FM DISTORTION CAUSED BY HEAD-TO-TAPE SPACING

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Summary Although it is well-known that frequency-modulated waves are distorted by systems with non-uniform frequency response, this distortion is difficult to calculate for most systems. The nature of the head-totape spacing transfer function allows the development of a closed form solution for FM distortion. Three methods of processing the voltage off the playback head are considered. Integrating the voltage results in amplitude-frequency distortion, but no harmonic distortion. Taking the voltage either directly off the head or differentiating it give both amplitude-frequency distortion and harmonic distortion. Experiments have verified the theoretical results.

Introduction Engineers often use frequency-modulation in magnetic tape recording systems because of its insensitivity to amplitude fluctuations. However, the playback process has a non-uniform frequency response which can distort the FM signal. Many linear systems distort FM signals, but in general there is no exact method to calculate such distortiona.[1] Fortunately, the Wallace[2] model for playback of signals recorded on magnetic tape allows the development of a closed form solution for FM distortion. The purpose of this paper is to present theoretical and experimental results for the FM distortion caused by head-to-tape spacing. Three methods of processing the voltage off the reproduce head are analyzed: 1) taking the voltage directly off the head, 2) integrating the voltage and 3) differentiating the voltage. Each method has a distinct pattern of FM distortion.

Analysis The analysis consists of determining system transfer functions which represent the effect of head-to-tape spacing in the playback process and finding the effects of these transfer functions on an FM signal.

Wallace's model for the head reproduce process gives the transfer function relating the reproduce or playback head output voltage to the tape magnetization as

$$Z(\omega) = j\omega\frac{(1 - e^{-\alpha\omega})}{\omega} e^{-\delta\omega} \qquad (1)$$

The $j\omega$ term results from the head voltage being proportional to the rate of change of flux. $(1-e^{-\alpha\omega})/\omega$ is the tape thickness term, and $e^{-\delta\omega}$ is the head-to-tape spacing term. The effects of eddy currents are neglected. If we assume a very thin tape, then (1) may be simplified by approximating $e^{-\alpha\omega}$ by $1-\alpha\omega$. $Z(\omega)$ becomes $j\alpha\omega e^{-\delta\omega}$ and after normalizing, we have

$$Z_1(\omega) = j\omega e^{-\delta\omega} \qquad (2)$$

If a thick tape is used, circuit playback equalization could be used to obtain a transfer function of the form of z_1. Integrating or differentiating the output voltage from the playback head would give transfer functions

$$Z_0(\omega) = e^{-\delta\omega} \qquad (3)$$

and

$$Z_2(\omega) = -\omega^2 e^{-\delta\omega}, \qquad (4)$$

respectively. The subscript on $Z(\omega)$ represents the number of derivative operators in tandem with the head-to-tape spacing term $e^{-\delta\omega}$.

To derive the effects of transfer functions $Z_{o-2}(\omega)$ on an FM signal., first let the tape magnetization be an FM signal of the form

$$i(t) = \text{Re}\left[S(t)\right] = \text{Re}\left[Ie^{j(\omega_0 t + m\sin\rho t)}\right] \qquad (5)$$

S(t) may be expanded into its spectral components as

$$S(t) = I\sum_{-\infty}^{\infty} C_n e^{j(\omega_0 + n\rho)t} \qquad (6)$$

where the C_n's are appropriate Bessel functions. When i(t) is passed through the playback head with transfer function $Z_o(\omega)$, the output voltage is

$$(e_{out})_0 = \text{Re}\left[I\sum_{-\infty}^{\infty} C_n Z_0(\omega_0 + n\rho)e^{j(\omega_0 + n\rho)t}\right]$$

$$= \text{Re}\left[I\sum_{-\infty}^{\infty} C_n e^{-\delta(\omega_0 + n\rho)} e^{j(\omega_0 + n\rho)t}\right] \qquad (7)$$

(7) can now be written as

$$(e_{out})_o = \mathrm{Re}[S(t + j\delta)] , \tag{8}$$

which is a result similar to that derived by Gerlach.[3] Transfer functions $Z_1(\omega)$ and $Z_2(\omega)$ can be substituted into (7), and the resulting output voltages are

$$(e_{out})_1 = \mathrm{Re}\left[\frac{d}{dt} S(t + j\delta)\right] \tag{9}$$

and

$$(e_{out})_2 = \mathrm{Re}\left[\frac{d^2}{dt^2} S(t + j\delta)\right] \tag{10}$$

respectively. Thus, we may work in the time domain using expressions (8)-(10) rather than the cumbersome frequency domain expansion of (7).

To determine the output voltages, we carry out the operations indicated for (8)-(10). Starting with (8), we have

$$\begin{aligned}(e_{out})_o &= \mathrm{Re}\left\{Ie^{j[\omega_o(t + j\delta) + m\sin p(t + j\delta)]}\right\} \\ &= Ie^{-(\omega_o\delta + m\sinh p\delta\cos pt)} \cdot \cos(\omega_o t \\ &\quad + m\cosh p\delta\sin pt),\end{aligned} \tag{11}$$

which contains both amplitude and frequency modulation. After passing this signal through an ideal limiter which limits the signal level to the troughs of the amplitude modulation, we have a signal with frequency modulation only,

$$Ie^{-(\omega_o\delta + m\sinh p\delta)} \cos(\omega_o t + m\cosh p\delta\sin pt) \tag{12}$$

Passing this signal through a frequency demodulator gives a demodulated output angle of

$$A_o(t) = m\cosh p\delta\sin pt \tag{13}$$

$\frac{dA_o}{dt}(t)$ is the corresponding demodulated FM signal.

When similar operations are performed on (9) and (10), the resulting expressions rapidly become very complicated. In general the output voltage consists of two FMed carriers in phase quadrature which are both amplitude modulated. By using trigonometric identities to combine these terms and assuming ideal amplitude limiting and frequency demodulating, we obtain for the respective demodulated output angles,

$$A_1(t) = m\cosh p\delta\sin pt - \tan^{-1}\frac{pm\sinh p\delta\sin\delta t}{\omega_o + pm\cosh p\delta\cos pt} \tag{14}$$

and

$$A_2(t) = m\cosh p\delta\sin pt + \tan^{-1}\frac{B}{C}$$

where

$$B = \rho m(\rho\cosh\rho\delta - 2\omega_o\sinh\rho\delta)\sin\rho t$$
$$- 2\rho^2 m^2\sinh\rho\delta\cosh\rho\delta\sin\rho t\cos\rho t.$$

and

$$C = \omega_o^2 + \rho m(2\omega_o\cosh\rho\delta - \rho\sinh\rho\delta)\cos\rho t$$
$$+ \rho^2 m^2[\cosh^2\rho\delta\cos^2\rho t - \sinh^2\rho\delta\sin^2\rho t] \quad (15)$$

Results (13)-(15) are the closed form expressions for the demodulated output angles obtained from playback of FM waves. As such, they contain the distortion terms caused by head-to-tape spacing.

Discussion of Results Examining the expressions for demodulated angle, we note that each has a distinct pattern. $A_o(t)$, the case corresponding to integrating the head output voltage, is in the simplest form. It shows that for a given head-to-tape spacing factor, δ, and modulating frequency, ρ, the amplitude of the demodulated signal is multiplied by the factor cosh(ρδ). This corresponds to amplitude-frequency distortion of the demodulated signal. However, there is no harmonic distortion introduced in this case. Fig. 1 shows a plot of coshρδ , which is the factor by which the signal amplitude is emphasized, as a function of ρδ in db. Experimentally measured points are also shown on Fig. 1. These show good agreement with the theory.

In view of the large value of coshρδ at high values of ρδ, one might well ask the following question. “Other factors being equal, will the amplitude of a given signal at frequency ρ be distorted by head-to-tape spacing more by direct recording or by FM recording?” In direct recording the signal would be attenuated by $e^{-\rho\delta}$ in FM recording it would be emphasized by the factor coshρδ. If we compare the reciprocal, $e^{\rho\delta}$ of the direct recording attenuation with coshρδ , we note that $e^{\rho\delta}$ is always greater than coshρδ, thus indicating that FM recording is always better with respect to this criterion. As a practical example suppose it is desired to record a 1 MHz signal in a system having a head-to-tape spacing loss of 6 db at that frequency. In direct recording, the signal voltage would be reduced by 0.5, in FM recording, the signal would be emphasized by 1.25.

Expressions $A_1(t)$ and $A_2(t)$, corresponding respectively to taking the voltage directly off the head and differentiating the voltage, contain terms which are the inverse tangents of functions which are periodic with the same period as sinρt. These inverse tangent functions can be expanded in Fourier series. When the expansion is done, it is found that there are significant amounts of higher harmonics present. Thus, there is harmonic distortion in the output as compared with $A_o(t)$ which has only amplitude-frequency distortion.

As an example, Fig. 2 shows the Fourier components for the demodulated FM signals $(dA_1 (t))/dt$ and $(dA_2(t))/dt$, for the case where m, the modulation index is 0.6, and the ratio of modulation frequency to carrier frequency, ρ/ω, is 0.5. These conditions correspond to a peak frequency deviation of 30% of the carrier frequency. The abscissa is the carrier attenuation in db caused by head-to-tape spacing. We note that the fundamental signal component has less variation for dA_1/dt while the second harmonic term is higher. Fig. 3 shows another case, modulation index m = 0.188 and modulating frequency over carrier frequency is 0.8 corresponding to a peak frequency deviation of 15% of the carrier frequency. Only the fundamental component is shown because the higher harmonics would be cut out by the low pass filter which is usually present in an FM demodulator. The variation of fundamental amplitude with carrier attenuation is similar in shape to the case shown in Fig. 2. However, the magnitude of the variation is greater for the higher modulating frequency.

Experiments have been carried out to measure the second harmonic distortion present in the demodulated output, dA_1/dt, which corresponds to taking the voltage directly off the head. The experimental system ran at 60 inches per second and used an FMed carrier of 80 kHz with a peak frequency deviation of 24 kHz or 30%. Modulating frequencies of 19 kHz and 33 kHz made identification of the second harmonic easy on a spectrum analyzer. Head-to-tape spacing was adjusted by shims. The experimental results shown in the table of Fig. 4 show very good agreement with theory. The low frequencies used in the experimental system allowed careful control of the head-to-tape spacing in the experiments, and are not typical of the frequencies now used in instrumentation and video recording. However, the results presented here are given in terms of the carrier attenuation in db caused by head-to-tape spacing and can be used for any FM system.

Sensitivity of a system to changes in head-to-tape spacing can be an important design consideration. Examining Figs. 1-3, we note that the three different methods of taking the voltage off the head all have different sensitivities with respect to changes in head-to-tape spacing which are functions of the carrier attenuation caused by head-to-tape spacing. Considering only the effect upon the fundamental amplitude, it appears that for low values of carrier attenuation, integrating the voltage off the head gives lowest sensitivity to variations in head-to-tape spacing. For medium attenuation, taking the voltage directly off the head seems best, and for high attenuation, taking the derivative off the head appears best.

Conclusion We have shown theoretically and experimentally the nature of FM distortion caused by head-to-tape spacing in tape recording systems. Closed-form expressions have been derived for the demodulated output angle resulting from passing an FMed wave through transfer functions

$$Z_0(\omega) = e^{-\delta\omega},$$

$$Z_1(\omega) = j\omega\, e^{-\delta\omega},$$

and $Z_2(\omega) = -\omega^2\, e^{\delta\omega}$ which represent the effect of head-to-tape spacing and respectively integrating, taking directly, or differentiating the voltage from the playback head.

This analysis applies to systems for which effects other than head-to-tape spacing such as tape thickness and skin effect are equalized. Such equalization is usually practiced in actual system design. In fact, the effect of a fixed value of head-to-tape spacing is itself often equalized. When this is done, the analysis presented here. is directly applicable to changes in head-to-tape spacing from the nominal value. A limitation is that full-equalization is restricted to a finite band width.

The closed form expressions are significant because they allow calculation of FM distortion for given system parameters. The results are applicable to design of FM tape recording systems, and thaw that harmonic distortion is absent in systems which integrate the voltage from the head. On the other hand, if sensitivity to variation in head-to-tape spacing is important, taking the voltage directly off the head or differentiating it may give better-system performance.

References

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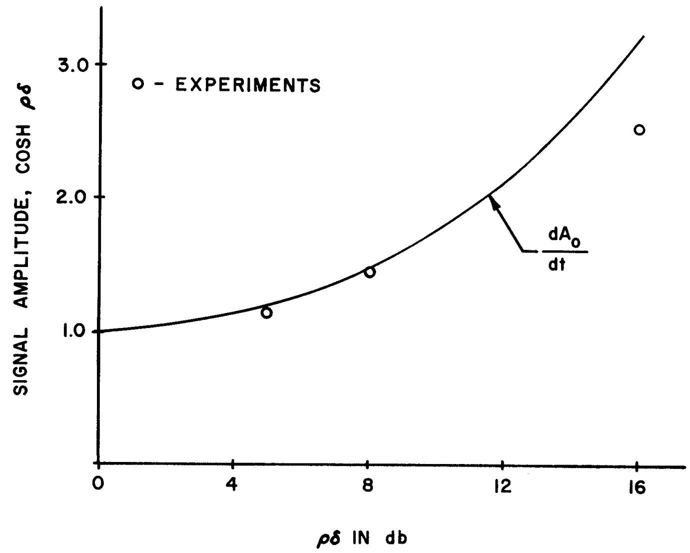


Fig. 1. Plot of amplitude-frequency distortion factor as a function of spacing loss, ρδ , for demodulated FM signal dA_0/dt (head voltage integrated).

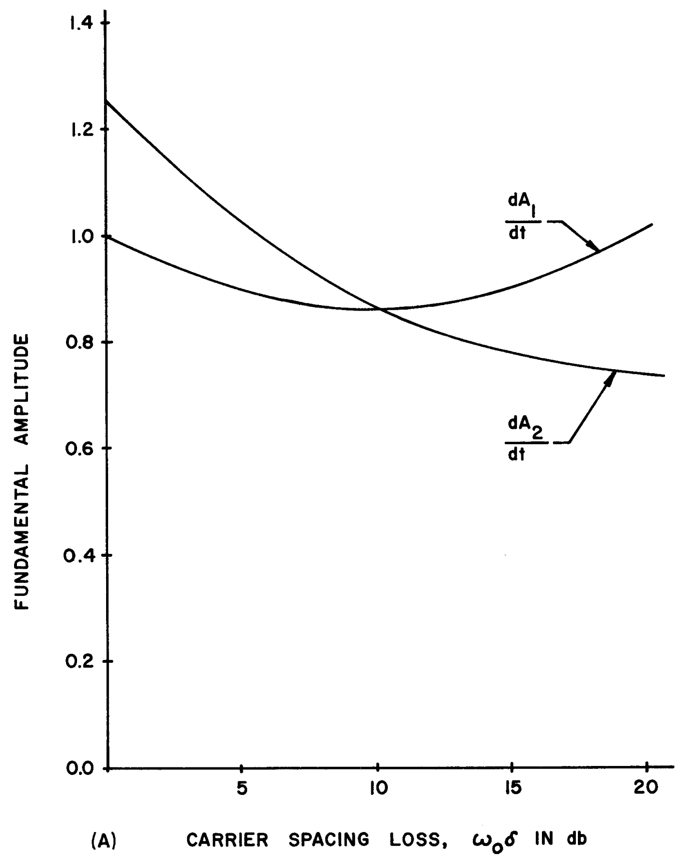
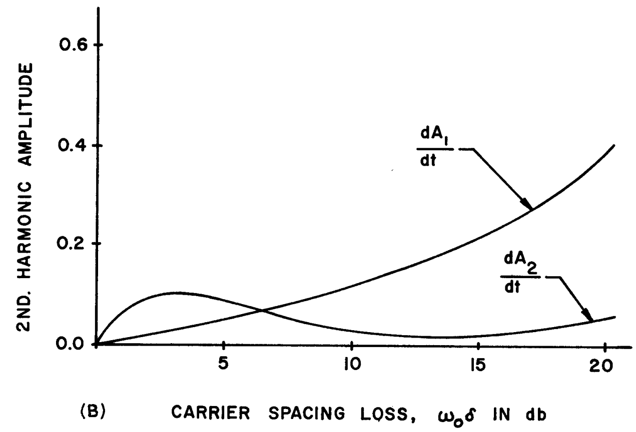


Fig. 2. Plot of Fourier components for demodulated FM signals, dA_1/dt (direct) and dA_2/dt (differentiated) as a function of carrier spacing loss. Modulating frequency/carrier frequency is 0.5. Peak frequency deviation is 30% of carrier frequency. (A) Fundamental (B) Second Harmonic.



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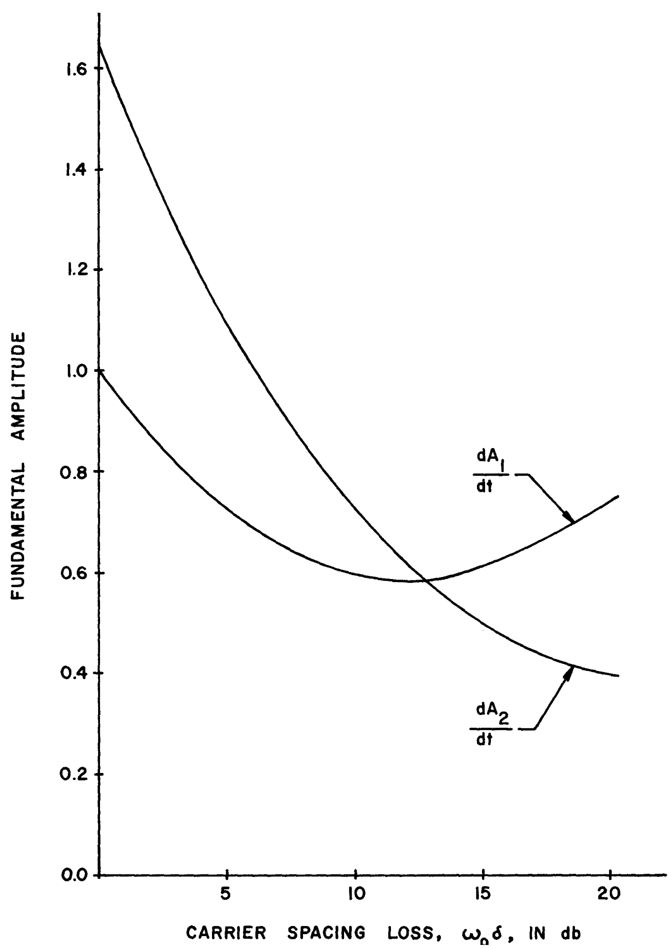

Fig. 3. Plot of Fundamental component for demodulated FM signals, dA_1/dt (direct) and dA_2/dt (differentiated) as a function of carrier spacing loss. Modulating frequency/carrier frequency is 0.8. Peak frequency deviation is 15% of carrier frequency.

SIGNAL FREQUENCY	CARRIER SPACING LOSS	SECOND HARMONIC	
		EXPERIMENTAL	THEORETICAL
19KHZ	10 db	2.5 %	2.2%
	12 db	3.0 %	2.7 %
33KHZ	10 db	9-10%	7.8 %
	12 db	10-11%	9.9%

Fig. 4. Measurements of second harmonic content in demodulated FM signal, dA_1/dt (direct).