ANALYSIS OF THE MODIFIED INTEGRATE AND DUMP DECISION DEVICE*

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Summary The purpose of this paper is to describe a system which exhibits better bit error rates for Frequency Shift Keying (FSK) signals than those now used to make the bit decision The system is similar to the popular Integrat6 and Dump device., but it is modified to take advantage of the information contained in the FM "clicks" resulting from the demodulation process to aid in making the proper bit decision. The paper is divided into four parts: First, a brief review of the Integrate and Dump Detector is presented. Then the "click" mechanism is described and such properties of this mechanism as the number of "clicks" in a channel are reviewed. Third, a method of using the information in the "clicks" to one's advantage is discussed. Fourth, and finally, the hardware needed to implement such a system is described in general and certain suggestions are made to improve the over-all decision making capabilities of the system.

General Discussion of the Integrate and Dump Detector and Frequency Shift Keying Frequency Shift Keying is a common way of transmitting Pulse Code Modulation (PCM) signals. Basically digital data is transmitted on frequency, f_1 , for a "one" and some other frequency, f_0 , for a "zero." The spread between the two frequencies is normally selected based on the bandwidth that is available to the system designer. To a certain extent, increasing the spread between the two frequencies improves the detection capability. If no noise is present, there is no problem in deciding which signal was sent regardless of the frequency spread between the two signals. When voltage, v_1 is present at the output, the detector signals a "one" was received and in a like manner, when voltage, V_0 , is present, signals a "zero" was received.

However, when zero-mean Gaussian noise is added to the channel, the problem of detecting the signal with noise is encountered. Table I outlines the correct and incorrect decisions that can be made by the detector. These decisions are somewhat clearer when

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TABLE I

TRUTH TABLE FOR BINARY DETECTOR

STATE	MESSAGE SENT	DECISION AT DETECTOR	CORRECT	INCORRECT
A	0	0	x	
B	0	1		x
C	1	0		x
D	1	1	x	

presented as their probability density curves. First, the probability density function of the signal is depicted in Fig. 1 for an FSK signal with two equiprobable states. Second, the probability density function of the Gaussian noise is shown in Fig. 2 for two different values of σ^2 The resultant probability density function for both high and low signal-to-noise ratios (SNR) are illustrated in Fig. 3(a) and 3(b), respectively, as the convolution of the probability density functions of the signal and the noise, since

$$\int_{-\infty}^{\infty} p_x(x)\,dx = 1 \qquad (1)$$

$$\int_{-\infty}^{\infty} p_y(y)\,dy = 1 \qquad (2)$$

$$p_z(z) = p_z(x + y) = p_x(x) * \cdot p_y(y) \qquad (3)$$

where * denotes convolution and x and y are statistically independent. Several things can be observed from Fig. 3. one way to improve the probability of making a correct decision is by increasing the SNR. Another way to accomplish the same goal is to increase the spread in frequencies between the two signals. A less obvious way to improve the chances of making a correct decision is to use the information in the FM "clicks" to aid in the decision-making process. This is only useful whenever there are numerous clicks in the output of the FM demodulator; however, this occurs when the input SNR is low so that the two states coincide. Therefore the appearance of the clicks occur at just the time when the bit decision is the most difficult to make. The two are definitely related and the clicks are partly responsible for the reduction in the bit error rate. The bit error rate is the measure of a digital system. It is defined as the probability that the amplitude of the noise over a bit period will exceed the amplitude of the signal in the same time period while having the opposite polarity. The noiseless integration of the signal in an Integrate and Dump Detector is shown in Fig. 4. The dotted path above the time axis illustrates the "one" while the one below the time axis illustrates the "zero." In normal operation the two paths are corrupted by noise and this accounts for the bit errors. The normal threshold for the Integrate and Dump Detector is the zero level and consequently when the path ends above the axis, the detector signals a “one” decision has been made and similarly whenever the path ends below the time axis, the detector signals a “zero.” obviously this decision is incorrect whenever the noise is sufficient to drive the signal and noise path into the opposite region. This probability can be calculated and the equations for the incorrect decisions are:

$$P(z<v_{th}|x = +\sqrt{s}) = \int_{-\infty}^{o} \frac{1}{2\sqrt{2\pi\sigma^2}} e^{-\frac{(z-\sqrt{s})^2}{2\sigma^2}} dz \tag{4}$$

which denotes the conditional probability of deciding a “zero” when a “one” was actually sent. This corresponds to State C in Table I.

$$P(z<v_{th}|x = -\sqrt{s}) = \int_{o}^{\infty} \frac{1}{2\sqrt{2\pi\sigma^2}} e^{-\frac{(z+\sqrt{s})^2}{2\sigma^2}} dz \tag{5}$$

which denotes the conditional probability of deciding a “one” when a “zero” was actually sent. This corresponds to State B in Table I.

The total probability of making a bit error is the sum of these two probabilities and since the two signals are symmetrical, the two probabilities are equal so that the total probability of a bit error, P_e , is:

$$P_e = \frac{2}{2\sqrt{2\pi\sigma^2}} \int_{o}^{\infty} e^{-\frac{(z+\sqrt{s})^2}{2\sigma^2}} dz \tag{6}$$

Through two changes of variables Eq. (6) becomes

$$P_e = \frac{1}{\pi} \int_{\sqrt{\frac{s}{2\sigma^2}}}^{\infty} e^{-\xi^2} d\xi \tag{7}$$

The complementary error function is defined as

$$\text{erfc } a \triangleq 1 - \text{erf} a = \sqrt{\frac{2}{\pi}} \int_{a}^{\infty} e^{-\,^{2}} d\xi \tag{8}$$

so that Eq. (7) can be rewritten as

$$P_e = \frac{1}{2} \text{ erfc } \sqrt{\frac{S}{2\sigma^2}} \tag{9}$$

FM Click Mechanism The FM click results from the action of increase as the FM discriminator. They appear at threshold and the CNR decreases. Schwartz (1966) attributes the occurrence of first threshold to the clicks. The general FM threshold theory has been well documented by Carson and Fry (1937), Crosby (1937), Blachman (1949), Lawson and Uhlenbeck (1950) and Middleton (1960). Generally these articles discuss the effects of both high and low SNR signals on FM discriminators. Specific discussions of FM threshold with particular attention to clicks has been done by Rice (1963) as previously mentioned, Cohen (1956), Schilling, Hoffman and Nelson (1967), Baghdady (1963), and Mazo and Salz (1966). Mazo and Salz have expanded on the works of the other authors in determining the effects of clicks on FSK in particular. Since many of the equations are well documented in the literature, only the essential equations will be reproduced.

When a signal and Gaussian noise are added together and discriminated by an ideal FM demodulator, clicks result at the output. The phase contours Γ_1 and Γ_2 are shown in Fig. 5 for the sum of signal and noise

$$\begin{aligned} Q \cos w_c t + I_n &= R \cos (w_c t + \theta) = \\ &= (Q + I_c) \cos w_c t - I_s \sin w_c t \end{aligned} \tag{10}$$

The phasor representation in Fig. 5 assumes that all the phasors are at the carrier frequency, $w_c t$, and that R(t) is the resultant envelope and θ(t) is the resultant phase of signal plus noise. As time progresses, the phase plot θ(t) is shown in Fig. 6(a) for the Γ contour and its derivative, θ'(t) is the output of an ideal discriminator for an input of θ(t). This is called by Rice (1963) a doublet and it has relatively little effect on the output of the low pass filter following the ideal discriminator because its area is very nearly zero.

The contour, Γ, produces a click and its phase and its output are shown in Fig. 6(b). Clicks can occur in either direction and this fact is illustrated in Fig. 7.

Carpenter (1968) has extended Rice's work by deriving the number of clicks with an unmodulated carrier with frequency offset $\phi(t) = 2\pi f_o$

$$\begin{aligned} N_+ = H_+(t) = \frac{r}{2}\Bigg\{ & \sqrt{1 + \left(\frac{f_o}{r}\right)^2} \text{ erfc}\left[\sqrt{CNR}\sqrt{1 + \left(\frac{f_o}{r}\right)^2}\right. \\ & - \frac{f_o}{r}\; e^{-CNR} \text{ erfc}\left[\frac{f_o}{r}\sqrt{CNR}\right]\Bigg\} \end{aligned} \tag{11}$$

which reduces to the equation for clicks when ϕ (t) = 0 which is Rice's (1963) equation for a symmetrical carrier.

$$N_+ = N_- = \frac{r}{2} \text{ erfc } \sqrt{CNR} \qquad (12)$$

The negative clicks for frequency offset ϕ (t) = $2\pi f_o$

$$N_- = H_-(t) = N_+ + f_o \, e^{-CNR} \qquad (13)$$

Therefore the excess number of clicks when the carrier has an offset ϕ (t) = $2\pi f_o$ is

$$\Delta N_- \cong f_o \, e^{-CNR} \qquad (14)$$

Likewise when the frequency offset is in the opposite direction, the predominance of clicks is in the positive direction. Therefore the excess number of clicks, when the carrier has an offset ϕ (t) = $-2\pi f_o$ is

$$N_+ \cong f_o \, e^{-CNR} \qquad (15)$$

Mazo and Salz (1966) sum up the situation as "the effect of the clicks on a modulated carrier is to tend to make the measured frequencies appear closer to the carrier frequency than the transmitted frequencies." "That is, confining oneself for the moment to only errors caused by clicks, frequencies transmitted higher than the carrier will be measured to be at that frequency or a lower one." Similarly, frequencies transmitted lower than the carrier are measured to be at that frequency or a higher one.

Combining the Two Effects This knowledge of the integrate and dump circuit and the nature of the FM clicks is used to propose a system that takes advantage of the click to aid in improving the bit decision process. Figure 8 gives a summary of the clicks and illustrates that the majority of the clicks are in opposition to the signal. For a positive signal at frequency, f_1 , the majority of the clicks are negative and vice-versa. Gaussian noise with zero mean can cause errors; however, because the mean is zero, the integration tends to average the Gaussian noise to zero. This is not true of clicks. The clicks tend to make a "one" appear to be a "zero" and by the same nature, a "zero" appear to be a "one." Therefore a new threshold voltage, $V_{THRESHOLD}$, could be chosen so that whenever the CNR indicates a strong signal, the bit decision as to whether a "one" or "zero" was sent is made in the integrate-and-dump detector circuit. on the other hand, when the CNR indicates the signal is below the threshold, then the bit decision is made by a device that detects clicks. The philosophy for selecting whether to accept the integrate-and-dump or the click detector for the bit decision is to calculate which detector has more energy and to take the bit decision from the device with more energy.

A simpler system to analyze and to build is picture in Fig. 9. The click detector detects positive and negative clicks and whenever one is detected, its effect on the integrate-and-dump system is nullified by injecting a pulse of 2π radians into the integrate-and-dump circuit.

Figure 9 is a block diagram of the modified integrate-anddump system for Frequency Shift Keying (FSK). The signal, s(t), consists of essentially rectangular pulses which frequency modulate a carrier, w_c , by plus and minus w_c in a continuous manner. The signal is transmitted through the Gaussian additive noise and the sum of the two is bandpass filtered to reject as much noise as possible while at the same time distorting the signal as little as possible. The sum of this signal and noise is input to the ideal discriminator where its ou put is fed to the integrate-and-dump circuit for a decision to be made on the recovered symbol. In parallel, the output of the discriminator is passed to the click detector where continuously positive and negative clicks are detected and simultaneously a 2π radian volt-second impulse is injected into the integrate-and-dump circuitry to nullify the effects of the clicks whenever they are detected. Perfect synchronization is assumed or in other words, the sampling times $t = 0$ and $t = T$ are assumed to be known precisely. Furthermore, the sampling is considered to be performed instantaneously.

The integrate-and-dump is defined by Downing (1964) to be equivalent to passing a time function through a linear filter having the voltage-frequency response function

$$\frac{\sin \pi fT}{\pi fT} \tag{16}$$

with the delay of T/2 seconds. The power spectrum of the averaged function is given by

$$G(f) = G_x(f) \frac{\sin^2 \pi fT}{(fT)^2} \tag{17}$$

The analysis is not intended to show that the system is optimum. Rather, its purpose is to show that the modified integrate-and-dump system gives better performance than the present widely used integrate-and-dump system.

Two svmbols are transmitted and at the output of the ideal FM discriminator, the $S_1(t)$ represents the one symbol and the $S_o(t)$ represents the zero. The two are bi-orthogonal which is d8fined to mean that the signals are the electrical inverse of one another. The noise out of the ideal discriminator is based on the model developed and supported by Rice (1963) where the noise is assumed to be separated into two independent components: one zero mean stationary Gaussian noise and the other additive click noise. The analysis further presumes that there is not intersymbol interference in the FM discriminator and this assumption infers that the bandwidth is infinite. The signal and noise into the integrate-and-dump circuit, assuming that a one was transmitted, is

$$(\xi|\text{one}) = \int_0^T S_1(t)\,dt + \int_0^T n(t)\;dt \tag{18}$$

Similarly the signal and noise, assuming that a zero was transmitted, is

$$(\xi|\text{zero}) = \int_0^T S_o(t)\,dt + \int_0^T n(t)\;dt \tag{19}$$

The noise consists of two random processes, the click noise and the Gaussian noise. However, the proposed system assumes that the clicks are perfectly detected and their effect is nullified leaving:

$$(\xi|\text{one}) = \int_0^T S_1(t)\,dt + \int_0^T n_g(t)\;dt \tag{20}$$

$$(\xi|\text{zero}) = \int_0^T S_o(t)\,dt + \int_0^T n_g(t)\;dt \tag{21}$$

where $n_g(t)$ is the Gaussian noise.

Papouyis (1965) indicates that $s = \int_a^b x(t)\,dt$ where x(t) is a random process having mean $\eta(t)$ and autocorrelation function $R(t_1, t_2)$ where the integral is defined as the mean square limit of a sum, then the expected value of the random variable is defined to be

$$E\{S\} = \int_a^b E\;\{x(t)\}\;dt = \int_a^b \eta(t)\;dt \tag{22}$$

and its variance is

$$\sigma_s^{\,2} = \int_a^b \int_a^b C(t_1,\; t_2)\,dt_1\;dt_2 \tag{23}$$

where $$C(t_1,\; t_2) = R(t_1,\; t_2) - \eta(t_1)\eta(t_2) \tag{24}$$

For the zero mean process

$$\sigma_s^{\,2} = \int_a^b \int_a^b R(t_1,\; t_2)\,dt_1\;dt_2 \tag{25}$$

Assuming white Gaussian noise at the input of the ideal discriminator, the output spectrum is

$$G_n(f) = \frac{2K^2 f^2}{A^2} N_o \frac{\text{watts}}{\text{cps}} \qquad \begin{array}{l} -B \le f \le B \\ |f| > B \end{array} \tag{26}$$

and the spectrum from the integrate-and-dump system

$$G_{n_{out}}(f) = \frac{2K^2}{A^2 \pi^2 T^2} N_o \sin^2 \pi f T \tag{27}$$

The variance can also be calculated according to Schilling, Hoffman, and Nelson (1967)

$$\sigma_s^2 = \int_{-\infty}^{\infty} G(f) \, |H(f)|^2 \, df \tag{28}$$

The probability of error, P_e, can then be calculated since the average value and the variance is known.

Klapper (1966) defines the probability of error with an integrate-and-dump system as

$$P_e = \frac{1}{2} \operatorname{erfc} \sqrt{3 \left(\frac{\Delta f}{Bv}\right)^2 \left(\frac{B_{if}}{2Bv}\right) \text{CNR}} \tag{29}$$

where Δf is the peak deviation of the input signal from the center frequency, B_v is the video bandwidth, and B_{if} is the input predetection bandwidth. For an integrate-and-dump system, Klapper further defines the probability of error in somewhat simpler terms as

$$P_e = \frac{1}{2} \operatorname{erfc} \sqrt{3D^2 \frac{E}{N_o}} \tag{30}$$

where E is the input energy per bit, N_o is the input noise power density and D is defined by Carson's Rule

$$2(1 + D)B_v = B_{if} \tag{31}$$

If the video bandwidth is one-half the bit rate, then

$$\text{CNR} = \frac{E}{N_o} \frac{1}{B_{if}T} \tag{32}$$

where T is the bit duration in seconds.

The modified integrate-and-dump system reduces the effect of the noise and consequently the probability of error will be improved.

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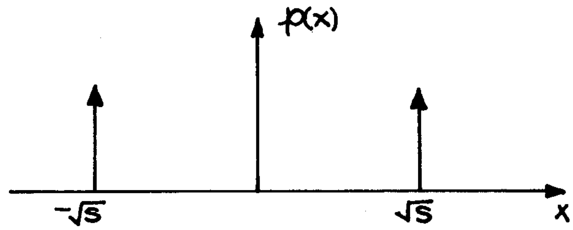

Figure 1. Probability Density Function Of the Signal

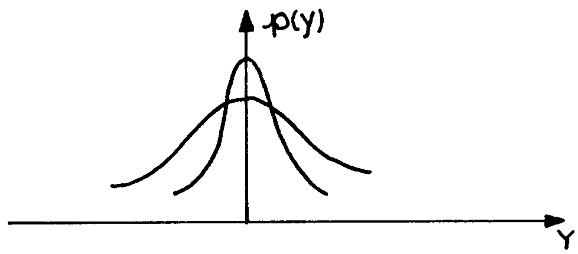

Figure 2. Probability Density Function for Noise

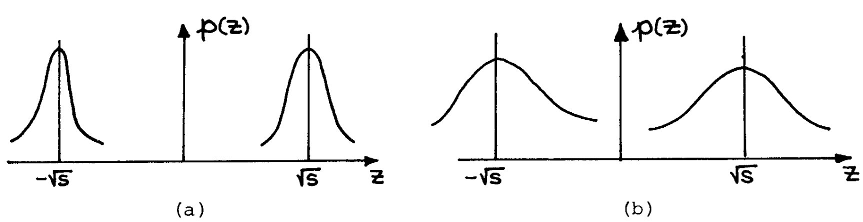

Figure 3. Probability Density Functions

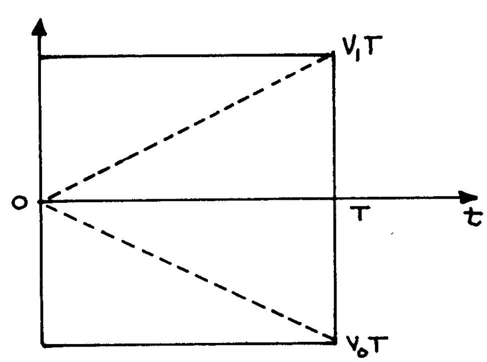

Figure 4. Path of Noiseless Integration

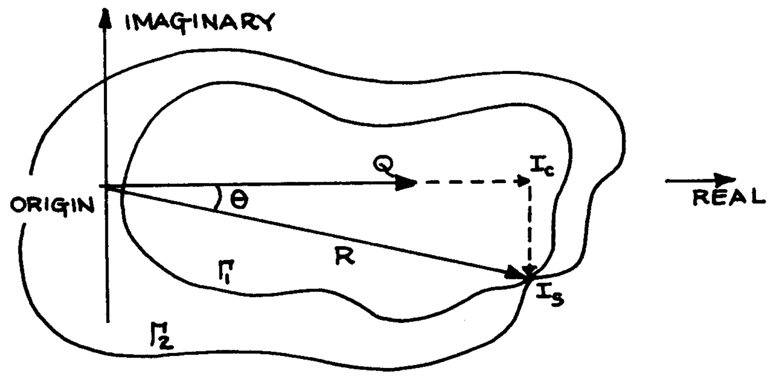

Figure 5. Contours of Resultant of Sum of Signal and Noise

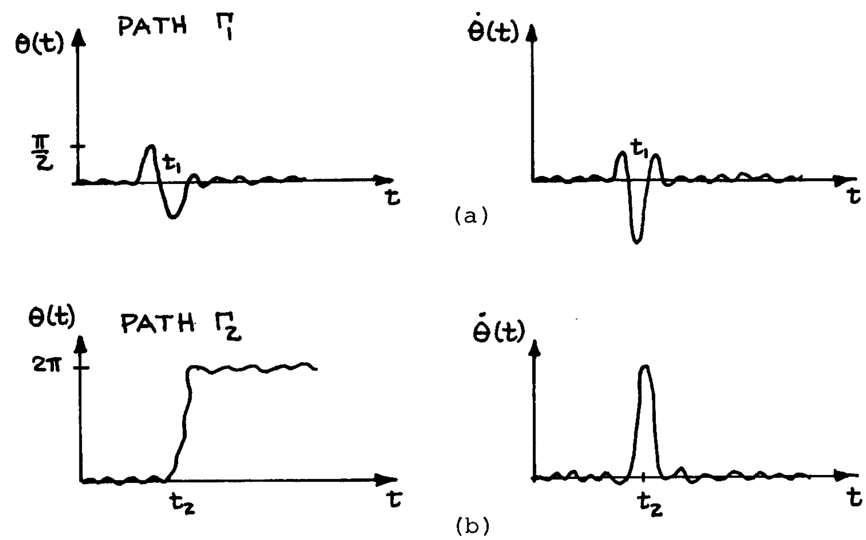

Figure 6. Input and Output of Ideal FM Discriminator For the Two Contours of Figure 5

Figure 7. Input and Output of Ideal FM Discriminator

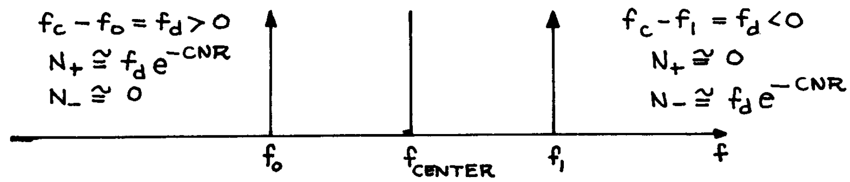

Figure 8. Summary of Positive And Negative Clicks

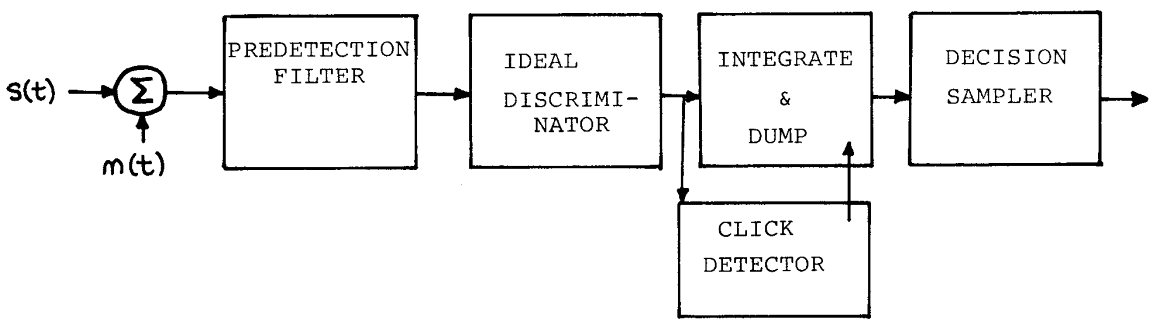

Figure 9. Modified Integrate and Dump System For Frequency Shift Keying (FSK)