A PHASE-LOCKED UHF TELEMETRY TRANSPONDER FOR MISSILE SCORING APPLICATIONS

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Summary  The Phase-Locked UHF Telemetry Transponder described in this article is part of the recently conceived Cooperative-Doppler Missile Scoring System. This system obtains the doppler curve of a missile relative to its target, without the use of special scoring equipment in the missile. This is accomplished through comparison of the telemetry information from the missile with a transponded signal from the target aircraft. The transponder is housed in the target aircraft and is responsible for transponding PAM/FM modulation from the telemetry band (2200-2290 MHz) to the scoring band (1760-1850 MHz), while preserving the phase of the modulation during the frequency translation. In order to accomplish this, true phase demodulation of the PAM/FM signal has been achieved through utilization of the phase-lock technique. Included in this article is an analytical discussion of the phase-lock loop design with derivations of the closed-loop transfer function and response bandwidth; Root-Locus analysis; Bode diagram; and dynamic range and phase response considerations.

Introduction  The Data Link Branch of the Functional Research Division of Naval Avionics was assigned the task of developing a Cooperative Doppler Missile Scoring System, as part of the recent VHF-UHF Telemetry Conversion Program. The AN/DRQ-4 (XAN-1) Telemetry Transponder discussed in this paper, is an integral part of the above system.

The Cooperative Doppler Scoring System is designed to obtain the doppler curve of a missile relative to its target, without the use of special scoring equipment in the missile. This is accomplished through direct comparison of the telemetry information from the missile, with a transponded scoring signal from the drone (target aircraft). The AN/DRQ-4 Transponder is housed in the target aircraft, and is responsible for the frequency translation and retransmission of the telemetry information at the scoring frequency.

The phase-lock technique was chosen, in the design of the AN/DRQ-4, as the best method to achieve an exact frequency translation of telemetry information at UHF
frequencies, while preserving the phase of the modulation; providing the necessary input-output frequency stability; and achieving the necessary dynamic range and system gain at UHF frequencies. The Voltage Controlled Oscillator (VCO) employed within the phase-lock loop is of the tunable, free-running variety which provides the modulation capability required by the modulation format. This free-running approach usually results in a UHF source which drifts approximately ± 2 MHz from the center frequency, over the operating temperature range. An automatic signal acquisition technique is incorporated in the loop to assure acquisition of the telemetry signal, regardless of the VCO center frequency. This consists of summing an internally generated triangular waveform into the tracking signal (i.e., closed-loop error signal), to cause the transponder output to search ± 3 MHz either side of the center frequency. When the system sweeps through the predetermined scoring frequency which agrees with the instantaneous incoming telemetry frequency, the system immediately phase-locks to the telemetry information and accurately translates the incoming information to the scoring frequency (transponder output). Note that the design of the tracking section’s response and the high ratio of tracking to search gains, assures a high probability of acquisition and good modulation capability, without the use of phase shifters and quadrature detectors to control the triangular search signal.

When the system is not phase-locked, the difference frequency produced by the detector creates a “beat-note” error signal at the output of the tracking filter. This signal provides a discriminator action to aid the phase-lock loop in reaching the pull-in threshold. This effect is further emphasized by the high loop gain, and the established values for the loop damping coefficient and natural loop frequency. The resulting (CW) pull-in range for the transponder is about one full megahertz; thus a high acquisition probability with a rapid lock-on rate under swept and/or transient conditions is assured.

**Transponder System Description**  
(See System Block Diagram; Figure 1) The transponder is basically a double heterodyne system utilizing the principle of the phaselock loop, as applied to the design of “locked-oscillators” for crystal-like frequency stability of UHF sources. In this application, the sampled output of the 3 watt VCO is the first local oscillator; that is, the oscillator which is being phase-locked. The purpose for utilizing this design concept is to achieve a translation of the telemetry information in its exact form, and to retransmit with sufficient power to assure reliable operation with the ground station over the desired range. Note that the retransmission of the telemetry signal is not a simple heterodyne translation to a lower frequency, but an actual remodulation and creation of a new duplicate of the telemetry information for transmission at a lower frequency. The use of the phase-lock technique makes the transponder a “closed-loop servosystem”, as compared with the simple “open-loop” heterodyne system employed in VHF transponders.
**RF Section - Block Diagram Description**  The sampled output from the VCO is used as the first local oscillator, and is heterodyned with the received telemetry information in the RF diode mixer. This produces a first IF frequency of 400-500 MHz. A one-half cubic-inch crystal oscillator with a fundamental frequency of 54-75 MHz is used to drive a times-six frequency multiplier and generate second local oscillator frequencies of 325-453 MHz. This oscillator has a frequency stability of fifty parts per million (0.005%), in order to assure stability of the frequency translation from the first IF frequency, to the second IF frequency (45 MHz). The second local oscillator signal generated by the frequency multiplier is heterodyned with the RF diode mixer output in the RF amplifier/transistor mixer, to produce a second IF frequency of 45 MHz. Note that this second IF frequency must always be 45 MHz, regardless of the telemetry and scoring frequencies selected; only the output frequency of the RF diode mixer and the first local oscillator frequency change. Thus the desired telemetry-scoring channel is determined by: a) selection of the proper crystal oscillator frequency for the X6 Multiplier; and b) tuning the VCO to the proper scoring frequency with a -3 VDC bias on the modulation input. (The RF amp/tran. mixer must also be peaked to the new first IF frequency.)

The 45 MHz IF signal is then amplified by the three-stage integrated-circuit IF amplifier to produce an IF signal of the proper magnitude for further processing. The intrinsic characteristics of this amplifier make it self-limiting. This self-limiting nature, and the inherent broadening of the passband under high signal levels; is responsible for the desired variation in the closed-loop damping coefficient. Thus, maximum sensitivity is maintained with maximum loop signal-to-noise for low signal levels at the telemetry input, without introducing destructive phase shifts during the missile-target intercept. The IF amplifier also has adjustable gain and output characteristics which are used to mate the RF Section to the Acquisition and Tracking Section, in such a manner as to achieve an 80 db dynamic range.

**Acquisition & Tracking Section**  The output of the IF amplifier enters the Acquisition & Tracking Section, where processing takes place to produce the “error signal” which modulates the VCO in the closed loop. A miniature balanced diode mixer is used as a phase detector to compare the IF signal with the frequency and phase of the reference oscillator. The output of this detector is a voltage proportional to phase difference, or the difference frequency, depending upon whether the loop is “locked” or “unlocked” respectively. The resulting phase-demodulated signal is an exact duplicate of the original information plus doppler frequency shift. A tracking filter is used at the output of the detector to control the closed-loop response of the system. The filtered signal is then combined with the search signal and amplified by the operational amplifier. The search signal is generated within the Acquisition & Tracking Section by an astable common-emitter multivibrator which controls constant current sources in a bidirectional Miller-integration circuit to create a triangular waveform. The high ratio of tracking to search gains of the summed signals is responsible for the ability of the system
to perform rapid transitions between the search and tracking modes of operation; and the linear sides of the triangular waveform immediately commence a linear frequency scanning of the system when a transition is made to the search mode. An emitter-follower driver stage is used at the output of the operational amplifier to provide RF isolation and sufficient current drive to the voltage-dependent input impedance of the VCO. It also allows increased slewing capability in the amplifier. It should be noted that the automatic nature of the aforementioned processes performed by the Acquisition & Tracking Section is responsible for the automatic operation of the transponder system.

**Voltage Controlled Oscillator (VCO)** The composite output signal produced by the Acquisition & Tracking Section modulates the VCO and produces the scoring output. In the two advanced developmental research models, the “error signal” was used to modulate a varactor diode used as a phase-switch in the planar triode cavity; thus modulating the frequency of the cavity. The sample output of the VCO acts as the local oscillator for the RF diode mixer; thus completing the closed loop.

From the closed-loop nature of the system, we see that any modulation present in the telemetry information will be reproduced exactly in the scoring output. Thus a constant mathematical difference is maintained between the instantaneous telemetry frequency and scoring frequency during phase-locked operation. The telemetry information is thus translated, in its exact form, to the lower frequency.

The entire transponder is powered by a DC-DC converter contained within the unit. This converter accepts unregulated 28 VDC supplied by the target drone, and produces highly regulated positive and negative fifteen volts to operate the RF components and printed circuit board containing the Acquisition & Tracking Section. It also supplies the high cathode voltage and filament current necessary for operation of the planar triode in the VCO.

**Frequency Stability & Error Contributed to Miss-Distance Determination** The long and short term frequency stability of the AN/DKQ-4 is solely dependent upon the performance of the two crystal oscillators used within the system, as a direct result of the phase-lock technique employed. Stability tests performed on the oscillators, along with various calculations involving the system configuration; have indicated that the worst-case system frequency drift during a typical missile-target-intercept is to be approximately 6 Hz. For the maximum relative missile velocities and minimum miss-distances indicated by system requirements, this represents a worst-case error in miss-distance determination of less than 1.4%. The probable error in miss distance determination contributed by the AN/DRQ-4 is estimated to be approximately 0.7%. Hence the error in miss-distance determination contributed by the transponder is compatible with a maximum error of 5%, for the complete MDI system.
**System Input Sensitivity**  The input sensitivity of the system is of the following form:

\[
\text{Input sensitivity} = \text{system compression point} - 10 \log_{10} \left[ \frac{K \log_{10} / \text{MIFL} - \text{DL}}{\text{MIFL} - \text{DL}} \right]
\]

where

MIFL = maximum IF output level (limiting level)
DL = detector level

The base of the log function must be found which yields the best logarithmic approximation to the actual curve. Careful analysis of this technique indicates that an excellent approximation may be obtained by using the base of the log function which causes the function to pass through the most probable value of detector input level for a maximum IF level of 0 dbm. This produces the most accurate approximation under the most probable operating conditions, with only slight error introduced (2-1/2% maximum) for extreme values of detector level.

Using the following empirical data, the “threshold extension” term may be evaluated:

Threshold Extension (TE) = 18 db below the system compression point
for MIFL = 0 dbm and DL = -5 dbm

Thus

\[
TE = 10 \log_{10} \left[ \frac{K \log_{10} / \text{MIFL} - \text{DL}}{\text{MIFL} - \text{DL}} \right]
\]

for MIFL and DL expressed in dbm’s,

\[
TE = 10 \log_{10} \left[ \frac{K \log_{10} \{\text{antilog}_{10} / \frac{\text{MIFL} - \text{DL}}{10}\}}{\text{MIFL} - \text{DL}} \right]
\]

converting to common logarithms,

\[
TE = 10 \log_{10} \left[ \frac{K/\text{MIFL} (\text{dbm}) - \text{DL} (\text{dbm})/10 \log_{10} (b)}{10 \log_{10} (b)} \right] = 18 \text{ db}
\]

thus for K = 20;

\[
\log_{10} (b) = \frac{2/\text{MIFL} (\text{dbm}) - \text{DL} (\text{dbm})/63}{0.159}
\]

and the resulting expression for system input sensitivity becomes:
Input sensitivity = system compression point - 10 \log_{10} \left[ \frac{12.6}{\text{MIFL (dbm) - DL (dbm)}} \right] = \text{IF amplifier compression point + loop SNR threshold + conversion loss - conversion gain - 10 \log_{10} \left[ \frac{12.6}{\text{MIFL (dbm) - DL (dbm)}} \right] } \text{dbm)

For an extreme range of values for the threshold extension term, the above expression has been found to match empirical data with a maximum error of approximately 2 db; or less than 2-1/2\%. of the total dynamic range.

The system sensitivity for the typical case described above is:

\[
\text{Input sensitivity} = -67 \text{ dbm} - 10 \log_{10} \left[ \frac{12.6}{5 \text{ db}} \right] = -85 \text{ dbm}
\]

Since the above expressions predict closed-loop performance from open-loop data, the presence of sources of noise and instability in the loop will degrade the system sensitivity from the predicted values. Hence, noise or instability of the modulator section Wthe VCO would degrade the equivalent loop SNR threshold with a corresponding alteration of the system compression point and system input sensitivity.

If the input-output characteristics of the IF amplifier and the combined characteristics of the diode and transistor mixers are utilized to graphically illustrate the system’s dynamic properties, the resulting curve validates the above predictions. It should be noted, however, that the exact location of the linear phase threshold is somewhat ambiguous with regard to experimental verification, since a noticeable phase shift typically occurs about one to three decibels below the system compression point.

This differential is intrinsic to the operation of the IF amplifier, and is dependent upon variations in input and output admittances of the integrated circuits; saturation characteristics of the current sources contained in the integrated circuits; and other factors including tuning of the IF stages. For the sake of simplicity, the linear phase threshold has been shown approximately equal to the system compression point.

The most important internal factors affecting input sensitivity are the RF section noise level, the VCO modulator stability and the stability of the Acquisition & Tracking Section. Any noise or instability of these components degrades the equivalent loop SNR threshold and the threshold extension term. Under this condition, the total system input sensitivity is degraded accordingly. Generally, the maximum degradation that could occur would reduce the input sensitivity to the system compression level (typically -67 dbm). Since reasonable tracking occurs above this signal level, even with high noise levels and poor loop stability, it is doubtful that an input sensitivity greater than the system compression level could ever be experienced.
**Dynamic Range & Phase Response**  A non-linear phase response in the RF section would produce delay distortion of the FM signal containing the information. Thus, it is necessary to provide sufficient RF bandwidth with minimum slope of the phase-versus-frequency response curve (i.e. minimum time delay) to assure minimum distortion of the modulation bandwidth. This requirement is, however, in direct conflict with the desired variation in loop damping coefficient for optimum acquisition probability and tracking, and the pass-band shifting characteristics of conventional automatic gain control techniques utilized to achieve large dynamic ranges.

The desired bandwidth control with a linear phase response (above the output compression point) and adjustable automatic limiting has been achieved through the design of an integrated circuit IF amplifier utilizing the outstanding phase properties of the single-ended differential amplifier configuration, and the applicability of individual limiting constant-current sources to this configuration. The amplifier thus provides the high gains needed for maintenance of sufficient tracking signal at low telemetry signal levels, a linear phase response above the system compression point for preservation of minimum loop phase error over a wide dynamic range, and automatic limiting and control of the loop damping coefficient to prevent saturation of the phase detector utilized to derive the loop error signal. Collaboration of these outstanding properties has resulted in minimum loop phase error and minimum doppler distortion at the critical missile - target intercept.

**Acquisition & Tracking Section**  The transponder acquisition & tracking section is responsible for acquisition of the telemetry signal and phase demodulation of the PAM/FM information contained therein. The second-order phase-lock loop requires sufficient loop gain to achieve maximum phase slope with minimum phase tracking error at the deviations and modulation rates imposed by the telemetry information. In order to maintain a small phase tracking error with full modulation index, it is necessary to provide an operational amplifier bandwidth much greater than the maximum modulation rate, and sufficient gain to maintain the tracking signal at modulation rates above the second breakpoint of the tracking filter. The second breakpoint of the tracking filter could not be extended to enhance the amplifier’s performance, since this would result in an insufficient loop damping coefficient and phase margin at the unity-gain bandwidth of the system.

With a wide operational amplifier bandwidth and a VCO modulation input bandwidth much greater than the maximum modulation rates, it is possible to control the natural loop frequency and damping coefficient through adjustment of the filters’ transfer function. Since the loop response is controlled only by the tracking filter, an adequate phase margin at unity gain is assured. With the timeconstants of the tracking filter fixed, the closed-loop will have an upper modulation limit which is roughly one to ten times the natural loop frequency, for a given damping coefficient.
In the transponder system, the damping coefficient must be made greater than the conventional 0.7 value typical of phase-lock systems in order to achieve very small tracking errors at modulation rates in excess of twice the natural loop frequency. In addition, the natural loop frequency must be correctly established to make the acquisition time compatible with the automatic acquisition system, and assure a high probability of acquisition with full modulation present on the telemetry signal. Hence, the natural loop frequency has been chosen by compromise with regard to the maximum modulation rates expected, the desired acquisition time, and assurance of a high probability of acquisition under full modulation. The loop damping coefficient has been established to provide the desired response time for the necessary modulation capability and the phase margin needed for loop stability at modulation rates approaching the unity-gain bandwidth.

In sinusoidal modulation test, the AN/DRQ-4 has proven capable of undistorted tracking up to a maximum modulation index of approximately 2.14 (about 750 KHz deviation) at the maximum modulation rate of 350 KHz. This typical value of 2.14 assumes a moderate received S/N ratio; values as high as 2.20 have been determined for exceptionally good signal-to-noise ratios and moderate telemetry signal levels. When these values are compared with the absolute maximum theoretical modulation index possible in a phase demodulation process (2.40), they indicate that the maximum modulation capability of the system is within approximately eight percent of the maximum capability that could be achieved if the loop SNR threshold and phase jitter were completely negligible.

**Phase-Lock Loop Design Considerations** - The operational amplifier gain \(K_{out}\), used with a given \(K_s\), \(K_d\) system product, determines the phase tracking-error within the loop. With the system gains established, the gain crossover point location (open-loop unity-gain bandwidth) determines the phase margin for system stability and the modulation capability. Within the closed-loop system, sufficient attenuation must be provided by the high frequency response of the tracking filter.

Assuming an FM system input of the form \(S \sin \omega t + p \sin \omega m t = S \sin (\omega t + \theta)\), where \(\theta\) is the instantaneous phase of the telemetry signal in the phase-locked condition, we may derive the following expression for the closed-loop transfer function at the VCO output:

\[
E(S) = \theta(S) - \theta_o(S) = 1/S \left[ E(S) K_d K_{out} F(S) K_o \right] = 1/S \left[ E(S) F(S) K_{o1} \right]
\]

where \(K_{o1}\) is the open-loop gain (assuming the opamp unity-gain bandwidth is much larger than the modulation bandwidth and the upper breakpoint of \(F(S)\))

\[
\theta_o(S) = \left[ \theta(S) - \theta_o(S) \right] \frac{F(S) K_{o1}}{S}
\]

\[
\frac{\theta_o(S)}{\theta(S)} = \frac{F(S) K_{o1}}{S + F(S) K_{o1}}
\]

Using \(F(S) = \frac{S + \omega b}{S + \omega d} \left( \frac{R_2}{R_1 + R_2} \right)\)
Closed-Loop Transfer Function

\[
\frac{\theta_e(S)}{\theta_i(S)} = \frac{K_{o1} (S + \omega b)}{S (S + \omega b) + K_{o1} (S + \omega b)} \left( \frac{R_2}{R_1 + R_2} \right)
\]

Derivation of System Parameters

We define the closed-loop transfer function as follows:

\[
H(S) = \frac{\theta_e(S)}{\theta_i(S)} = \frac{F(S) K_{o1}}{S + F(S) K_{o1}} \quad \text{where} \quad F(S) = \frac{\tau_2 S + 1}{(\tau_1 + \tau_2)S + 1}
\]

\[
H(S) = \frac{(\tau_2 S + 1) K_{o1}}{(\tau_1 + \tau_2) S + 1} = \frac{\tau_2 S + 1}{S^2 (\tau_1 + \tau_2) + S(1 + \tau_2 K_{o1}) + K_{o1}}
\]

Closed-Loop Transfer Function

Hence,

\[
H(S) = \frac{\frac{\tau_2 K_{o1}}{\tau_1 + \tau_2} S + \frac{K_{o1}}{\tau_1 + \tau_2}}{S^2 + \left(\frac{1 + \tau_2 K_{o1}}{\tau_1 + \tau_2}\right) S + \frac{K_{o1}}{\tau_1 + \tau_2}}
\]

where the denominator of the closed-loop transfer function corresponds to the characteristic equation of a general second-order linear system; i.e.,

\[
H(S) = \frac{\left[2 \delta \omega n - (\tau_1 + \tau_2)\right] S + \omega f^2}{S^2 + 2 \delta \omega n S + \omega f^2}
\]

From the above,

Natural Loop Frequency

\[
\omega_n = \left(\frac{K_{o1}}{\tau_1 + \tau_2}\right)^{\frac{1}{2}} = 913 \times 10^3 \text{ radians/sec} \quad \left( f_n = 145.4 \text{ KHz} \right)
\]

Loop Damping Coefficient

\[
\delta = \frac{1 + \tau_2 K_{o1}}{2 \omega_n (\tau_1 + \tau_2)} = \frac{1}{2} \left(\frac{K_{o1}}{\tau_1 + \tau_2}\right)^{\frac{1}{2}} \frac{\tau_2 K_{o1} + 1}{K_{o1}} = \frac{\omega_n}{2} \left[ \frac{\tau_2}{K_{o1}} \right] = 0.83
\]
It should be noted that the calculated values for natural loop frequency and loop damping coefficient are achieved with full signal level at the detector input. This implies that the system is operating above the output compression point, and that the maximum loop damping coefficient is achieved under this condition. For low signal levels, the equivalent loop gain and loop damping coefficient are reduced from the values indicated above. Under this condition, system stability is maintained because a lower loop damping coefficient is permissible with reduced loop gain. In fact, the loop damping coefficient closely approaches the 0.707 theoretically optimum value for optimum acquisition and tracking properties, at telemetry levels near the system’s value of input sensitivity. The system is thus made unconditionally stable for all signal levels, with the maximum loop gain limited by the automatic limiting properties of the IF amplifier.

In order to determine the closed-loop performance of the system while tracking a telemetry signal, we will derive the closed-loop response bandwidth directly from the closed-loop transfer function.

For the values calculated above:

\[ 2 \delta w_n = 378.9 \times 10^3, \text{ and } \tau_1 + \tau_2 = 63.8 \times 10^{-6} \]

Thus, \( 2 \delta w_n \gg \tau_1 + \tau_2 \) and the closed-loop transfer function may be approximated as follows:

\[
H(S) = \frac{\left[2\delta w_n - (\tau_1 + \tau_2)\right] S + w_n^2}{S^2 + 2\delta w_n S + w_n^2} \approx \frac{2\delta w_n S + w_n^2}{S^2 + 2\delta w_n S + w_n^2} \quad \text{for} \quad 2 \delta w_n \gg \tau_1 + \tau_2
\]

The closed-loop response bandwidth is derived as follows:

\[
\left| H(j\omega) \right|^2 = \frac{1}{2} \quad \text{for half-power point (3 dB bandwidth)}
\]

where \( \omega = \omega_3 \text{ db} \).
Thus,

\[
\left| \frac{2j\delta w_n^2 + w_n^2}{-w_3^2 + 2j\delta w_n w_3 + w_n^2} \right|^2 = \frac{1}{2}
\]

\[
\frac{\omega_n^4 + 4\delta^2 w_n^2 w_3^2}{\left(\omega_n^2 - w_3^2\right)^2 + 4\delta^2 w_n^2 w_3^2} = \frac{1}{2}
\]

\[
2\omega_n^4 + 8\delta^2 w_n^2 w_3^2 = \omega_n^4 - 2\omega_n^2 w_3^2 + w_3^2 + 4\delta^2 w_n^2 w_3^2
\]

\[
\omega_3^2 \omega_n^2 = (2\delta^2 + 1) = \omega_n^4
\]

\[
\omega_3 = \left( 2\delta^2 + 1 + \frac{1}{1 + (2\delta^2 + 1)^2} \right)^{\frac{1}{2}}
\]

**Closed-Loop Response Bandwidth**

\[
\omega_3 = \omega_n \left[ 2\delta^2 + 1 + \sqrt{1 + (2\delta^2 + 1)^2} \right]^{\frac{1}{2}}
\]

Using the value of $\omega_n$ and $\delta$ damping calculated above:

\[
\frac{\omega_3 \text{db}}{\omega_n} = 2.225 \quad \Rightarrow \quad \omega_3 \text{db} = 2.03 \times 10^6
\]

($f_{3db} = 323.5 \text{ KHz}$)

For modulation rates between the closed-loop bandwidth and the unity-gain bandwidth, the pole-zero configuration of the system indicates a slope of the response curve of minus 6 db/octave. This change in slope above the upper breakpoint of the tracking filter is provided by the first breakpoint of the operational amplifier, and is necessary to establish the unity-gain bandwidth and proper phase margin. With a slope of -6 db/octave, the closed-loop response at the maximum modulation rate of 350 KHz is approximately:

\[
|H(j\omega)|_{350 \text{ KHz}} \approx -3 \text{ db} - \left( \frac{350 - 323.5}{323.5} \right) 6 \text{ db/oct.}
\]

\[
\approx -3.5 \text{ db @ 350 KHz}
\]

Thus, the closed-loop response is down approximately 3.5 db at the maximum modulation rate.
**System Root-Locus Analysis**  The locus of complex conjugate closed-loop poles is shown in the following root-locus plot, along with the locations for maximum loop gain and damping coefficient. Since poles and zeros nearest the imaginary axis are dominate roots of the characteristic equation, the farther the complex conjugate (closed-loop) poles from the imaginary axis, the greater the loop damping. Location of the closed-loop poles to the left of the upper breakpoint, (zero) of F(S), indicates a loop damping coefficient greater than one-half. The dependence of conjugate pole location upon input telemetry level is also illustrated (see Figure 3).

**Closed-Loop Bode Diagram**  Using the approximation validated above for the closed-loop transfer function, the low and high frequency asymptotic approximations to the closed-loop system response may be derived as follows:

\[
H(S) = \frac{\left[2\delta w_n - (\tau_1 + \tau_2)\right] S + w_n}{S^2 + 2\delta w_n S + w_n^2} \approx \frac{2\delta w_n S + w_n^2}{S^2 + 2\delta w_n S + w_n^2}
\]

for \(2\delta w_n \gg (\tau_1 + \tau_2)

\[
H(S) = \frac{2\delta}{\omega_n} S + 1
\]

**Low Frequency Asymptote**

Thus, \(20 \log_{10} |H(j\omega)| = 20 \log_{10} \left[\left(\frac{2\delta\omega}{\omega_n}\right)^2 + 1\right]^{\frac{1}{2}} - 20 \log_{10} \left[\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \left(\frac{2\delta\omega}{\omega_n}\right)^2\right]

\[
\left(\frac{2\delta\omega}{\omega_n}\right)^2 \right) = 10 \log_{10} \left[\left(\frac{2\delta\omega}{\omega_n}\right)^2 + 1\right] - 10 \log_{10} \left[\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \left(\frac{2\delta\omega}{\omega_n}\right)^2\right]
\]

for \(\omega \ll \omega_n \left(\frac{\omega}{\omega_n} \ll 1\right)

\[
20 \log_{10} |H(j\omega)| \approx 10 \log_{10} (1) - 10 \log_{10} (1) = 0 \text{ db}
\]

\( \Rightarrow \) straight line asymptote with zero slope
High Frequency Asymptote

For $\omega \gg \omega_n \left( \frac{\omega}{\omega_n} \gg 1 \right)$;

$$20 \log_{10} |H(j\omega)| \approx 20 \log_{10} \left[ \frac{2\delta\omega}{\omega_n} \right] - 10 \log \left[ \left( \frac{\omega}{\omega_n} \right)^4 + \left( \frac{2\delta\omega}{\omega_n} \right)^2 \right]$$

$$\approx 20 \log_{10} \left( \frac{2\delta\omega}{\omega_n} \right) - 40 \log_{10} \left( \frac{\omega}{\omega_n} \right)$$

but, $\frac{2\delta\omega}{\omega_n} \approx \frac{\omega}{\omega_n}$ for $\frac{\omega}{\omega_n} \gg 1$,

thus, $20 \log_{10} |H(j\omega)| \approx -20 \log_{10} \left( \frac{\omega}{\omega_n} \right)$

=> straight line asymptote with slope

-20 db/decade (i.e., -6 db/octave)

The approximate intersection of low and high frequency asymptotes is:

$$-20 \log_{10} \left( \frac{\omega}{\omega_n} \right) = 0 \text{ db} \Rightarrow \omega = \omega_n$$

**Conclusion**  In conclusion, we have shown that phase demodulation of the PAM/FM telemetry signal makes it possible for the AN/DRQ-4 UHF telemetry transponder to accurately translate S-band telemetry information to the scoring band while preserving the phase of the modulation during the translation. Hence the scoring signal is an accurate duplicate of the original telemetry information, plus the doppler shift due to the relative velocity between missile and target.
Figure 1. AN/DRQ-4 UHF Transponder Block Diagram

Figure 2. System Dynamic Range
Figure 3. Closed-Loop Root-Locus Plot

Figure 4. Bode Diagram; Approximate Closed-Loop Response Vs. Frequency