EFFECTS OF MULTIPATH ON TELEMETRY DATA TRANSMISSION

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Summary. - Spurious amplitude and phase modulation effects occur in the received telemetry signal when both the direct waves and the indirect waves enter the receiving antenna. In this paper, a simple multipath model is used which has a single direct wave and a single indirect wave. The indirect wave differs from the direct wave by a time delay and a constant amplitude. With such a simple model, the effects of multipath on telemetry data transmission can be fully examined because of the mathematical simplicity. First an FM system operating above the threshold is considered. The intermodulation distortion and the degradation in the data signal-to-noise ratio due to multipath are both examined. Next we consider two digital systems, namely the noncoherent FSK and the PCM/FM with the discriminator detection. Both systems utilize a split-phase baseband signal. And the error rates depend on the multipath time delay. Comparison is made between the split-phase and the NRZ (non-return-to zero) baseband signals. Available performance improvement from using the predetection diversity combining and the synchronization problem are then discussed. Finally several methods to improve the telemetry data are suggested.

Introduction. - In the study of the telemetry data from two recent Apollo Range Instrumentation Aircraft (ARIA) flights, the anomalies displayed on the ARIA tapes are identified as due to the relatively high flight altitude and the multipath propagation [1]. Due to the high flight altitude, there is an appreciable time delay between the direct and the indirect paths. The resulting effects of the multipath propagation on the telemetry data are examined in this paper. Three systems are considered: the continuous FM system, the noncoherent FSK, and the digital FM with the discriminator detection. For convenience in analysis, we use a simple multipath model which has a single direct wave and a single indirect wave. The indirect wave differs from the direct wave by a time delay and a constant amplitude.

The oscillograms of the predetection and the post-detection waveforms reported by Hedeman and Nichols [1] show the following anomalies: (1) the large intermodulation products in an FM system, and (2) the high bit error rates during the periods of high signal strength in the PCM systems. This paper analyzes and verifies these anomalies and suggests methods to improve the telemetry data transmission when the effects of multipath become significant.

The Continuous FM System. - Let ω_C and ω_D be the carrier and the data signal frequencies respectively, and $\Delta\omega_D$ be the frequency deviation. We assume that only one indirect wave of amplitude b is present at the same carrier frequency as the direct wave of amplitude a. The direct FM wave is

$$a(t) = a \cos \left(\omega_C t + \frac{\Delta\omega_D}{\omega_D} \sin\omega_D t\right) \tag{1}$$

and the indirect wave is

$$b(t) = b \cos \left[\omega_C (t - \Delta t) + \frac{\Delta\omega_D}{\omega_D} \sin\omega_D (t - \Delta t)\right] \tag{2}$$

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where Δt is the time difference between the direct and the indirect waves. Here Δt is considered to be constant. The vector addition of the direct and the indirect waves is shown in Fig. 1. In this figure, vector $\bar{a}$ represents the indirect wave, vector $\bar{b}$ represents the indirect wave and vector $\bar{c}$ the resultant. Note that $\bar{c}$ is modulated in phase and amplitude relative to $\bar{a}$ due to the relative motion of transmitter and receiver. The difference in the instantaneous frequencies of (1) and (2) is denoted as $\Delta F(t)$,

$$\Delta F(t) = \Delta\omega_D(\cos\omega_D t - \cos\omega_D(t - \Delta t)) \tag{3}$$

which is periodic with frequency ω_D. The angle between $\bar{a}$ and $\bar{b}$ is given by,

$$\phi = 2\pi \int \Delta F(t)dt = 2\pi \frac{\Delta\omega_D}{\omega_D} (\sin\omega_D t - \sin\omega_D(t - \Delta t)) + \phi_o \tag{4}$$

where $\phi_o = \omega_c \Delta t$ is the "average" phase angle between $\bar{a}$ and $\bar{b}$ determined by the path difference, $\Delta R = f_c \lambda \Delta t$. Here λ is the wave length of the telemetry carrier. If $\Delta R/\lambda = 1/4$, for example, then $\phi_o = \pi/2$. Let

h_1 = height of transmitter above the reflecting plane,

h_2 = height of receiver above the reflecting plane,

R = radio range.

Assume that the earth acts like a smooth horizontal reflector and use the flat earth approximation. Then if $R \gg (h_1 + h_2)$, the path difference between the two waves is,

$$\Delta R = \frac{2h_1h_2}{R} \tag{5}$$

Thus as h_1, h_2, and R change, ϕ_o changes. If we assume that the rate of change of ϕ_o is small compared to $\Delta F(t)$, then Figure 1 shows the average ϕ_o, which moves slowly around the circle with a rapid phase jitter superimposed on the motion. In Figure 2, $\phi_o = 0, n2\pi$, n being a positive integer, and the maximum rate of change of $\theta(t)$ occurs. The instantaneous frequency modulation of the resultant $C(t)$ is $\frac{1}{2\pi}\frac{d\theta(t)}{dt}$ Hz/sec. so that the maximum multipath frequency modulation occurs for $\phi_o = 0, n2\pi$. The vector sum $\bar{C}$ is a nonlinear operation (law of cosine) so that when different baseband frequencies are superimposed, sum and difference frequencies are generated and intermodulation results.

For illustrative purpose, we consider a continuous FM system with three sinusoidal modulating waveforms denoted as,

$$K_1 \cos\omega_1 t + K_2 \cos\omega_2 t + K_3 \cos\omega_3 t \tag{6}$$

where f_1, f_2 and f_3 are not harmonically related (e.g. f_1 = 20 KHz,

f_2 = 36 KHz, f_3 = 52 KHz). then

$$\Delta F(t) = \sum_{i=1}^{3} K_i [\cos\omega_i t - \cos\omega_i(t - \Delta t)] \tag{7}$$

The peak-to-peak excursion of ϕ, the angle between $\bar{a}$ and $\bar{b}$, is

$$\phi = 2\pi \int \Delta F(t)dt = \sum_{i=1}^{3} \frac{2\pi K_i}{\omega_i} [\sin\omega_i t - \sin\omega_i(t - \Delta t)] + \phi_0 \quad (8)$$

In the remainder of this paper, we assume $\phi_0 = 0$. The peak-to-peak excursion of each term $2\Delta\phi_i$ is $4\pi K_i/\omega_i$ radians, $i = 1,2,3$. If each $\Delta\phi_i$ is not too large and $b \ll a$, the resulting frequency modulation, denoted as $\Delta q(t)$, is

$$\Delta q(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt} \simeq - \frac{b}{a-b} \frac{1}{2\pi} \frac{d\phi(t)}{dt}$$

$$= \frac{2b}{a-b} \sum_{i=1}^{3} K_i \sin \frac{\omega_i \Delta t}{2} \sin \left(\omega_i t - \frac{\omega_i \Delta t}{2}\right) \quad (9)$$

By using the vector $\bar{a}$ as reference, and taking into account the direct wave frequency modulation, the resultant signal corresponding to vector $\bar{c}$ is

$$c(t) \cos \left(\omega_c t + 2\pi \int \left(\sum_{i=1}^{3} K_i \cos\omega_i t\right)dt + 2\pi \int \Delta q(t)\, dt\right)$$

$$= c(t) \sum_{\ell=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} J_\ell(\beta_1) J_m(\beta_2) J_m(\beta_3) \cos(\omega_c + \ell\omega_1 + m\omega_2 + n\omega_3)t \quad (10)$$

where

$$\beta_i = \frac{K_i}{\omega_i} \sqrt{\left(1 - \frac{2b}{a-b} \sin^2 \frac{\omega_i \Delta t}{2}\right)^2 + \left(\frac{2b}{a-b} \sin \frac{\omega_i \Delta t}{2} \cos \frac{\omega_i \Delta t}{2}\right)^2} \quad (11)$$

So there are desired frequency components $\omega_c \pm \ell\omega_1$, $\omega_c \pm m\omega_2$, and $\omega_c \pm n\omega_3$ and undesired frequency components (intermodulation products),

$\omega_c \pm \ell\omega_1 \pm m\omega_2$, $\omega_c \pm m\omega_2 \pm n\omega_3$, $\omega_c \pm \ell\omega_1 \pm n\omega_3$,
and $\omega_c \pm \ell\omega_1 \pm m\omega_2 \pm n\omega_3$.

Figures 3(a) and 3(b) show the multipath effects in the ARIA telemetry on a typical mission. The PCM was split-phase at 4.8 Kb/sec. The continuous FM channels were 2 cω, 4 cω, and 6 cω frequencies at 20, 36, and 52 KHz respectively. The receiving antenna provided simultaneous right and left circular polarization. The vehicle antenna pattern was such that the left hand circular and right hand circular components were alternately larger. The indirect ray has a reversal of circular polarization sense upon reflection. For example, when the output of the left hand feed is smaller than that of the right hand feed, it suffers more from the multipath and vice versa. This is seen in both the AM (signal strength) and FM (intermodulation distortion). At the time of the record, the range, R, was 26 nm, aircraft altitude, h_1, was 6 nm and the altitude of the vehicle was 15 nm. Direct calculation gives $\Delta t = 35\ \mu$sec. which is about twice the period of the highest subcarrier frequency. Thus if the amplitude of the reflected wave is comparable to the amplitude of the direct wave, severe intermodulation distortion occurs as shown in Figures 3(a) and 3(b). To illustrate, Eq. (9) shows that the amplitude of $\Delta q(t)$ due to a single channel can be as large as $2bK/(a-b)$ when the delay is an odd integer of half the subcarrier period. Here K is the carrier deviation of that channel. If the delay is 40 µsec., for example, the 36 KHz channel is delayed about 3/2 periods. Thus if K = 15 KHz for this channel, and $b/a \simeq 0.5$, then the amplitude of $\Delta q(t)$ due to this channel alone is about 30 KHz. If all three channels are considered, then Eq. (10) indicates that the undesired signal components can be very significant.

The data signal-to-noise ratio is now examined. The received signal can be written as

$$c(t)\ \cos(\omega_C t + 2\pi \int(\sum_{i=1}^{3} K_i \cos\omega_i t)dt + 2\pi\int\Delta q(t)dt) + n(t) \tag{12}$$

where $n(t)$ is the additive white noise with one-sided predetection noise spectral density N_o watts/Hz. The demodulated signal-plus-noise is proportional to

$$\Sigma K_i \cos\omega_i t + \Delta q(t) + n_D(t)$$
$$= \sum_{i=1}^{3} K_i\ [\cos\omega_i t + \frac{2b}{a-b} \sin \frac{\omega_i \Delta t}{2}\ \sin(\omega_i t - \frac{\omega_i \Delta t}{2})] + n_D(t) \tag{13}$$

where $n_D(t)$ is the demodulated FM noise.

Assume that the three subcarriers are separated by bandpass filters of bandwidth W centered at each subcarrier. When the carrier signal-to-noise ratio is above the threshold, the demodulated output (i.e. the data) has the signal-to-noise ratio

$$(\frac{S}{N})_{D(\text{multipath})} = 3\ (\frac{\omega_D \beta}{W})^2 . \frac{1}{2}\ \frac{\widehat{c^2(t)}/2}{N_o W} \tag{14}$$

where ω_D corresponds to a subcarrier frequency ω_1 and β is given by Eq. (11) with the subscript removed. And $\widehat{c^2(t)}$, the average value of $c^2(t)$, is given by

$$\widehat{c^2(t)} = a^2 + b^2 - 2\ ab\ \widehat{\cos\phi(t)} \tag{15}$$

where, for small $\phi(t)$, $\cos\ \phi(t) \simeq 1 - \frac{1}{2}\phi^2(t)$ (16)

and $\phi(t)$ is given by Eq. (4) with $\phi_o = 0$.

If there is no multipath, the data signal-to-noise ratio is

$$(\frac{S}{N})_{D(\text{no multipath})} = 3\ (\frac{K}{W})^2 \cdot \frac{1}{2}\ \frac{a^2/2}{N_o W} = \frac{3}{4}\ \frac{K^2 a^2}{N_o W^3} \tag{17}$$

The ratio of the data signal-to-noise ratio with and without multipath is, with the use of Eqs. (11) and (16),

$$\frac{(\frac{S}{N})_{D(\text{Multipath})}}{(\frac{S}{N})_{D(\text{no multipath})}} = [\ (1 - \frac{b}{a})^2 + \frac{b}{a}\ \widehat{\phi^2(t)}\ ] \cdot [(1 - \frac{2b}{a-b} \sin \frac{\omega_D \Delta t}{2})^2 + (\frac{2b}{a-b} \sin \frac{\omega_D \Delta t}{2} \cos \frac{\omega_D \Delta t}{2})^2] \tag{18}$$

The right hand side of Eq. (18) will always be less than one for our assumptions of small $\phi(t)$ and $b<a$. Hence the multipath effect always reduces the signal-to-noise ratio of the telemetry data in the FM system. The amount of degradation depends on the time delay and the amplitude ratio of the indirect and direct waves. Performance analysis below FM threshold is the same as that of the conventional FM system. More performance degradation due to multipath is expected when the FM system operates below the threshold.

<u>Split-Phase/FSK Noncoherent Reception.</u> - The structure of the split-phase

signal for a mark or a space consists of frequency f_1 transmitted during the first half of the signal duration T, and frequency f_2 during the second half, or vice versa. At a particular time instant, the direct wave is $a \sin\omega_i t, \omega_i = 1,2$ and the indirect wave is $b \sin\omega_i(t - \Delta t)$, $i = 1,2$. The received signal is

$$a \sin \omega_i t + b \sin \omega_i(t - \Delta t) + n(t) \tag{19}$$

where $a \sin \omega_i t + b \sin \omega_i(t - \Delta t)$

$$\simeq (a + b \cos\omega_i \Delta t) \sin \omega_i t - b \cos \omega_i t \sin \omega_i \Delta t$$

$$\simeq R(t) \sin (\omega_i t + \theta_i(t))$$

and $R^2(t) = a^2 + b^2 + 2ab \cos \omega_i \Delta t$

$$\theta_i(t) = \tan^{-1} \frac{-b \sin\omega_i \Delta t}{a + b \cos\omega_i \Delta t}$$

The receiver will select randomly with equal probability mark or space if the signal detected at each half bit interval has the same frequency, i.e. f_1 followed by f_1 or f_2 followed by f_2. Thus the error probability is

$$Pe = \frac{1}{2} e^{-\frac{a^2 + b^2 + 2ab \cos\omega_1 \Delta t}{2\sigma_n^2}} \cdot \frac{1}{2} e^{-\frac{a^2 + b^2 + 2ab \cos\omega_2 \Delta t}{2\sigma_n^2}}$$

$$+ \frac{1}{2} \cdot \frac{1}{2} e^{-\frac{a^2 + b^2 + 2ab \cos\omega_1 \Delta t}{2\sigma_n^2}} \left(1 - \frac{1}{2} e^{-\frac{a^2 + b^2 + 2ab \cos\omega_2 \Delta t}{2\sigma_n^2}}\right)$$

$$+ \frac{1}{2} \left(1 - \frac{1}{2} e^{-\frac{a^2 + b^2 + 2ab \cos\omega_1 \Delta t}{2\sigma_n^2}}\right) \cdot \frac{1}{2} e^{-\frac{a^2 + b^2 + 2ab \cos\omega_2 \Delta t}{2\sigma_n^2}}$$

$$= \frac{1}{4} e^{-\frac{a^2 + b^2}{2\sigma_n^2}} \left[e^{-\frac{ab \cos\omega_1 \Delta t}{\sigma_n^2}} + e^{-\frac{ab \cos\omega_2 \Delta t}{\sigma_n^2}}\right] \tag{20}$$

where σ_n^2 is the noise variance. Eq. (20) depends on the frequencies f_1 and f_2 and the time delay Δt. The smallest Pe occurs when $\omega_1 \Delta t$ and $\omega_2 \Delta t$ are both an even integer multiplied by 2π, $\omega_1 \neq \omega_2$, i.e.

$$Pe_{min} = \frac{1}{2} e^{-\frac{(a + b)^2}{2\sigma_n^2}} \tag{21}$$

The largest Pe occurs when $\omega_1 \Delta t$ and $\omega_2 \Delta t$ are both an odd integer multiplied by 2π $\omega_1 \neq \omega_2$, i.e.

$$Pe_{max} = \frac{1}{2} e^{-\frac{(a-b)^2}{2\sigma_n^2}} \tag{22}$$

Assume that $\omega_1 < \omega_2$ without loss of generosity. Both Eqs. (21) and (22) require that $(\omega_2 - \omega_1)\Delta t$ be an even integer multiplied by 2π, a very unlikely event for the assumption of a very small Δt. Eq. (21) is most unlikely because it requires that $\omega_1 \Delta t \geq 4\pi$.

For an NRZ baseband format, the error probability is

$$Pe = \frac{1}{2}\left[\frac{1}{2} e^{-\frac{a^2 + b^2 + 2ab\cos\omega_1\Delta t}{2\sigma_n^2}} + \frac{1}{2} e^{-\frac{a^2 + b^2 + 2ab\cos\omega_2\Delta t}{2\sigma_n^2}}\right]$$

$$= \frac{1}{4} e^{-\frac{a^2+b^2}{2\sigma_n^2}}\left[e^{-\frac{ab\cos\omega_1\Delta t}{\sigma_n^2}} + e^{-\frac{ab\cos\omega_2\Delta t}{\sigma_n^2}}\right]$$

which is the same as EQ. (20) for the split-phase baseband format.

Discriminator Detection of PCM/FM. - The energy of an NRZ baseband signal is concentrated over a bandwidth equal to the bit rate. The energy of a split-phase baseband signal, however, spreads over a bandwidth equal to twice the bit rate. The binary signal (split-phase) is filtered by a premodulation filter with bandwidth about twice that required for an NRZ baseband signal. If we use a periodic signal of period T to approximate the split-phase baseband signal, then the fundamental frequency of the split-phase signal is also equal to twice the bit rate. And the premodulation filter bandwidth is capable of passing the first harmonic. Without multipath, the modulating signal at the output of the premodulation filter is

$$\dot{D}(t) = \Delta\omega\cos\omega t \tag{23}$$

where $\omega = \frac{2\pi}{T}$. The IF filter bandwidth is chosen in accordance with the Carson's rule.

$$B_{IF} = 2(\beta + 1)f$$

where the modulation index β is defined as $\beta = \Delta f/f$. Assume that a rectangular IF filter of bandwidth B_{IF} is used, and the Gaussian shaped low-pass filter 3dB cutoff frequency is taken as 1/T. The received signal is approximately,

$$c(t)\cos\left[\omega_c t + 2\pi\int\dot{D}(t)dt + \frac{b}{a-b}\phi(t) + \theta_m(t)\right] + n(t) \tag{24}$$

$$\text{where } \phi(t) = \frac{\Delta\omega}{\omega}\left[\sin\omega t - \sin(\omega t - \omega\Delta t)\right] \tag{25}$$

and $\theta_m(t)$ is the random phase jitter due to the multipath fading. The modulating signal with multipath is

$$\dot{D}_m(t) = \dot{D}(t) + \frac{2b\Delta\omega}{a-b}\sin\frac{\omega\Delta t}{2}\sin(\omega t - \frac{\omega\Delta t}{2}) \tag{26}$$

We can define the frequency deviation index with multipath as

$$\beta_m = \beta\sqrt{(1 - \frac{2b}{a-b}\sin^2\frac{\omega\Delta t}{2})^2 + (\frac{2b}{a-b}\sin\frac{\omega\Delta t}{2}\cos\frac{\omega\Delta t}{2})^2} \le \beta \tag{27}$$

which is similar to Eq. (11). It is important to note that multipath reduces the frequency deviation which now depends on Δt and b/a. The received signal-to-noise ratio, or the signal energy per bit to noise density ratio, is $\overline{c^2(t)}/2\sigma_n^2$, where σ_n^2 is the noise variance. From Eq. (25),

$$\overline{\phi^2(t)} = 2\beta^2\sin^2\frac{\omega\Delta t}{2} \tag{28}$$

Also from Eq. (25), the ratio of the received signal-to-noise ratios with and without multipath is $(1 - \frac{b}{a})^2 + \frac{b}{a}\,\overline{\phi^2(t)}$. If $a = 4b$, $\overline{\phi^2(t)} = 1\ \text{rad}^2$ (maximum allowed), then the degradation due to multipath is a factor of 13/16 in the signal-to-noise ratio.

By the use of Eqs. (15) and (16), we can write

$$\frac{\widehat{c^2(t)}}{2\sigma_n^2} = \frac{(a-b)^2}{2\sigma_n^2}\left[1 + \frac{2ab\ \beta^2 \sin^2 \frac{\omega\Delta t}{2}}{(a-b)^2}\right] \tag{29}$$

where $\frac{(a-b)^2}{2\sigma_n^2}$ is the minimum received signal-to-noise ratio and the inverse of the bracket term is the "fading depth."

The error probability consists of two parts: one due to the Gaussian noise and the other due to the spike noise at the output of the FM discriminator. The error rate analysis is the same as that of Schilling et.al. [2] because c(t) is a deterministic waveform. The error due to the Gaussian noise is given by

$$P[\text{error GN} \mid +] = \frac{1}{2}\ \text{erfc}\ \frac{0.813\ \beta_m \sqrt{\beta_m + 1}}{\sqrt{f_o(\beta_m)}} \sqrt{\frac{\widehat{c^2(t)}}{2\sigma_n^2}} \tag{30}$$

where

$$f_o(\beta_m) = \text{erf}\ (0.837(\beta_m + 1)) - 0.945(\beta_m + 1)e^{-0.7(\beta_m + 1)^2}$$

The error due to the spike noise is

$$P[\text{error neg. spike} \mid +] \tag{31}$$

$$= \begin{cases} \frac{\beta_m}{\pi} \exp\left(-\frac{\widehat{c^2(t)}}{2\sigma_n^2}\right) \cdot \sin \frac{\pi F}{2T}, & 0 \le \frac{F}{T} \le 1 \\ & (0.734 \le \beta_m \le 4.24) \\ \frac{\beta_m}{\pi} \exp\left(-\frac{\widehat{c^2(t)}}{2\sigma_n^2}\right), & \frac{F}{T} \ge 1\ (\beta_m \le 0.734) \end{cases}$$

where

$$\frac{F}{T} = \begin{cases} 0.755\ [\ln \frac{3\sqrt{2}}{\beta_m}]^{1/2}, & \beta_m \le 4.24 \\ 0, & \beta_m > 4.24 \end{cases}$$

The mark and the space are assumed to be equally probable. The total error probability is the sum of Eqs. (30) and (31)

The basic difference between the split-phase baseband signal and the NRZ baseband signal is in ω, and thus β_m. The modulating signal frequency of the split-phase signal is twice that of the NRZ signal. For a given β and small $\omega\Delta t$, β_m of the split-phase signal is slightly less than β_m of the NRZ signal. Hence for a given β, the split-phase signal does not perform better than the NRZ signal if $\beta_m < 0.734$. On the other hand, for a given β_m, Eqs. (30) and (31) are also valid for the NRZ signal, and the two signals perform the same. The spectral characteristic of the split-phase signal, however, makes it very attractive from the synchronization viewpoint to be discussed later.

Predetection Diversity Combining. - Diversity combining is an effective way to improve the system performance in the multipath fading environments. Combining can be performed either before or after detection and the signals may be selected or added either linearly or in variable proportions. Apart from the obvious advantages of operational simplicity, the predetection diversity combining exhibits lower distortion under multipath conditions, because of the linear combining process, and the higher improvement in the demodulated signal-to-noise ratio as compared with the post-detection combining. Test results [1] have shown that in some areas, post-detection combining is more effective than pre-detection combining and vice versa. In this paper, the error rates for the predetection diversity combining with noncoherent reception and with the discriminator detection of PCM/FM are considered.

First we examine the noncoherent reception of split-phase/FSK. Here, we consider a simple case of two diversity branches. The received data is the sum of $a_1 \sin\omega_i t + b_1 \sin\omega_i(t - \Delta t_1) + n_1(t)$ and $a_2 \sin\omega_i t + b_2 \sin\omega_i (t - \Delta t_2) + n_2(t)$. The combined signal is,

$$(a_1 + a_2 + b_1 \cos\omega_i \Delta t + b_2 \cos\omega_i \Delta t_2)\sin\omega_i t - (b_1 \sin\omega_i \Delta t_1 + b_2 \sin\omega_i \Delta t_2)$$

$\cos\omega_i t$, which has an amplitude R_i, $i = 1,2$,

$$R_i^2 = (a_1 + a_2)^2 + 2(a_1 + a_2)\,(b_1 \cos\omega_i \Delta t_1 + b_2 \cos\omega_i \Delta t_2) + b_1^2 + b_2^2 + 2b_1 b_2 \cos\omega_i(\Delta t_1 - \Delta t_2) \tag{32}$$

The noise terms $n_1(t)$ and $n_2(t)$ have the same noise powers. The result of the noise combining is

$$n(t) = n_1(t) + n_2(t) \tag{33}$$

which has twice the original noise power. Then, according to Eq. (20), the error probability is given by,

$$Pe = \frac{1}{4}\left[e^{-\frac{R_1^2}{4\sigma_n^2}} + e^{-\frac{R_2^2}{4\sigma_n^2}}\right] \tag{34}$$

where R_1 and R_2 are given by Eq. (32). As expected, the smallest error probability occurs when $a_1 = a_2$, $b_1 = b_2$, and $\Delta t_1 = \Delta t_2$. In this case, the error probability is given by

$$Pe_{min} = \frac{1}{4} e^{-\frac{a^2 + b^2}{\sigma_n^2}}\left[e^{-\frac{2ab\cos\omega_1\Delta t}{\sigma_n^2}} + e^{-\frac{2ab\cos\omega_2\Delta t}{\sigma_n^2}}\right] \tag{35}$$

which is a 3dB improvement in the signal-to-noise ratio as compared with Eq. (20).

Next we shall examine the discriminator detection of PCM/FM. Consider two multipath fading signals,

$$c_1(t) \cos\left[\omega_c t + \theta_{m1}(t) + \frac{b}{a-b}\phi_1(t) + 2\pi\int \dot{D}_1(t)dt\right]$$

and $$c_2(t) \cos\left[\omega_c t + \theta_{m2}(t) + \frac{b}{a-b}\phi_2(t) + 2\pi\int \dot{D}_2(t)dt\right]$$

where $\phi_1(t)$ and $\dot{D}_1(t)$ have the time delay Δt_1 and $\phi_2(t)$ and $\dot{D}_2(t)$ have the time delay Δt_2. To simplif the analysis, assume $\Delta t_1 = \Delta t_2$ so that $\phi_1(t) = \phi_2(t) = \phi(t)$ and $\dot{D}_1(t) = \dot{D}_2(t) = \dot{D}(t)$. The combined signal is

$$c(t) \cos\left[\omega_c t + \theta_{m1}(t) + \frac{b}{a-b}\phi(t) + 2\pi\int \dot{D}(t)dt + \tan^{-1}\frac{c_2 \sin(\theta_{m2} - \theta_{m1})}{c_1 + c_2\cos(\theta_{m2} - \theta_{m1})}\right] \tag{36}$$

where $$c(t) = \sqrt{c_1^2(t) + c_2^2(t) + 2c_1(t)c_2(t)\cos(\theta_{m2} - \theta_{m1})} \tag{37}$$

The noise jitters due to the multipath fading, θ_{m1} and θ_{m2}, have the same statistical characteristics and they both are small. Hence

$$\tan^{-1} \frac{c_2 \sin(\theta_{m2} - \theta_{m1})}{c_1 + c_2 \cos(\theta_{m2} - \theta_{m1})} \simeq \frac{c_2(\theta_{m2} - \theta_{m1})}{c_1 + c_2 \cos(\theta_{m2} - \theta_{m1})}$$

which has a nearly zero mean value and a variance given approximately by,

$$2\left(\frac{c_2}{(c_1 + c_2)}\right) \sigma^2_{\theta m1} < \frac{1}{2} \sigma^2_{\theta m1}$$

The received signal is then, from Eqs. (36) and (33),

$$c(t)\cos[\omega_C t + \theta_m(t) + \frac{b}{a-b} \phi(t) + 2\pi \int \dot{D}(t)dt] + n(t) \tag{38}$$

where

$$\theta_m(t) = \theta_{m1}(t) + \frac{c_2(\theta_{m2} - \theta_{m1})}{c_1 + c_2 \cos(\theta_{m2} - \theta_{m1})}$$

is still a random phase term. Thus we have exactly the same form of the signal-plus-noise expression as Eq. (24) except that $c(t)$ and $n(t)$ are now different. Let σ_n^2 be the noise variance of $n_1(t)$ or $n_2(t)$. The received singal-to-noise ratio is

$$\frac{\widehat{c^2(t)}}{4\sigma_n^2} \simeq \frac{1}{4\sigma_n^2} [\, \widehat{(c_1(t) + c_2(t))^2} - \widehat{c_1(t)c_2(t)}\, \sigma^2_{\theta m1} \,]$$

$$< \frac{\widehat{(c_1(t) + c_2(t))^2}}{4\sigma_n^2} \tag{39}$$

The largest improvement in the received signal-to-noise ratio is obtained when $c_1(t) = c_2(t)$. The maximum (but unattainable) improvement is a factor of 2(3dB) by using the predetection IF diversity combining with two diversity branches. The error probability can be written by inspecting Eqs. (30) and (31) as

$$Pe = \frac{1}{2} \operatorname{erfc} \frac{0.813\beta_m\sqrt{\beta_m + 1}}{\sqrt{f_o(\beta_m)}} \sqrt{\frac{\widehat{c^2(t)}}{4\sigma_n^2}} \tag{40}$$

$$+ \begin{cases} \dfrac{\beta_m}{\pi} \exp\left(-\dfrac{\widehat{c^2(t)}}{4\sigma_n^2}\right) \sin \dfrac{\pi F}{2T}, & 0 \le \dfrac{F}{T} \le 1 \\ & (0.734 \le \beta_m \le 4.24) \\ \dfrac{\beta_m}{\pi} \exp\left(-\dfrac{\widehat{c^2(t)}}{4\sigma_n^2}\right), & \dfrac{F}{T} \ge 1 \ (\beta_m \le 0.734) \end{cases}$$

where β_m and $\widehat{c^2(t)}/4\sigma_n^2$ are given by Eqs. (27) and (39) respectively.

In both Eqs. (34) and (40), we have noted that the maximum but practically unattainable improvement by using two predetection diversity combining branches is 3dB in the signal-to-noise ratio. For a very small Δt, such improvement can be closely obtained. The two simple cases discussed in this section fully illustrate the effectiveness of using the predetection diversity combining against the multipath fading that arises in the telemetry data transmission.

<u>Synchronization Study</u>. - Loss of synchronization occurs frequently in the test record [1] of the 4. Kb/sec split-phase PCM/FM. From the synchronization viewpoint we can show that the split-phase baseband format is

better than the NRZ format. The power spectra of PCM/FM with NRZ and the split-phase baseband signals have been examined by the author [3]. Unlike the NRZ format, the split-phase PCM/FM spectrum has a discrete component centered at the carrier frequency. The existence of this carrier makes it easier for frequency acquisition and tracking. Futhermore, this carrier may be removed from the modulated signal by a narrow-band filter and used for frequency synchronization. The forced transition at every bit period of the split-phase baseband format results in more average number of zero crossings as compared with the NRZ format. Thus a lower error rate in bit-sync is expected with the split-phase format. Practically various techniques are available to operate so well that we can assume that the data degradation caused by the synchronization process is negligible. A possible exception is the "deep fade" case at which the received signal strength is so small that a second-order phase-locked loop may "track" the noise to frequencies outside the pull-in range. The deep fade may occur when the change in the time delay is the largest and the strengths of the direct and the indirect waves are comparable.

When the range is the smallest, the change in the time delay is the largest since

$$\frac{d\Delta t}{dt} = \frac{\text{constant}}{R^2} \frac{dR}{dt} ,$$

which is inversely proportional to the square of the range. Thus, in consistent with the test record [1], the communication link drops in and out of synchronization more often at a closer range than at a longer range.

Suggested Methods of Data Improvement. - We have shown through a simple model that the multipath interference due to the earth scattered (indirect) signals can have serious effects on the telemetry data transmission. So the multipath effects must be taken into consideration in mission planning. Our model suggests the following three posibilities [1] to improve the data: (1) Fly the aircraft at sufficiently low altitudes that the time delay due to multipath is only a small portion of the bit period if the link is a pure PCM. For a given information bandwidth and time delay, the required bit period may be fairly large (which implies a low data rate telemetry). We may run into the wideband case, i.e. the information bandwidth and the bit duration product is greater than unity. If we can reduce the time delay, then the system can operate at a higher data rate. For the continuous FM system, the time delay must be a small fraction of the period of the highest subcarrier frequency. (2) Attempt to keep the first antenna null in the direction of the indirect wave. (3) Use simultaneous right and left hand circular polarization with predetection and/or post-detection real time combining. There are other possibilities that may not be as effective as mentioned above. For example, we can use the narrow-band filter technique to suppress the multipath effect in a narrow-band channel. But the indirect wave is centered at the same carrier frequency as the direct wave. The indirect wave cannot be removed by the filter without decreasing the direct wave signal power.

Concluding Remark. - Specular component is much more important than the diffused components in a typical multipath problem. The simple model employed in this paper takes into account the specular component only. The error performance evaluated in this paper should serve as an important lower bound of the actual performance in the multipath environment. This model is particularly feasible for the telemetry systems considered in this paper. And the analysis based on this model fully verifies the test results.

References

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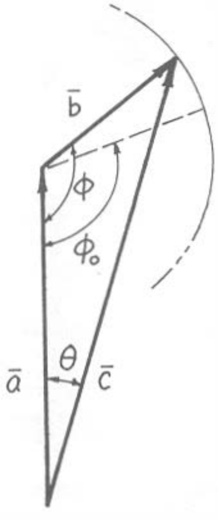

Fig. 1 - Vector Diagram Representing the Effect of a Single Multipath Component

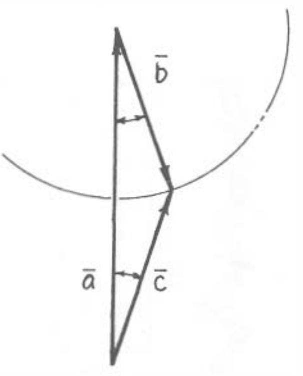

Fig. 2 - Vector Diagram Representing Maximum Frequency Modulation Resulting from a Single Multipath Component

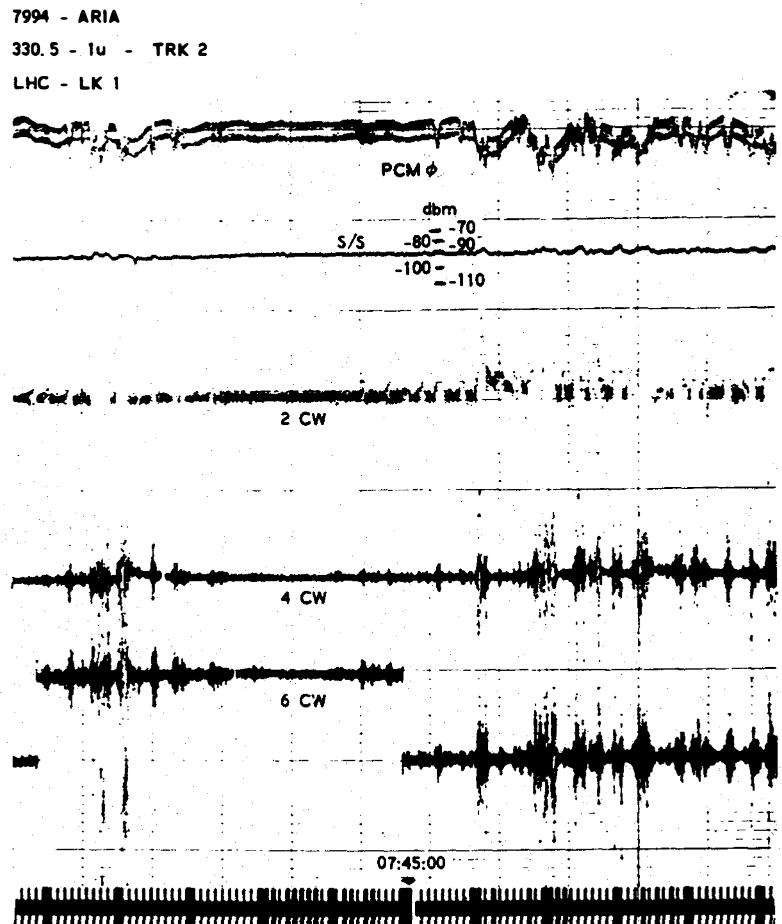

Fig. 3(a) - Oscillogram of Post-Detection Waveforms from ARIA on Mission 7994 for Right-hand Circular Polarization Link 1

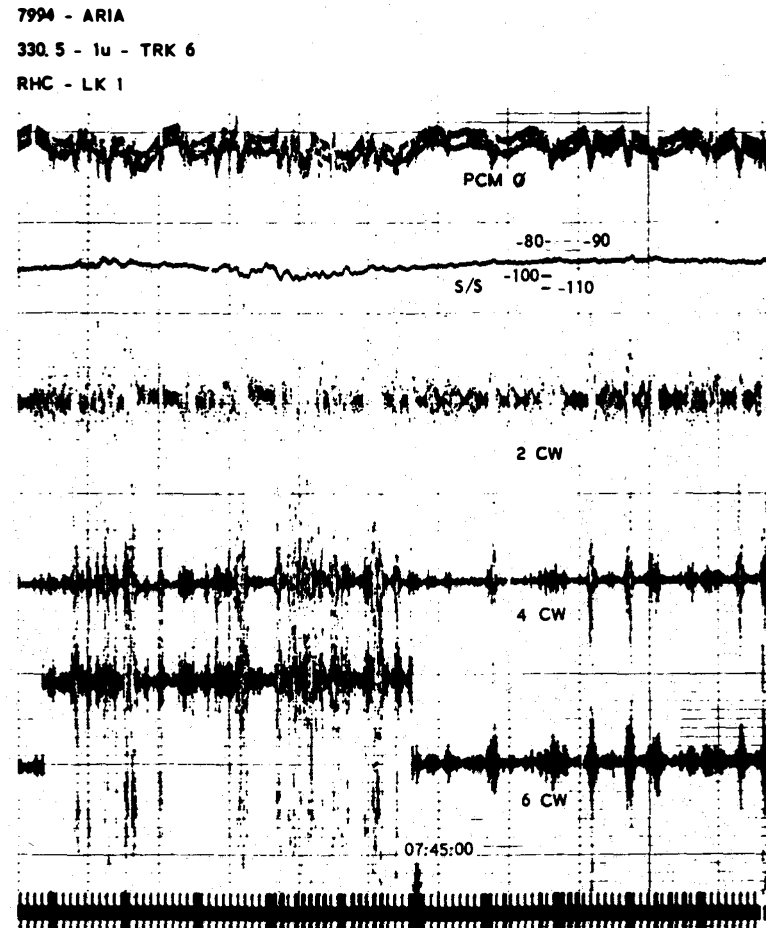

Fig. 3(b) - Oscillogram of Post-Detection Waveforms from ARIA on Mission 7994 for Left-hand Circular Polarization Link 1