USE OF AN ERROR MODEL AND A SIMULATION PROGRAM TO SUPPORT TECHNICAL MANAGEMENT

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Summary This paper contains a discussion of various computer programs and their interconnection with an “error model” which have been developed and are being used by TRW, to form a very useful tool for technical management of a missile development and testing program.

General aspects and requirements of the simulation and of some subroutines are outlined. A review of possible error sources is made emphasizing their effect on the frequency tracking performance of a typical instrumentation system such as the FPQ-6 radar operating with a radar transponder installed on the target.

I. Simulation; General Aspects An accurate, yet flexible, system simulation can be a valuable management tool in making and supporting technical decisions with regard to the design and testing of any complex system structure. The value of such a simulator lies, primarily, in its ability to evaluate at reasonable cost:

- The adequacy of any of the various subsystems or total system to fulfill system requirements (data quality).
- Tradeoffs between concepts and individual parameters within any given concept.
- Test methods and the accuracy of same.
- Hardware design requirements and specifications.
- Need for and adequacy of support systems and requirements.

In connection with the Minuteman III program, TRW has been in process of designing such a simulator on which the outcome of the operational and flight test phase of this missile may be predicted. The major objectives of this simulator as to management will be discussed in detail in a companion paper entitled: J. R. Warren: Instrumentation Systems Engineering Management.

In planning this simulation, we have taken advantage of various computer programs that have been developed for use on past portions of the Ballistic Missile program, using appropriate programs either directly or in modified form; and have supplemented these with new analysis and programs where necessary.

To enjoy the flexibility required to make the simulator a truly useful tool, the simulation of each subsystem under the cognizance of the user must, in themselves, have the flexibility required to accept the actual subsystem hardware.

Those portions of the system not under direct control of the user must also be accurately simulated so as not to introduce operational anomalies which could effect the simulation results.

The simulation may be broken into two major parts: the six axis simulation of the <u>missile flight</u>; and the simulation of the <u>range radar</u> <u>instrumentation system</u> in use at the Western Test Range. The primary new development in our program have been made with respect to the radar instrumentation system, and specifically with respect to the FPQ-6 radar and the radar transponder.

The simulation of this portion is further broken down into three major areas: the total effects and accuracy of radar due to systematic errors and errors introduced as a result of noise degradation of the transmitted/received signal; errors introduced through the <u>transponder</u> due to transponder systematic errors and missile maneuvers and errors introduced due to the <u>propagation path</u> over which measurements are made. In each area, we have attempted to include every conceivable error source, independent of the magnitude of the error introduced. Appendix B accounts for the major error sources which are to be considered.

While many of the error sources that have been included introduce error components which are very small with respect to the required accuracy, the neglect of such errors in the analysis can and would destroy any user confidence in the simulation results.

Each area of the range instrumentation simulation may require several subprograms; for instance, in evaluation of the errors introduced by the propagation path of the radar signal, the effects of path bending due to atmospheric refraction must be accounted for. To accomplish this requires a concise knowledge of the variation of the atmospheric properties with longitude, latitude and altitude, as well as the statistical quality of such variation. Thus the need for an “atmospheric model” as a subprogram in the simulation.

Another example is furnished by the transponder or “beacon” installed on the missile. Its primary function is to respond to a sequence of pulses forming a specified interrogating code with a delayed coherent pulse of increased power, thereby increasing the S/N ratio and tracking accuracy of the interrogating radar. Unfortunately, as far as frequency (or velocity) tracking is concerned, the beacon is subject to various phase instabilities which will be interpreted as frequency deviations by the coherent signal processor on the ground, resulting in velocity errors.

As the main contributors to phase instabilities we may mention short time instabilities of the local oscillator due to injection locking or mechanical vibrations and the effect of S/N ratio at the input. These instabilities have been investigated in detail by various subcontractors and by TRW theoretically and specifications on beacon performance have been set. The beacon test set which is being built to evaluate beacon performance under actual environmental conditions may very well be considered as a subprogram in the form of a very accurate analogue computer. This test set will also be used to independently exercise the beacon operation.

A major area of concern is the effect on the response signal of the transponder due to missile maneuvers, these are twofold: first, since the omnidirectional antenna for the transponder will most likely consist of two or more “slot” type radiators located symmetrically around the body of the missile, several signal phase reversals will be encountered in the antenna radiation pattern in a small angular region around the mid point between the individual slot radiators. Such phase reversals may appear in the radar as rapid “jumps” in doppler shift and in fact may cause the velocity tracking loop to lose lock. The second and minor effect is the doppler shift variation resulting from the alternate receeding and approaching of the antenna as the missile rolls on its major axis. This latter effect is a function of missile roll rate, antenna placement on the missile, and incident angle of interrogation by the measurement radar. Thus, to adequately assess these effects, the simulator must contain a fine grain model of the transponder antenna phase radiation patterns as well as a precise model of the missile dynamics. The theoretical assessment of this type of error is still in progress.

The individual errors encountered in the instrumentation system may be looked at as components of a total error vector. These errors have to be compiled using such well known compendiums as Barton’s, Berkowitz’s or Skolnik’s and the results of numerous investigations which have been or are being performed by RCA, Motorola, Boeing and TRW.

We will now review the computer programs.

II. Review of Computer Programs In order to show how one can achieve these goals, we have prepared a simulation Block Diagram (Figure 1) showing how we --

starting from a preconceived Missile Flight Plan -- arrive at an estimate of the uncertainty in position and velocity at any time during its flight. In essence, Figure 1 is a computer schedule; it contains a number of available computer programs and other special computing tasks which were (or have to be) added to account for the effects of the C-band beacon and propagation effects.

A specific missile flight program is conceived and used to predict the missile trajectory; the computer output is called the state vector $S_L(t)$ as a function of time t. The subscript L indicates that its components (range, elevation and azimuth and their derivatives [R, $\dot{R}$... E, $\dot{E}$ ... A, $\mathring{A}$] are referred to the launch point. A revised version of this "Trajectory Generation Program" described in Appendix A includes the roll, pitch and yaw angles of the missile (or attitude).

Since the missile is tracked by a number m of radar stations on the ground the trajectory information is needed in terms of the local coordinate system of the tracking radars. This -is obtained in the "Geometry Transformation" block. The output, state vector $S_m(t)$, is different for each station. The missile attitude becomes the local "Aspect Angles". The $S_m(t)$ vector will be needed in all subsequent calculations. It is one of the inputs to the "Error Model" block. This block, together with the previously mentioned geometry-transformation, forms an available computer program called "Tracker Model Program", the details of which are given in Appendix A. In order to use it for our purposes, some modifications are necessary which are obtained by the subroutines indicated by the 3 blocks underneath.

As pointed out previously, the individual calculated errors can be looked at as the components of a total error vector $\epsilon_m(t)$, as a function of time, and different for each radar station (hence, subscript m).

Finally, the BET (Best Estimate of Trajectory) Program solves for the best estimate of position uncertainty at a given time on the reference trajectory, using the given instrumentation geometry and weighing by random error variances. Less weight is given to those stations which have larger RMS errors (see Appendix A, c).

We return now to the Error Model and restrict ourselves to problems relating to Doppler-frequency tracking. For the purpose of this discussion it is convenient to subdivide the error budget as shown in Table 1.

a) Constant errors, indicated as inputs in the form of constant error coefficients K_{cm}, for the various m stations.

b) Errors dependent on missile dynamics. These errors are defined by error constants K_{im} such as velocity or acceleration constants. They lead to time (or trajectory) dependent errors and have been used extensively in the available computer program; with the exception of loop transient errors.

c) “Fluctuation Errors” are caused by receiver noise, and their magnitude depends on the S/N ratio. The latter is determined partly by the beacon antenna gain for the proper aspect angles, partly by the missile range. The ISMAP (“Instrumentation Margin Analysis Program” (see Appendix A for detail) together with a Radar Range calculation, will develop the SIN ratio as a function of time. This information will be recorded on tape and will be fed to the “Tracker Model Program” as a first modification. Also included here is “Beacon Noise”. This term evaluates Doppler fluctuations caused by roll of the missile (spin of the beacon antennas). These fluctuations are determined by the rate of change of the carrier phase, $d\phi/dt = f(t)$, and will be obtained on tape, from experimental antenna measurements.

d) The effects of propagation have been separated out, although they too are partly of a fluctuating nature, resulting from turbulence in the troposphere and ionosphere. A computer subroutine must be provided for the calculation; with the results recorded and fed into the available Tracker Model Program. Additional bias terms can be estimated and compensated for within the troposphere only.

Table 1
Radar Doppler Tracking Errors

Type	Description
a) Constant errors	Oscillator stabilities Transmitter spectrum spread Computer errors (lowest significant bits) Tracking Loop: Discriminator Drift Readout granularity Beacon Spectrum Spread
b) Errors dependent on missile dynamics	Position errors Velocity errors Acceleration errors Jerk errors Loop transient errors Maneuver errors

c)	Fluctuation errors	Receiver noise error Beacon Noise error
d)	Propagation error	Beam-bending and turbulence

Appendix A: Computer Programs-Details (a) Trajectory Generation Program. This program generates a time history of position, velocity and acceleration satisfying the equation of motion with gravity and drag used in this case as the only forces. A simple ARDC atmosphere and fixed ballistic coefficient was used in the drag model. This trajectory is used to generate tracking system error models in the subsequent program.

The trajectory generation program also generates the variational equations for separation conditions; that is, the matrix of partial derivatives

$$\frac{\partial\left(P_{(t_i)},\ V_{(t_i)}\right)}{\partial(\delta P_o,\ \delta V_o)} \overset{\Delta}{=} B_o(t_k)$$

where δP_o and δV_o are perturbations in the separation position and velocity. This is a 6 x 6 matrix. This matrix is derived from free-flight (gravity) terms only, but is evaluated around the trajectory which considers drag, so that the error will be small for high -β vehicles.

This matrix will enable the analyst to evaluate the perturbation in the trajectory at any time t_i following separation for given values of δP_o , δV_o. These perturbations are used in the subsequent programs to deteomine how well a specific instrumentation array could determine δP_o, δV_o from data.

(b) Tracker Model Program This program determines the time history of tracker errors given the reference trajectory and tracker location. For example, a radar range error could be modelled as

$$R = K_c + (\Delta T)(\dot{R})$$

where K_c represents a bias error (constant) and ΔT a timing error. A unit ΔT causes an error in R proportional to range rate, k. The tracker modelling program calculates all necessary terms (such as R) needed to represent the modellable errors of the tracking system.

So that all of the error sensitivity partials may be in the tracking system coordinates, the Tracker Model Program also transforms the separation condition variational equations that show the perturbation in tracker measurements (R, A, E, say) for unit separation δP_o, δV_o values. Combining the transformed separation variational matrix with the tracking model error matrices, a composite matrix $B(t_i)$ is generated giving the sensitivities of all tracker measurements to errors in separation conditions and tracker errors.

$$B(t_i) = \frac{\partial (R, A, E)}{\partial (\delta P_o, \delta V_o, \text{tracker error})}$$

(c) BET Program This program solves for the best estimate of position uncertainty at a given time on the reference trajectory given instrumentation geometry and weighing (by random error variances).

The BET functions by using the values computed in the Tracker Model Program of

$$B_Q = \frac{R, A, E, R, \text{etc.}}{\delta X, Y, Z}$$

$$B_K = \frac{\delta R, A, E, R}{\delta K_i} \qquad K_i = \text{error coefficients}$$

and then computing a trajectory covariance matrix (uncertainty in position and/or velocity).

(D) The ISMAP Program This program was designed to compute received signal strengths as a function of missile position (or time) and missile attitude.

In the geometry transformation block, it transforms the missile trajectory from launch point coordinates to radar coordinates; this now includes the missile attitude.

In the next block it solves the radar equation for received signal strength. For this it needs as inputs various ground station parameters (such as transmitted power, antenna gains) as well as beacon reply power. The beacon antenna gain has been stored in the memory as a function of aspect angles, at intervals of 2 degrees, usually in dB above omnidirectional. This input has been obtained experimentally for a given beacon antenna configuration; it is measured for either left or right hand circular polarization, or a composite of both.

The output of this program is on tape and in printed form. It lists, for each of up to 5 ground stations, at specific time intervals: missile aspect angle, location and attitude; system margin or gain (dB).

A few options have been added to the original de-sign, For more detail the reader is refered to the companion paper: L. D. Foust, A Computerized Data Management for the Minuteman Instrumentation System.

Appendix B: Survey of Error and Noise Sources.

I.) FPQ-6 Transmitter	A)	Frequency Source Stability, spectrum
	B)	Modulator Noise, spectrum
	C)	Amplifier Phase characteristics, effects on coherency
	D)	Plumbing Effects of mismatch and losses
	E)	Antenna Type, illumination, gain, polarization
II.) Propagation path	A)	Atmospheric effects
	B)	Ionospheric effects
	C)	Meteorological effects
	D)	Multipath effects
	E)	Anomalies
	F)	Rocket flame effects
III.) Transponder		
Receiver	A)	Phase characteristics, noise
	B)	Detection (threshold, code recognition)
	C)	Delay
Transmitter	D)	Power amplifier (distortion, output phase char.)
Transmitter and Receiver	E)	Plumbing (losses, mismatch)
	F)	Antenna (gain, polarization, phase)
IV.) FPQ-6 Receiver	A)	Antenna (gain, feed geometry polarization, temperature)
	B)	Plumbing
	C)	IF and Video (noise, phase characteristics, limiting, FM Detector)
	D)	Velocity tracker implementation (processing computer errors, granularity, transient response)
V.) Missile Maneuvers	A)	Antenna gain changes
	B)	Antenna phase changes
	C)	Antenna polarization changes

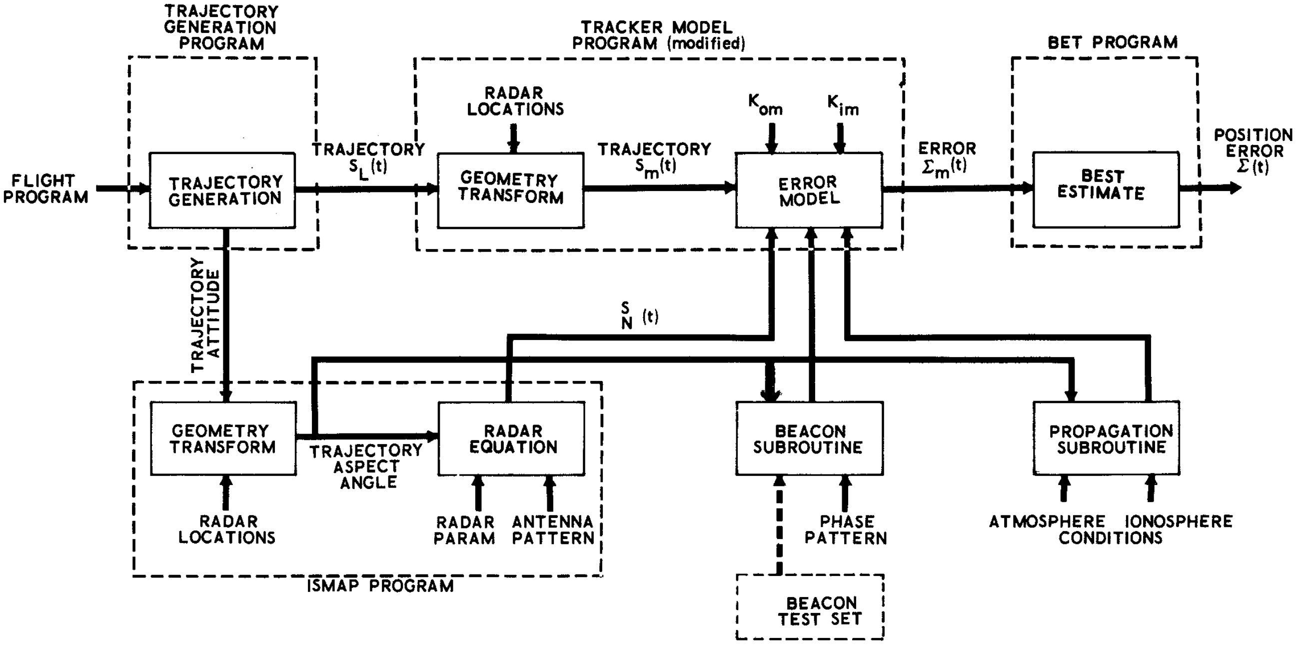

FIGURE 1 SIMULATION BLOCK DIAGRAM