BIT ERROR PERFORMANCE ANALYSIS OF BPSK IN THE PRESENCE OF MULTIPATH FADING

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ABSTRACT

The presence of multipath fading has been shown to degrade the performance of a wireless channel. This paper quantifies the effects of multipath interference on signal performance based on the estimated parameters of the multipath signal. Theoretical results are compared with actual results obtained through the Advanced Range Telemetry (ARTM) program.

KEY WORDS

Channel Performance, Bit Error Rate Analysis, Multipath Fading, ARTM.

INTRODUCTION

Multipath interference occurs when reflected replicas of a transmitted signal arrive within the main lobe of the receive antenna gain pattern. In general, the multipath reflections arrive asynchronously and with random relative phase to the line of sight, and therefore cause random periods of constructive and destructive interference. During periods of destructive interference the effective signal to noise ratio decreases, resulting in an increase in the probability of bit error. Likewise when the signals interfere constructively, the effective signal to noise ratio is improved and the resulting bit decisions are more accurate.

The characteristics of multipath fading are dependent upon the geometry of the channel and are beyond the scope of this paper. However for a given "multipath pattern," the effects of the multipath on signal performance can be determined. Based on this information an appropriate mitigation technique can be selected to increase the positive effects of multipath interference and minimize the negative results of interference.

In this paper the probability of bit error for a BPSK modulated signal is derived based on the characteristics of a 2-Ray multipath channel. The theoretical results are compared to channel sounding data collected from measurements provided by the ARTM program.

CHANNEL MODEL

An aeronautical telemetry channel is by nature a dynamic system. The characteristics of the channel change rapidly as the aircraft position changes relative to the receiving station. Multipath fading is dependent upon the geometrical relationship between the aircraft's antenna, the receiving station's antenna, and the physical terrain below. To simplify the system model, the communications channel can be approximated over a short time interval as a linear time-invariant system.

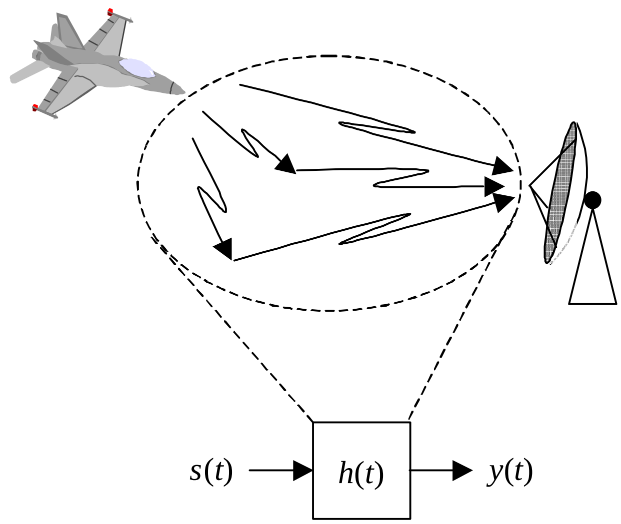

Figure 1: Linear time-invariant aeronautical telemetry channel.

We can model the aeronautical telemetry channel with an impulse response *h(t)* and a transfer function H(f) as shown in Figure 1. When a signal *s(t)* is input to this system, the channel output y(t) is given by the convolution of *s(t)* with *h(t)*. In the frequency domain, the channel output Y(f) = H(f)S(f).

Rice et al [1], showed that an aeronautical telemetry channel may be accurately modeled as one line-of-sight signal plus one or two strong specular reflections that arrive with some small delay relative to arrival of the line-of-sight signal. The relative delay was shown to generally be less than 500 nanoseconds. The relative amplitude of the multipath reflections was strongest for reflections arriving with a short delay of less than 100 ns. Such a channel can be modeled mathematically as

$$h(t) = d(t) + \Gamma e^{j(2pf_0 t + q)} d(t - t) \qquad (1)$$

where $0 \le \Gamma \le 1$ is the strength of the multipath signal relative to the line-of-sight signal, θ is the phase shift caused by the signal's reflection, τ is the delay of the specular reflection and $2pf_0 t$ is the phase shift caused by the delay (f_0 is the carrier frequency). For our analysis we will combine the phase shifts due to the reflection and the delay into a single parameter $g = 2pf_o t + q$.

PERFORMANCE UNDER MULTIPATH FADING

Multipath reflections are *often* undesirable because they can corrupt the transmitted signal and decrease the signal performance as measured by the probability of bit error. The characteristics of the multipath signal, such as the relative amplitude (Γ), phase (γ) and delay (τ), determine the overall effect that the multipath will have on the bit error rate. When a single multipath reflection is present at the receiving antenna, the received signal can be modeled including the effects of additive noise as

$$r(t) = s(t) + \Gamma\cos(g)s(t - t) + n(t) \tag{2}$$

Incorrect bit decisions can be caused by multipath interference and by noise in the channel and receiver. In general, a high signal to noise ratio will decrease the likelihood of a bit error caused by noise in the channel. However, the presence of multipath interference can degrade a high signal to noise ratio, resulting in decreased signal performance. The effects of multipath interference can be seen in a comparison of eye patterns.

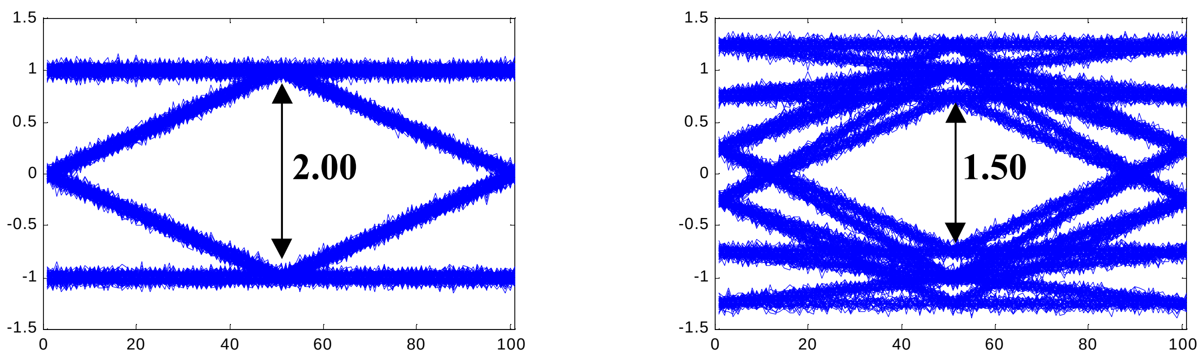

Figure 2: Normal Eye Pattern for BPSK signal in Gaussian noise (left) and Eye Pattern of multipath corrupted signal where $\Gamma = 0.25$ and $\tau = 2.5*T_b$ (right).

The eye diagram shows that the noise margin of the system has decreased by a factor proportional to the relative amplitude of the multipath signal [2]. Consequently the sensitivity of the receiver to timing errors has also increased. As the relative amplitude of the multipath signal increases, the "eye" will close even further, indicating an even higher probability of bit errors.

A standard BPSK demodulator makes its bit decisions based on the output of a matched filter sampled at the bit time. For a signal *s(t)* that is corrupted by a single multipath reflection, the output of a matched filter receiver with impulse response $h(t) = f^*(-t)$ sampled at the bit time is given by

$$x(T_b) = \int_0^{T_b} s(t)f(t)dt + \int_0^{T_b} \Gamma\cos(g)s(t - t)f(t)dt + \int_0^{T_b} n(t)f(t)dt \tag{3}$$

Depending on the characteristics of the multipath interference, the multipath will add either constructively or destructively to the line of sight signal. Constructive interference will increase the signal to noise ratio and improve the quality of the bit decisions, while destructive interference will decrease the signal to noise ratio and increase the likelihood of bit errors.

Case 1: τ < T_b

If the relative delay τ is constrained to be less than the bit time, T_b, then it is assured that the multipath interference will be constructive for at least a portion of the bit time as long as $-p/2 \leq g \leq p/2$. Assuming equally likely symbols, the previously transmitted bit and the current bit will be identical 50% of the time. When this occurs, the multipath interference will be constructive for the entire symbol time. In this case, the matched filter response due to the multipath interference would be

$$\int_0^{T_b} \Gamma\cos(g)s(t-t)f(t)\,dt = \pm\sqrt{E_b\left(\Gamma\cos(g)\right)^2} \tag{4}$$

When the current bit and the previous bit are opposite, the multipath component of the matched filter output is

$$\int_0^{T_b} \Gamma\cos(g)\,s(t-t)\,f(t)dt = \pm\sqrt{E_b\left(\Gamma\cos(g)\left(1-2t/T_b\right)\right)^2} \tag{5}$$

If the response of the matched filter due to the noise term $n(t)$ is represented by the zero mean Gaussian random variable $W \sim N(0, s^2)$, the sampled output of the matched filter is

$$x(T_b) = \left\{\begin{array}{ll} \pm\sqrt{E_b\left(1+\Gamma\cos(g)\right)^2} + W, & \textit{Identical Bits} \\ \pm\sqrt{E_b\left(1+\Gamma\cos(g)\left(1-2t/T_b\right)\right)^2} + W, & \textit{Opposite Bits} \end{array}\right\} \tag{6}$$

When the noise power is high enough to perturb the output, $x(T_b)$, beyond the decision boundary, a bit error occurs. For example, the probability of bit error when two binary "0's" are transmitted sequentially can be expressed as

$$\begin{aligned} P(E) &= \Pr\left\{-\sqrt{E_b\left(1+\Gamma\cos(g)\right)^2} + W > 0\right\} \\ &= \Pr\left\{W > -\sqrt{E_b\left(1+\Gamma\cos(g)\right)^2}\right\} \end{aligned} \tag{7}$$

The probability of bit error is obtained by integrating the upper tail of the probability density function of the noise distribution.

$$P(E\,|\,0,0) = \int_0^{\infty} \frac{1}{\sqrt{2ps^2}} \exp\left\{\frac{-(x-m)^2}{2s^2}\right\}dx \tag{8}$$

where $m = -\sqrt{E_b\left(1+\Gamma\cos(g)\right)^2}$ and $s^2 = N_o/2$.

This integral can also be expressed in terms of the Q function

$$P(E\,|\,0{,}0) = P(E\,|\,1{,}1) = Q\left(\sqrt{\frac{2E_b \cdot (1+\Gamma\cos(g))^2}{N_o}}\right) \tag{9}$$

The result of equation 9 also holds for the case when 2 binary “1’s” are transmitted sequentially. When the current bit and the previous bit happen to be opposite, the probability of error can be shown to be

$$P(E\,|\,0{,}1) = P(E\,|\,1{,}0) = Q\left(\sqrt{\frac{2E_b \cdot (1+\Gamma\cos(g)\cdot(1-2t/T_b))^2}{N_o}}\right) \tag{10}$$

Under the assumption that all bits are equally likely, application of the law of total probability yields the probability of error for BPSK when $\tau < T_b$:

$$P(E) = \frac{1}{2}Q\left(\sqrt{\frac{2E_b \cdot (1+\Gamma\cos(g))^2}{N_o}}\right) + \frac{1}{2}Q\left(\sqrt{\frac{2E_b \cdot (1+\Gamma\cos(g)\cdot(1-2t/T_b))^2}{N_o}}\right) \tag{11}$$

Case 2: τ > Tb

If the relative delay is greater than the bit time there is the additional possibility that multipath interference will be completely destructive during an entire symbol time. When this occurs, the receiver will experience the largest possible drop in the effective signal to noise ratio. This makes the bit decisions most sensitive to noise. It can be shown that these errors will account for a majority of the total number of bit errors. The probability of error in this situation is given by

$$P(E) = Q\left(\sqrt{\frac{2E_b \cdot (1-\Gamma\cos(g))^2}{N_o}}\right) \tag{12}$$

When the multipath interference is completely constructive, the probability of error can be shown to be

$$P(E) = Q\left(\sqrt{\frac{2E_b \cdot (1+\Gamma\cos(g))^2}{N_o}}\right) \tag{13}$$

This is identical to the case above where two identical symbols are received in sequence.

When a multipath ray interferes constructively more than it interferes destructively during a single bit time, the probability of bit error is

$$P(E) = Q\left(\sqrt{\frac{2E_b \cdot \left(1 + \Gamma\cos(g)/T_b \cdot \left(2((t))_{T_b} - T_b\right)\right)^2}{N_o}}\right) \tag{14}$$

Similarly, when the amount of constructive interference is less than the amount of destructive interference during a single bit time, the probability of bit error is

$$P(E) = Q\left(\sqrt{\frac{2E_b \cdot \left(1 - \Gamma\cos(g)/T_b \cdot \left(2((t))_{T_b} - T_b\right)\right)^2}{N_o}}\right) \tag{15}$$

If we assume that the conditions represented by equations 12, 13, 14, and 15 are equally likely to occur in a random binary signal, the total probability of error when $\tau > T_b$ is

$$\begin{aligned} P(E) = {} & \frac{1}{4}Q\left(\sqrt{\frac{2E_b \cdot (1 + \Gamma\cos(g))^2}{N_o}}\right) + \frac{1}{4}Q\left(\sqrt{\frac{2E_b \cdot \left(1 + \Gamma\cos(g)/T_b \cdot \left(2((t))_{T_b} - T_b\right)\right)^2}{N_o}}\right) \\ & + \frac{1}{4}Q\left(\sqrt{\frac{2E_b \cdot (1 - \Gamma\cos(g))^2}{N_o}}\right) + \frac{1}{4}Q\left(\sqrt{\frac{2E_b \cdot \left(1 - \Gamma\cos(g)/T_b \cdot \left(2((t))_{T_b} - T_b\right)\right)^2}{N_o}}\right) \end{aligned} \tag{16}$$

SIMULATION OF THEORETICAL RESULTS

Computer simulations were used to verify the derivation of the probability of bit error under multipath fading conditions. A NRZ BPSK signal was corrupted by simulated multipath interference with known relative amplitude, Γ, phase, γ, and delay, τ. This composite signal was subsequently demodulated using a standard matched filter receiver. The probability of error was calculated as the number of bit errors divided by the total number of transmitted bits. In all simulations a set of 100,000 random bits was transmitted and received.

In the first simulation, the characteristics of the simulated multipath interference, (Γ, τ, γ) were held constant while the signal to noise ratio was allowed to vary from 3 to 10 dB. The probability of bit error for this configuration was plotted along with that of an uncorrupted BPSK signal. As shown in Figure 3, when the signal is corrupted by a single multipath reflection with parameters ($\Gamma = 0.85$, $\tau / T_b = 0.80$, $\gamma = \pi/8$) the performance was decreased by approximately 3 dB. To compensate for the decreased performance of the telemetry channel due to this specific multipath interference, an additional 3 dB of link margin must be added. As the relative amplitude of the multipath signal increases, the signal performance will deteriorate even further.

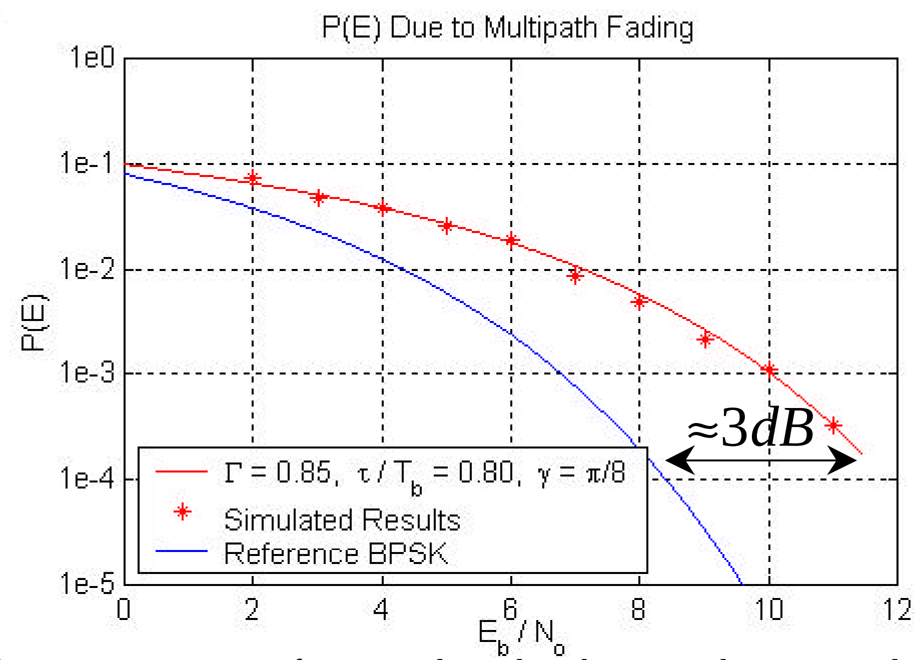

Figure 3: Performance comparison of BPSK and a multipath corrupted BPSK signal.

The second simulation measured the effect of the relative amplitude, Γ, and the relative phase, γ, on the signal performance as the relative delay of the multipath signal varied from 0 to 2 times the bit time. For this test, the signal to noise ratio was constant at a value of 5 dB. Figure 4 shows the results for two signals. The amplitude and phase of the two multipath signals are held constant at $\Gamma_1 = 0.75$, $\gamma_1 = \pi/8$ and $\Gamma_2 = 0.95$, $\gamma_2 = 0$. Because the amount of constructive interference was greater than the amount of destructive interference for $\tau / T_b < 0.5$, the overall signal performance improved dramatically due to the increased signal energy provided by the multipath reflection. However for $\tau / T_b > 0.5$ the performance rapidly decreased as the destructive interference lowered the effective SNR.

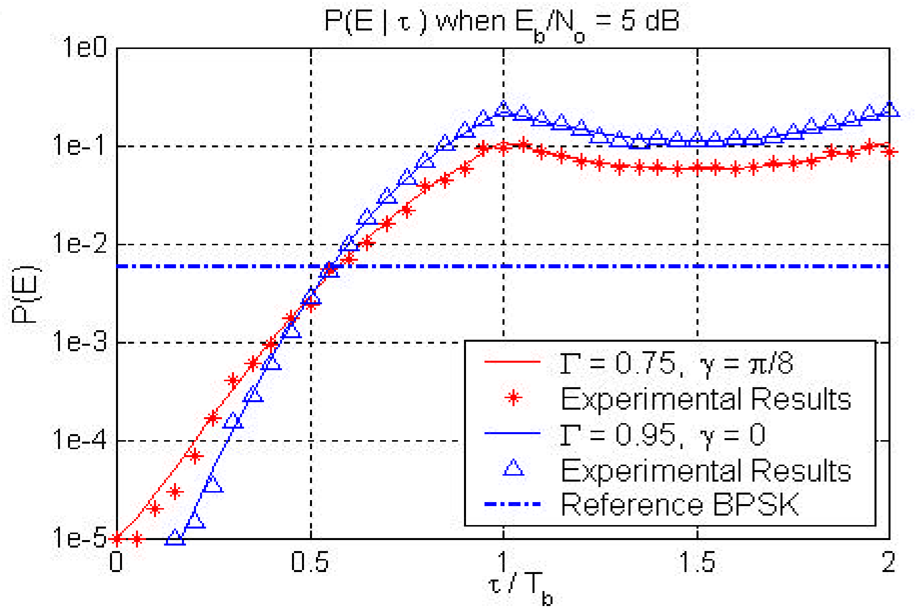

Figure 4: Probability of bit error as a function of the relative delay of the multipath signal.

The third simulation explored the effect of a variation in the relative phase shift on the overall signal performance. As seen in Figure 5, when the multipath signal arrives in phase with the line of sight signal, there is an increase in the total signal power for $\tau / T_b < 0.5$ which correspondingly increases the signal performance. However, if the multipath signal arrives out of phase with the line of sight signal, there will be an attenuation of the total signal power for $\tau / T_b < 0.5$ resulting in a dramatic drop in performance.

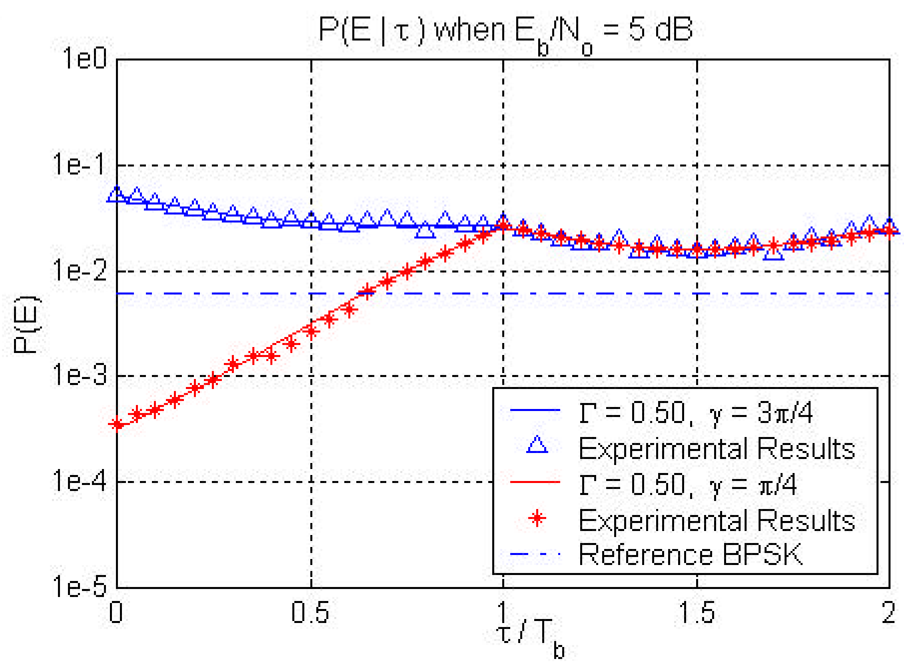

Figure 5: Probability of bit error as a function of the relative delay and phase of the multipath signal.

ARTM CHANNEL SOUNDING RESULTS

The ARTM test flights maintained an extremely high signal to noise ratio so that all bit errors would be independent of the additive noise and a direct result of multipath interference. Rice et al. [1] conducted channel sounding experiments at Edwards Air Force Base and estimated the characteristic parameters of the multipath fading that occurred. The actual probability of bit error was correlated with the multipath characteristics at specific times. The results indicate that while large amplitude reflections cause the most severe increases in bit error rates, even relatively small multipath reflections can cause significant increases in the bit error rate. This conclusion is supported by the dependence of the bit error rate on the relative phase and delay of the multipath reflection.

Two characteristic examples of the ARTM channel sounding data are shown below. These examples illustrate the effect of multipath fading on the spectrum of the received signal. Figure 6 displays the results of a "long-delay" multipath reflection with a relatively small amplitude. Figure 7 is typical of a "short-delay" multipath signal with significant relative amplitude.

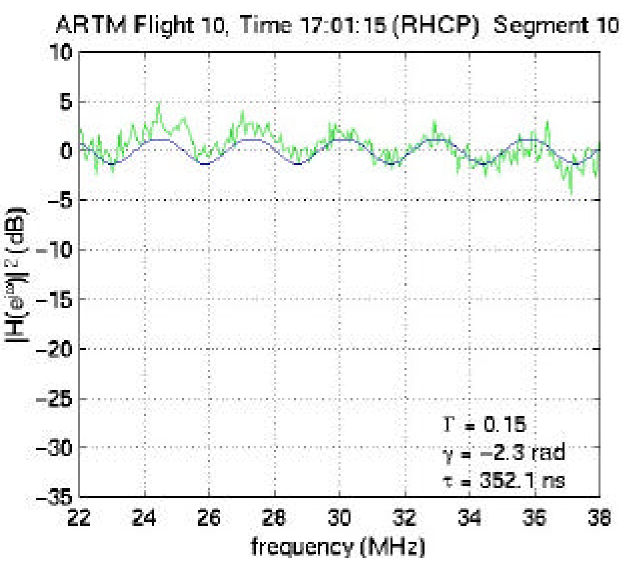

Figure 6: "Long-delay" multipath fading.

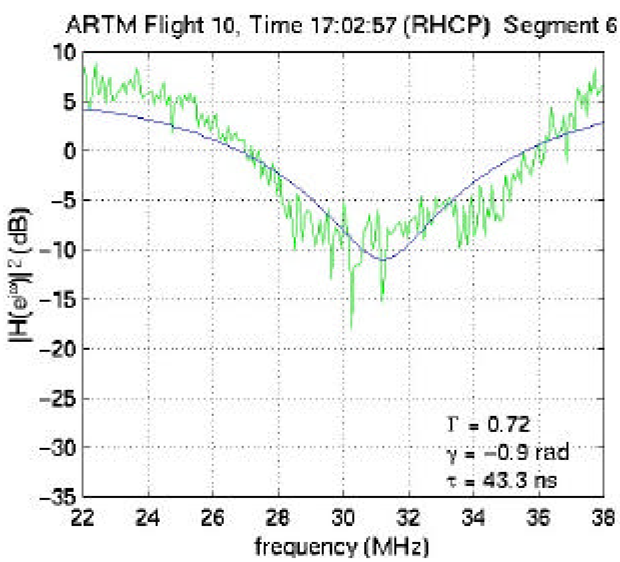

Figure 7: "Short-delay" multipath fading

Based on the estimated multipath characteristics of the channel sounding data in figures 6 and 7, equations (11) and (16) were used to estimate of the probability of bit error for these two channels. The signal to noise ratio, E_b/N_o, was estimated based on the range of the aircraft during the fade events. The calculation of the theoretical bit rate does not account for the effects of symbol timing offset and carrier phase recovery. For these reasons the calculated bit error rate will be an approximation to the actual bit error rate that one could expect in a real communications system. Table 1 shows the agreement between the estimated bit error rate and the observed bit error rate.

Table 1: Comparison of measured bit error rate and calculated bit error rate for multipath fading characterized by figures 6 and 7.

	Γ	γ	τ	Measured Bit Error Rate	Estimated Bit Error Rate
Figure 6	0.15	-2.3 rad	352.1 ns	5.50×10^{-6}	6.31×10^{-6}
Figure 7	0.72	-0.9 rad	43.3 ns	5.00×10^{-3}	5.70×10^{-3}

CONCLUSIONS

A closed form solution has been derived to estimate the probability of error for a BPSK signal corrupted by a single multipath reflection. The probability of bit error can be calculated as a function of the characteristics of the multipath fading: relative amplitude, phase and time delay. Computer simulations verified the theoretical results. The probability of bit error is highly sensitive to the relative phase and time delay of the multipath arrival for a given relative amplitude. Empirical channel sounding results obtained from the ARTM channel sounding project showed excellent agreement of a real world BPSK telemetry system with the derived bit error model.

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