CALCULATING POWER SPECTRAL DENSITY IN A NETWORK-BASED TELEMETRY SYSTEM

Scott Brierley
The Boeing Company
Integrated Defense Systems
Launch and Satellite Systems

ABSTRACT

Calculating the power spectral density (PSD) at the transducer or data acquisition system offers advantages in a network-based telemetry system. The PSD is provided in real time to the users. The conversion to PSD can either be lossless (allowing a complete reconstruction of the transducer signal) or lossy (providing data compression). Post-processing can convert the PSD back to time histories if desired. A complete reconstruction of the signal is possible, including knowledge of the signal level between the sample periods. Properly implemented, this method of data collection provides a sharp anti-aliasing filter with minimal added cost. Currently no standards exist for generating PSDs on the vehicle. New standards could help telemetry system designers understand the benefits and limitations calculating the power spectral density in a network-based telemetry system.

KEY WORDS

Power Spectral Density, Lossy Data Compression

INTRODUCTION

A useful and convenient format to review certain types of data is provided by the power spectral density (PSD). This format is often used to evaluate accelerometer and microphone data but can by used on other types of transducers. The power spectral density can be calculated using a Fast Fourier Transform (FFT).

GUIDELINES FOR GENERATING A PSD

The FFT is a computationally easy way to calculate the discrete Fourier transform of a waveform. First, N samples of data are collected. The FFT algorithm operates on a number of samples that is a power of 2 (2, 4, 8, 16, etc). If N is not a power of 2, a sufficient number of zero samples must be added to the end to make N a power of 2.

The FFT algorithm output is a set of N real numbers and N imaginary numbers. In other words, putting N numbers through the FFT algorithm results in 2N numbers out. However, N-2 of these numbers corresponds to complex conjugates and can be eliminated. The DC term and the highest frequency term have no imaginary part, so the imaginary part of those two terms may also be eliminated. The net result is N numbers into the FFT algorithm result in N numbers out.

The magnitude and phase are calculated from the real and imaginary parts of the numbers as follows:

$$Magnitude = \sqrt{(real\ \ part)^2 + (imaginary\ \ part)^2}$$

$$Phase = \tan^{-1}\left(\frac{imaginary\ \ part}{real\ \ part}\right)$$

The FFT is equivalent to running the data through a bank of N/2 bandpass filters. In addition, the N samples are averaged to produce the zero frequency (DC) term. Since the highest frequency is equal to the sample rate divided by 2, and there are N/2 bandpass filters, the spacing between each bandpass filter is the sample rate divided by N. Therefore, to increase the frequency resolution, N is increased.

USING A PSD AS AN ANTI-ALIAS FILTER

Data must be sampled at greater than twice the highest frequency in the signal. Any frequency higher than twice the sample rate will appear as a lower frequency (an alias frequency) and contaminate the data. It is not possible to discriminate between these aliased frequencies and real data without a-priori knowledge of the signal, so these aliased frequencies contaminate the data.

An anti-alias filter is typically used to filter out any frequencies higher than twice the sample rate. Because realizable analog filters do not have an infinitely sharp rolloff, the data must be sampled at higher than twice the highest frequency of interest. Typically, the data are sampled at 2.5 to 5 (or more) times the highest frequency of interest. This represents a waste of bandwidth because the frequency content of the sampled data between the highest frequency of interest and ½ the sample rate is contaminated with aliased frequencies and is unusable.

An effective solution to this problem is to:

(1) Filter the data with a simple analog anti-alias filter.
(2) Sample the data at a sample rate much higher than the highest frequency of interest.
(3) Run the sampled data through an FFT algorithm.
(4) Eliminate any spectral components higher than the highest frequency of interest.

Filter specifications may be very loose, as long as the filter has sufficient rejection to filter out any alias frequencies between 0 Hz and the highest frequency of interest. This allows a low-cost filter to be used that has few poles poles and loose tolerances on the cutoff frequency. For example, suppose that data are required to be collected from DC to 2000 Hz, and that the data are sampled at 20,000 samples per second. Any frequency component higher than 18 kHz will alias down to 2000 Hz or lower, and must be filtered out.

By calculating the PSD, the bit rate is reduced by 40% to 80% compared to a system sampled at 2.5 to 5 times the highest frequency of interest, along with a potential cost reduction by using lower cost anti-alias filters.

RECONSTRUCTING TIME HISTORIES

If the phase is retained, the time history can be reconstructed by calculating the inverse FFT as follows:

$$V(t) = \sum_{m=0}^{N-1} (Magnitude_m)\cos(m\omega_o t + Phase_m)$$

Where

$\omega_o = 2\pi(Sample\ Rate)/N$

$N = number\ of\ samples\ processed\ by\ the\ FFT$

$Magnitude_m$ is the magnitude of the mth spectral component

$Phase_m$ is the phase of the mth spectral component

Note in the equations above that there is no restriction that time (t) must be a time that a sample was taken. In other words, sending PSDs instead of the sampled data facilitates a complete reconstruction of the original data, including recovery of the data between the sample points

DATA COMPRESSION

Random vibration or acoustic data frequently have an approximately gaussian amplitude distribution. The high degree of randomness makes the usual data compression algorithms ineffective. Generating PSDs provides a method of data compression.

If a reconstruction of the time history of the data is not required, the phase information need not be retained. This provides an immediate reduction of the bit rate by one-half.

Knowing that a particular spectral component exceeds the maximum predicted PSD is more important than knowing that a spectral component is less than the maximum predicted. Therefore the data can be compressed as follows:

(1) Run the FFT algorithm on the data.
(2) Divide each spectral component by the maximum predicted level at that frequency.
(3) Eliminate the spectral components that have the lowest levels relative to the predicted level.

If the network supports a variable data rate, only the spectral components that exceed a given percent of the maximum predicted level are sent down. Otherwise, a fixed number of spectral components are eliminated each time the PSD is calculated. Additional word(s) must be added to the data stream to show which spectral components were deleted.

Calculating and transmitting the overall RMS level enables calculation of the average level of the spectral components that were eliminated. The peak G level could also be transmitted if the peak value is of interest.

A filtered version of the time history can still be calculated if the phase is retained for the remaining spectral components.

SHOCK RESPONSE SPECTRUM

If the accelerometer is intended to measure a shock event, calculating a shock response spectrum (SRS) instead of a PSD may provide an even higher level of data compression.

Shock events can produce high-frequency components, which require a high sample rate. For example, monitoring a shock over a typical frequency range of 100 Hz to 10,000 Hz would require a sample rate of 25,000 to 50,000 samples per second or higher.

A typical shock transient from the sudden application or release of loads decays in about 5 to 15 ms. The window over which the shock transient is measured should be at least this long. To ensure that the data-collection period includes a complete shock pulse, overlapping measurement windows could be used. For example, every 20 ms the data over the previous 40 ms could be analyzed.

The SRS is typically calculated at intervals of at least 1/6-octave intervals. Over the frequency range of 100 to 10,000 Hz, this would require calculation of about 41 data points. Assuming that the SRS is calculated every 20 ms, the sample rate is reduced to 2050 samples per second.

The data can be compressed even more by sending down only the components that are higher than the predicted SRS, similar to that done for PSD plots.

Because different time histories can produce the same SRS, the time history cannot be recovered from the SRS.

CONCULSIONS

Calculating PSDs at the sensor or in the data acquisition system has several advantages in a network-based telemetry system. The PSD provides a meaningful display of many types of data, helping an understanding of the health of the system both in real-time and in post-launch data review. Calculating PSDs has the potential for reducing system cost by allowing the use of lower cost anti-alias filters. The primary advantage of calculating and transmitting PSDs is providing data compression that may not be obtainable by other means.