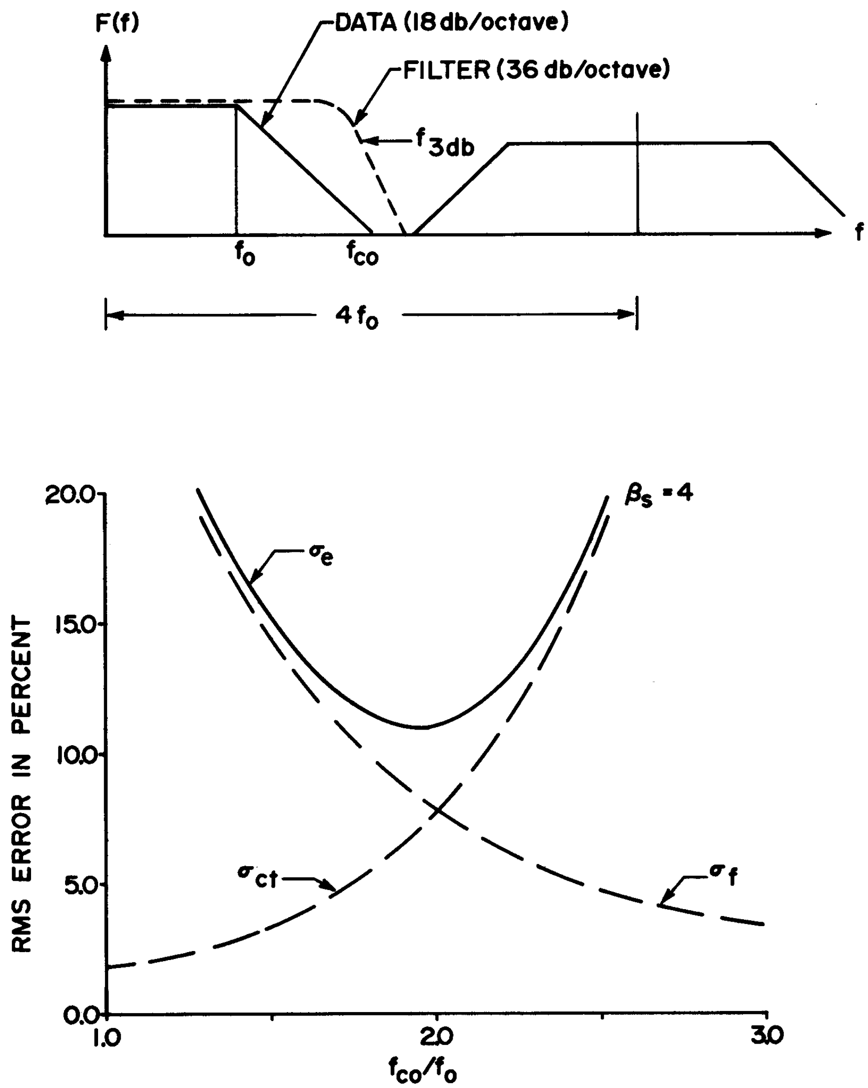

Figure 4 - Illustrative Example of Waveform Distortion

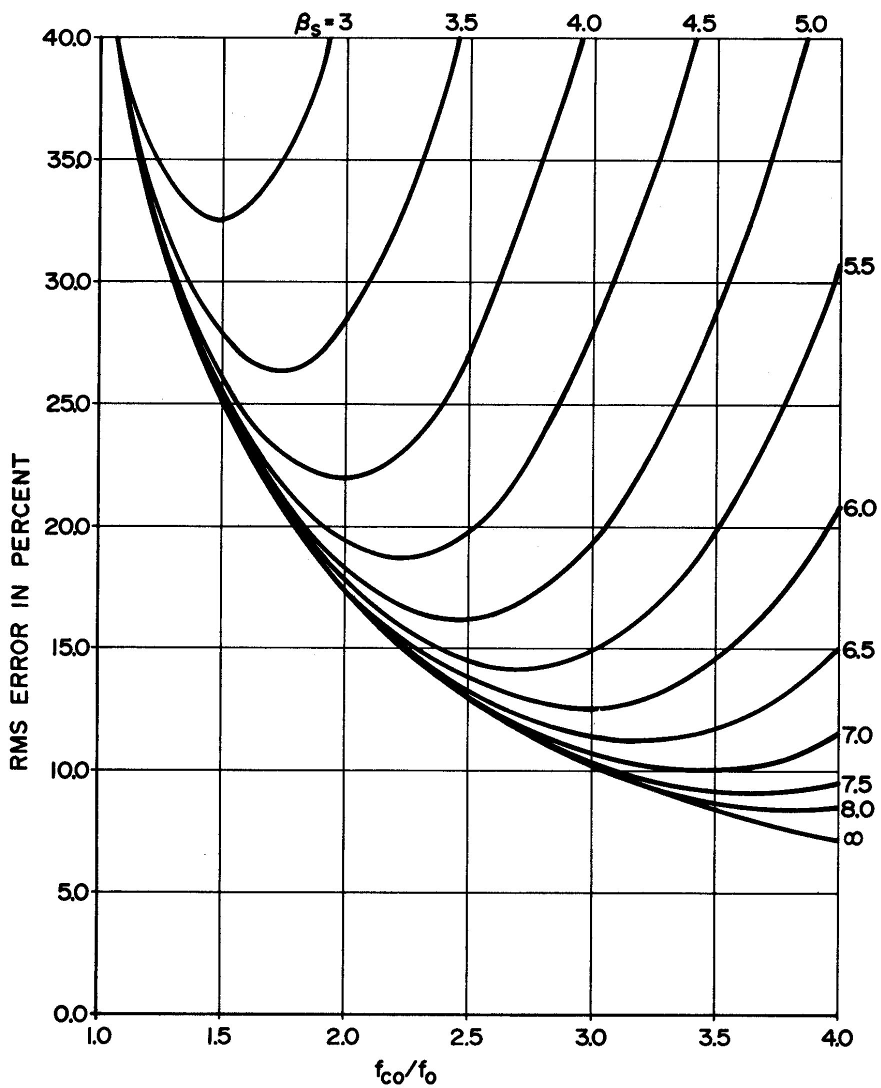

Figure 5 - Waveform Distortion for Second-Order Data and Sixth-Order Filtering

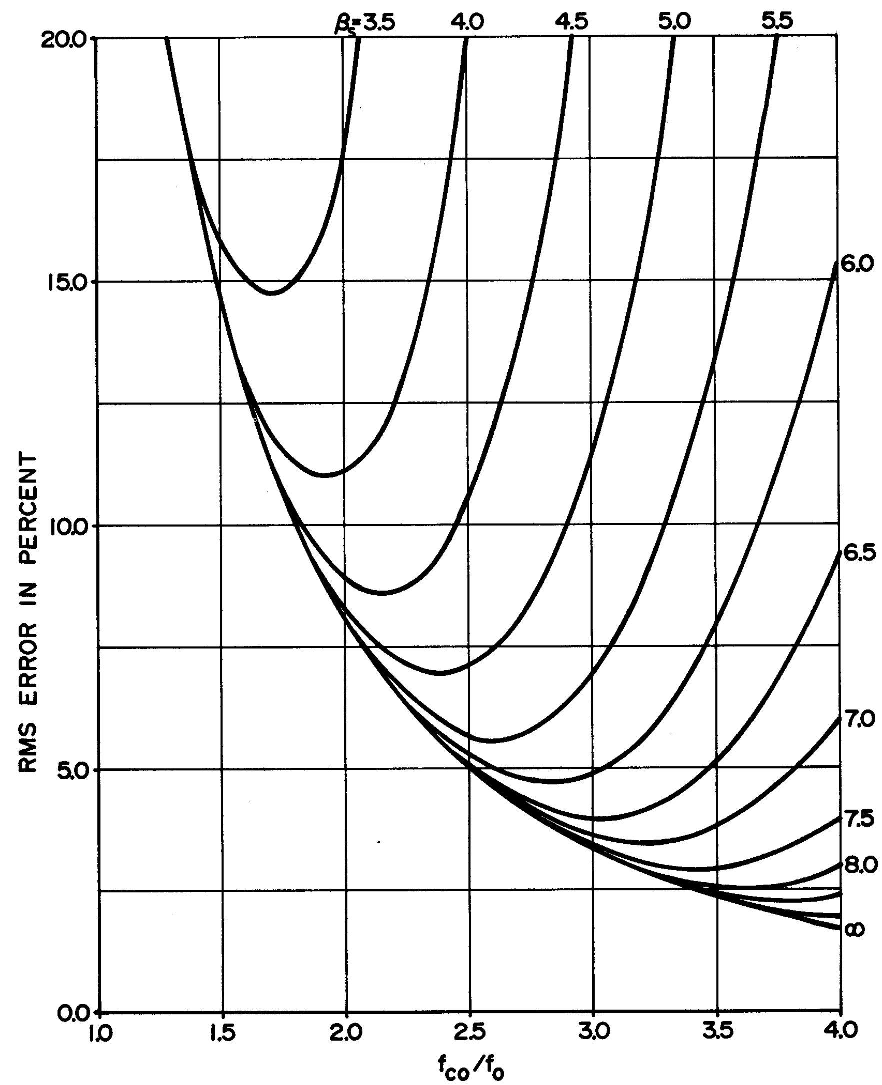

Figure 6 - Waveform Distortion for Sixth-Order Data and Sixth-Order Filtering

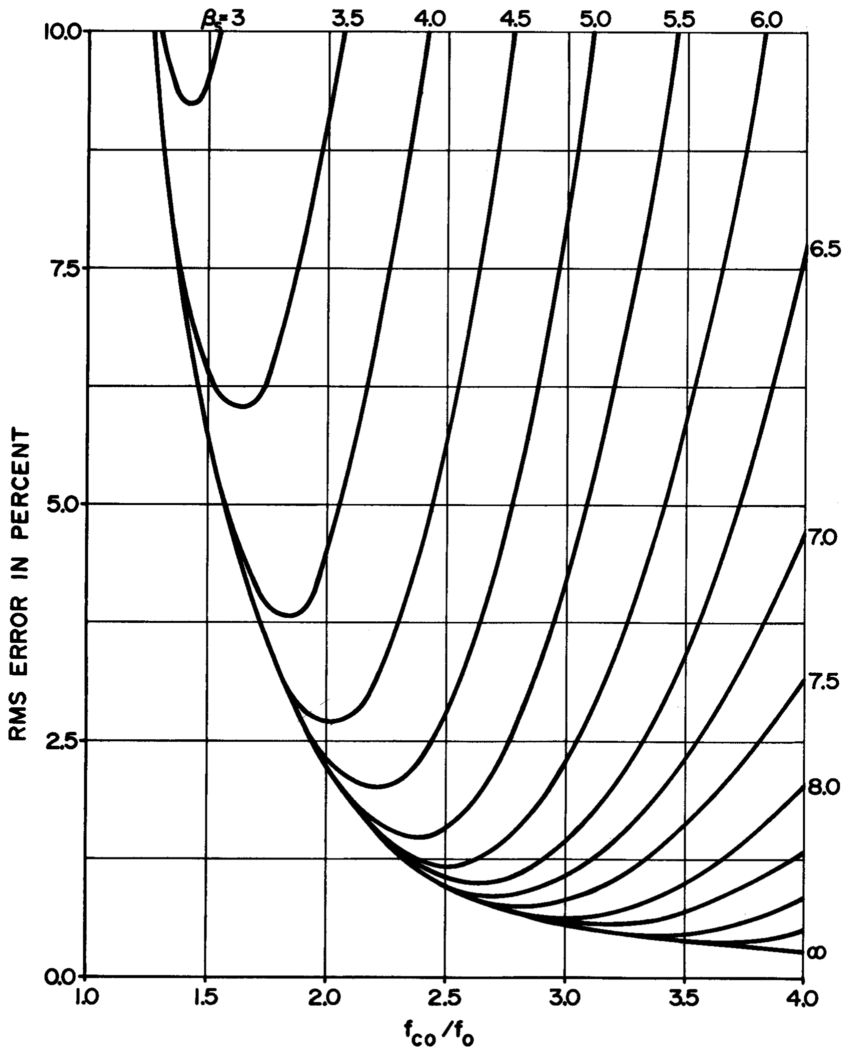

Figure 7 - Waveform Distortion for Sixth-Order Data and Sixth-Order Filtering

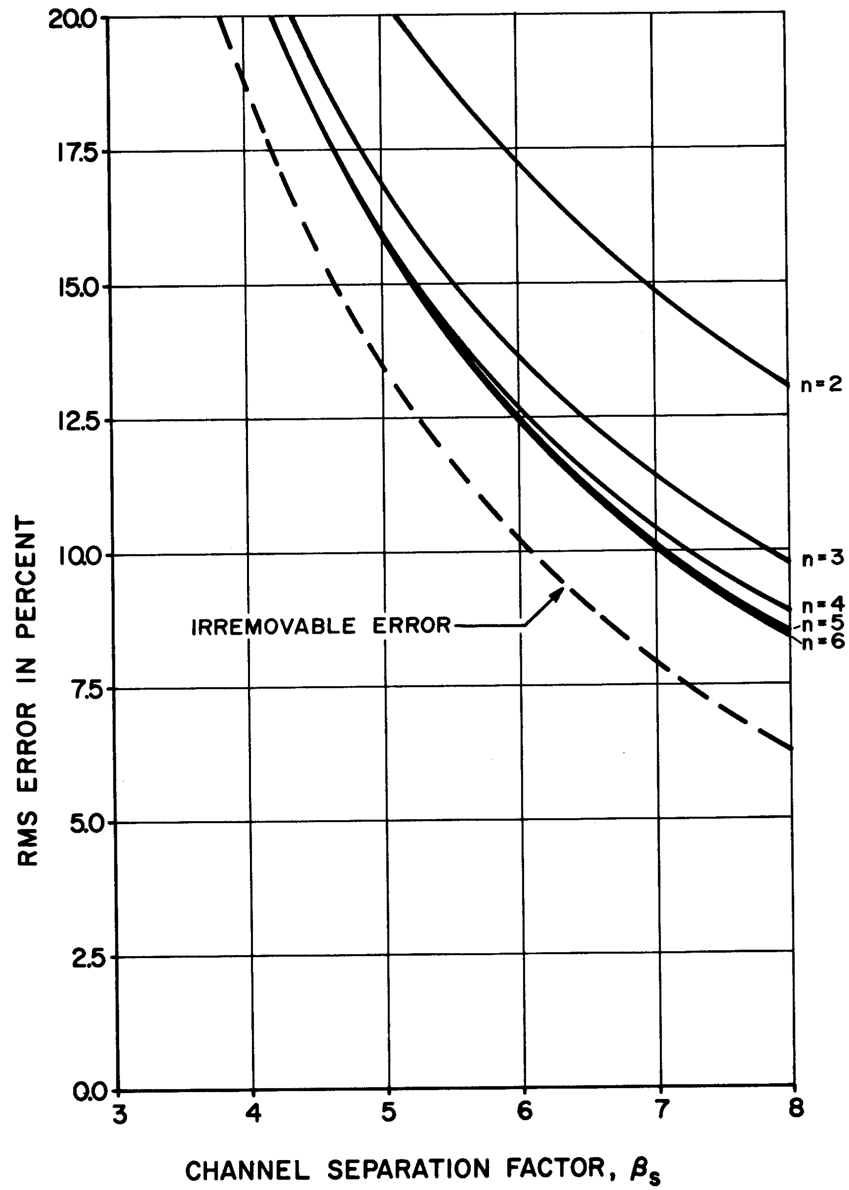

Figure 8 - Minimum Waveform Distortion for Second-Order Data

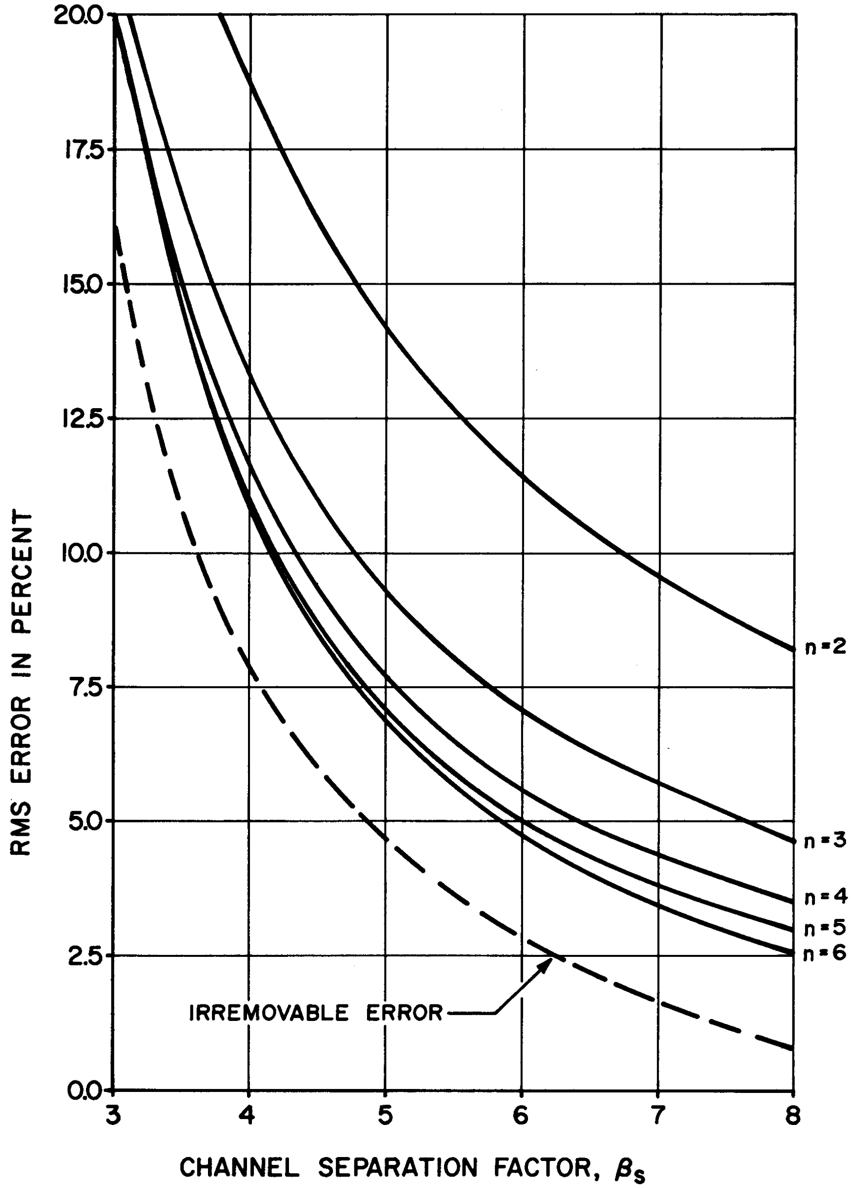

Figure 9 - Minimum Waveform Distortion for Third-Order Data

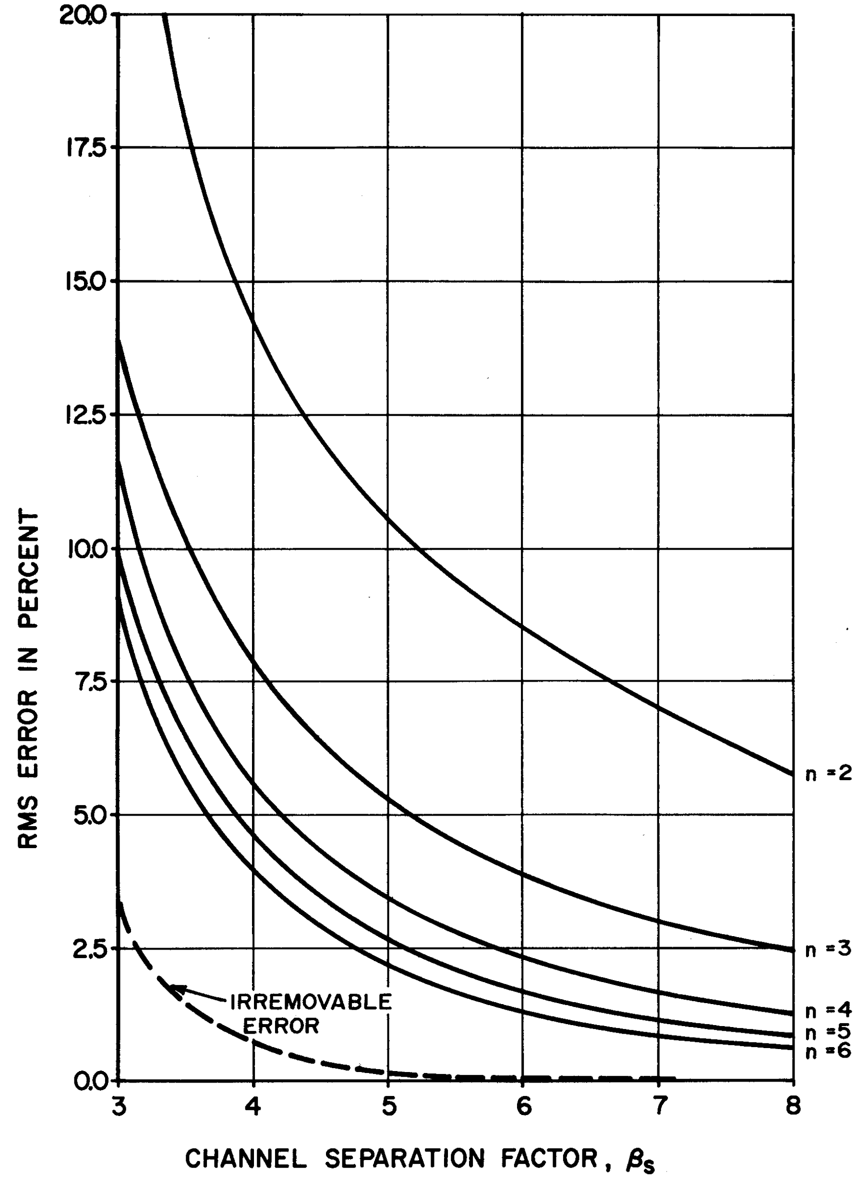

Figure 10 - Minimum Waveform Distortion for Sixth-Order Data

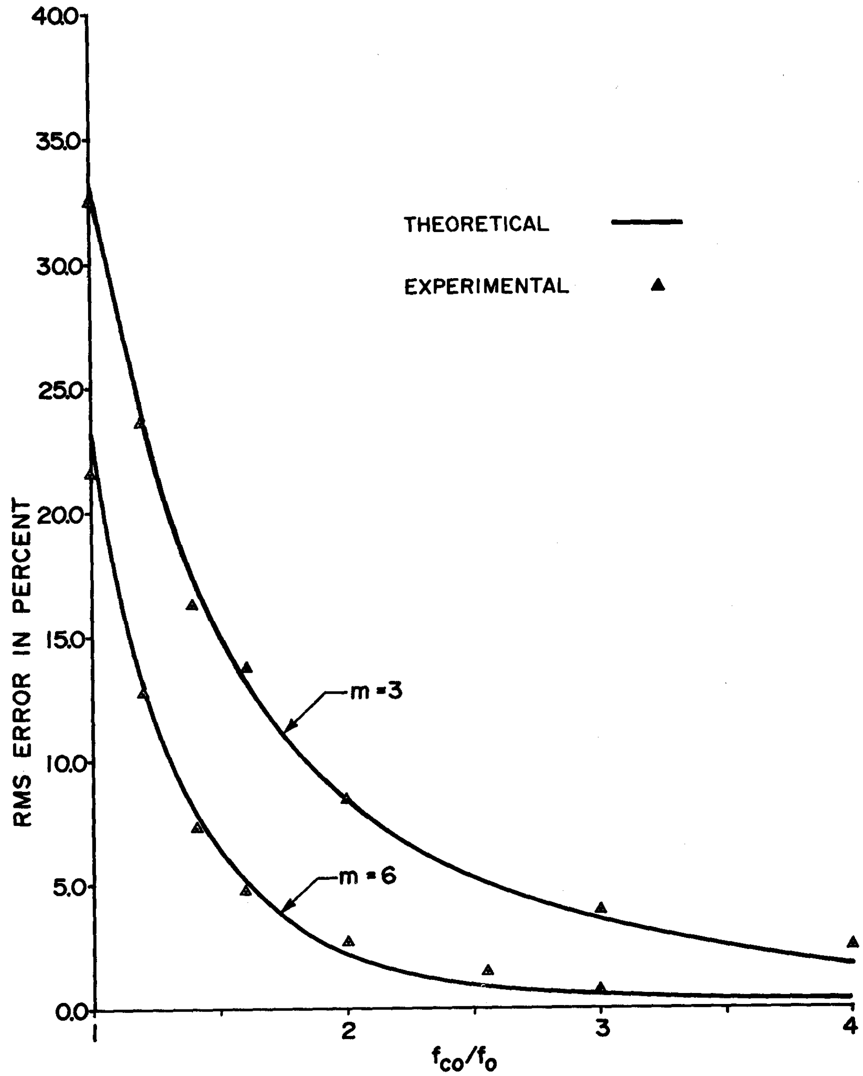

Figure 11 - Channel Filter Distortion for Sixth-Order Filtering