OPTIMIZATION OF REFERENCE WAVEFORM FILTERS IN COHERENT DELAY LOCKED LOOPS

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ABSTRACT

In this paper, a new coherent correlation-loop architecture for tracking direct-sequence spread-spectrum signals is proposed. In the proposed correlation loop model, the mean-square tracking error is minimized by varying the cross-correlation function between the received signal and the locally generated signal. The locally generated signal is produced by passing a replica of the transmitted signal through a linear time-invariant filter, which is termed the VCC filter. The issue of bandwidth of a correlation loop is addressed and a bandwidth definition for comparative purposes is introduced. The filter characteristics to minimize the tracking errors are determined using numerical optimization algorithms. This work demonstrates that the amplitude response of the VCC filter is a function of the input signal-to-noise ratio (SNR). In particular, the optimum filter does not replicate a differentiator at finite signal-to-noise ratio as is sometimes assumed. The optimal filter characteristics and the knowledge of the input SNR can be combined to produce a device that has very low probability of loosing lock.

KEY WORDS

PN code tracking, DLL, time tracking loops

INTRODUCTION

Direct-sequence (DS) spread-spectrum communications systems are extremely useful for mitigating the effects of intentional interference, preventing undesired detection and demodulation, and in code division multiple access systems. To exploit the benefits of a DS spread-spectrum signal, the receiver must be able to synchronize the locally generated pseudo-noise (PN) sequence with the incoming PN sequence. Delay locked loops (DLL) are widely used in DS-systems for PN code tracking.

The general structure of correlation loops such as phase-locked loops (PLL) and DLLs is illustrated in Figure 1. The tracking performance of a correlation loop is a function of input SNR, loop filter,

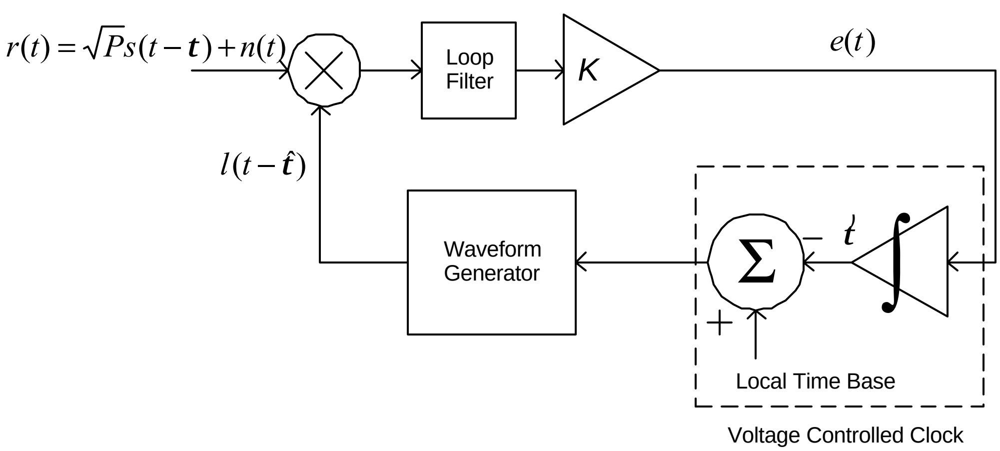

Figure 1: Generalized Correlation Loop

loop gain and the cross correlation function between the transmitted signal and $s(t)$ and the reference signal $l(t)$, $R_{XL}(\tau)$. The optimization of $R_{XL}(\tau)$ has been studied [1,2], but these results are not widely used. This is due in part to the difficulty of producing an arbitrary waveshape with the local reference generator. Following the approach of [3], we restrict our attention to waveform generators that are a replica of the transmitted waveform generator followed by a linear time-invariant filter, $H(f)$, as shown in Figure 2.

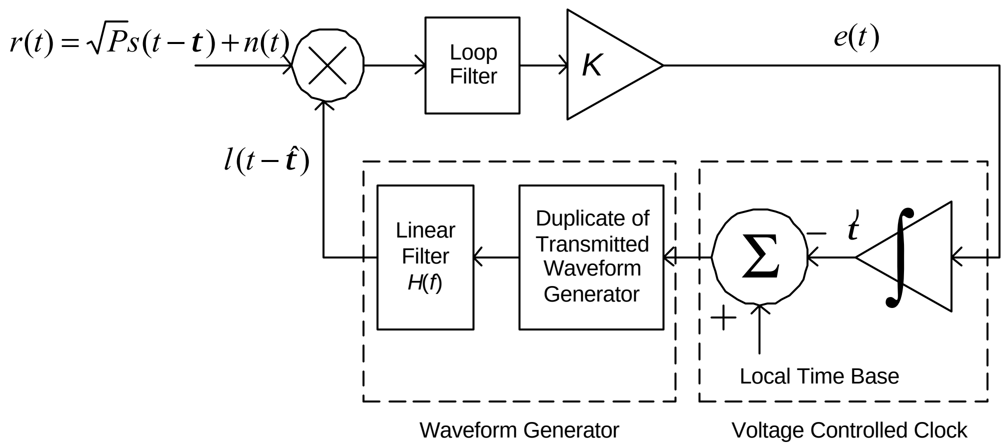

Figure 2: Modified Correlation Loop

The cross-correlation function $R_{XL}(\tau)$ can be controlled by altering $H(f)$ which is called the VCC filter. In a conventional early-late delay-locked loop (ELDLL) this filter approximates a differentiator. A differentiator is also used in a PLL, but it is usually combined with the voltage-controlled clock and called a voltage controlled oscillator (VCO).

The results of Spilker and Magill [4] show that the maximum-likelihood optimum-tracking discriminator for an arbitrary wideband signal is a multiplier which forms the product of the received signal plus noise and the derivative of the receiver generated replica of the transmitted signal. In contrast to this, Layland [2], and Kosbar [3] showed that the important aspect is not the derivative but the resultant ninety-degree phase shift. As will be shown later, this can also be shown using the fact that the characteristic function of a correlation loop should be an odd function. The results of [5] show the performance of a first-order correlation loop using a Hilbert transformer (90-degree phase shifting filter) as the VCC filter. This paper extends the work of [5] to determine the optimum form of the VCC filter in order to minimize the mean-square tracking error (MSTE) of a first-order correlation loop.

ANALYSIS

In this paper, we will focus our attention to first-order correlation loops, i.e. correlation loops with the loop filter replaced by a unity gain. In the modified correlation loop shown in Figure 2, the received signal $r(t) = \sqrt{P}s(t-\tau)+n(t)$ is a delayed version of the originally transmitted signal $s(t)$ that has been corrupted by additive white Gaussian noise (AWGN). Signal $s(t)$ is assumed to be unity power and $n(t)$ has a double-sided power spectral density of $N_o/2$ W/Hz. The correlation loop generates an estimate, $\hat{\tau}$ for the channel delay, τ, by cross correlating $r(t)$ with the locally generated signal $l(t)$. Following the approach of [3], we can derive the non-linear equivalent model of a first-order correlation loop as follows: From the modified correlation loop shown in Figure 2, the control function, $e(t)$, can be expressed as

$$e(t) \;=\; \sqrt{P}K\, s(t\text{-}\tau(t))\; l(t\text{-}\hat{\tau}(t)) \;+\; K\, n(t)\; l(t\text{-}\hat{\tau}(t)). \tag{1}$$

With the assumption that signals $s(t)$ and $l(t)$ are periodic with period T, and constant channel delay $s(t)$ and $l(t)$ can be expanded using the Fourier series (FS) expansion. The FS expansion of $s(t)$ and $l(t)$ are

$$s(t) \;=\; \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_o t} \;=\; \sum_{n=-\infty}^{\infty} x_n e^{j\phi_n} e^{jn\omega_o t} \tag{2}$$

$$l(t) \;=\; \sum_{n=-\infty}^{\infty} L_n e^{jn\omega_o t} \;=\; \sum_{n=-\infty}^{\infty} l_n e^{j\varphi_n} e^{jn\omega_o t} \tag{3}$$

where $\omega_o = 2\pi / T$. The FS coefficients can be found in the conventional manner. By ignoring self-noise and double frequency terms and using (2) and (3), (1) can be rewritten as

$$e(t) \;=\; \sqrt{P}K \sum_{n=-\infty}^{\infty} X_n L_n^* e^{-jn\omega_o(t-\hat{\tau}(t))} \;+\; Kn(t) \sum_{m=-\infty}^{\infty} L_m e^{jm\omega_o(t-\hat{\tau}(t))}. \tag{4}$$

The above expression can be depicted more compactly using the cross-correlation function of $s(t)$ and $l(t)$, $R_{XL}(\tau)$, which is defined as

$$R_{XL}(\tau) = \frac{1}{T}\int_0^T l(t)s(t-\tau)dt \tag{5}$$

Substituting (2) and (3) into (5) yields

$$R_{XL}(\tau) = \sum_{n=-\infty}^{\infty} X_n L_n^* e^{-jn\omega_o(t-\hat{\tau}(t))}. \tag{6}$$

Now (4) can be expressed in a more simplified manner using (6) as

$$e(t) \;=\; \sqrt{P}K\, R_{XL}(\tau\text{-}\hat{\tau}) \;+\; K\, n'(t) \tag{7}$$

where

$$n'(t) \;=\; n(t)l(t-\hat{\tau}(t)) = \; n(t)\sum_{m=-\infty}^{\infty} L_m e^{jm\omega_o(t-\hat{\tau}(t))}. \tag{8}$$

From these results, the baseband non-linear-equivalent model of the first-order correlation loop can be represented as shown in Figure 3. This is similar to the non-linear model of the conventional phase-locked loop model with a non-sinusoidal nonlinearity. Using the Fokker-Planck technique, it is possible to show that the probability density function (p.d.f.) of the steady-state timing error process, $p(\tau_e)$, where $\tau_e = \tau - \hat{\tau}$, must satisfy the differential equation [6]:

$$0 = \frac{d}{d\tau_e}\left[\alpha R_{XL}(\tau_e)p(\tau_e) + \frac{d}{d\tau_e}p(\tau_e)\right] \tag{9}$$

where α is a function of the transmitted and locally generated signal powers, the loop gain and the noise power spectral density.

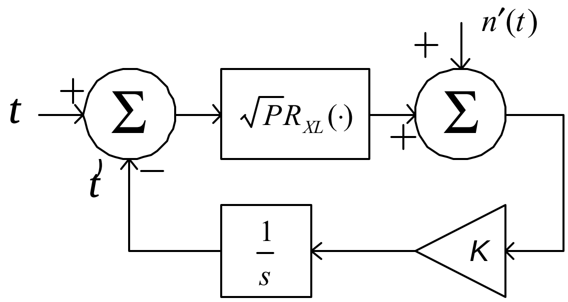

Figure 3: Non-Linear-Equivalent Model the First-Order Correlation Loop

Since the process $n'(t)$ is not stationary, it is difficult to calculate the statistics of $n'(t)$. The noise term $n'(t)$ can, however be shown to be approximately white and Gaussian [7], if $n(t)$ is assumed white and Gaussian [1] and [6], and is assumed in this work.

From Figure 3, it is clear that the cross-correlation function between $s(t)$ and $l(t)$, $R_{XL}(\tau)$, or the characteristic function, influences the performance of the correlation loop. The linear time-invariant filter (VCC filter), $H(f)$ shown in Figure 2 directly influences the waveform $l(t)$ and hence the cross-correlation function, $R_{XL}(\tau)$. That is, the linear filter, $H(f)$, can be used to control the characteristic function of the loop.

Since the objective of a correlation loop is to adjust the delay estimate $\hat{\tau}$ to that of the incoming signal, τ, the characteristic function, $R_{XL}(\tau)$ needs to distinguish whether τ is larger than $\hat{\tau}$ or vice versa. To satisfy the above requirement, the characteristic function to has be an odd function of the phase error $\phi = \tau - \hat{\tau}$ [1,8]. That is,

$$R_{XL}(\tau) = -R_{XL}(-\tau). \tag{10}$$

Substituting (6) in equation (10) yields

$$\sum_{n=-\infty}^{\infty} X_n L_n^* e^{-jnw_o t} = -\sum_{n=-\infty}^{\infty} X_n L_n^* e^{jnw_o t} = -\sum_{n=-\infty}^{\infty} X_{-n} L_{-n}^* e^{-jnw_o t} . \tag{11}$$

The above condition can be satisfied if and only if

$$X_n L_n^* = -X_{-n} L_{-n}^* \quad \text{for each } n. \tag{12}$$

Since X_n and L_n are FS coefficients of real signals, $s(t)$ and $l(t)$, the following properties exist:

$$|X_n| = |X_{-n}| \quad \text{and} \quad \arg(X_n) = \text{-}\arg(X_{-n}) \tag{13}$$

and

$$|L_n| = |L_{-n}| \quad \text{and} \quad \arg(L_n) = \text{-}\arg(L_{-n}) \tag{14}$$

Now by using the above properties and (12) yields

$$e^{j(f_n - y_n)} = -e^{-j(f_n - y_n)} = e^{j(p - (f_n - y_n))} \tag{15}$$

To satisfy the above condition it is necessary to have

$$f_n - y_n = \frac{p}{2} + mp \tag{16}$$

where $m = 0, \pm 1, \pm 2, \Lambda$. That is, it is necessary to have a ± 90 degree phase shift between all the harmonics of the VCC filter output, $l(t)$, and transmitted signal, $s(t)$. Thus, an optimum VCC filter should impose a 90-degree phase shift on all the frequency components of the waveform generator output.

The performance of a correlation loop with a Hilbert transformer compared to a conventional early-late delay-locked loop (ELDLL) is presented in [5]. In [5], it is shown that the performance of a correlation loop with the Hilbert transformer is superior at moderate to low signal-to-noise ratios.

SIMULATION MODELING AND OPTIMIZATION

Correlation loops are non-linear devices which makes mathematical analysis extremely difficult if not impossible. Generally the analysis is made on linear or approximately linear regions of the non-linearity present in correlation loop. A linearized version of the loop is based on the selected linear region. Most of the available parameters, which give a measure of the performance on correlation loops, such as bandwidth, tracking error variance, are defined using these linearized models. Such an approximation is not possible in the case of correlation loop with a Hilbert transformer [5]. Thus, computer simulations were used to determine tracking performance of correlation loops in this work. Numerical optimization algorithms were used in determining the optimum form of the VCC filter.

A. Comparison of Correlation Loops

The knowledge of the bandwidth of a correlation loop is important when it is examined for tracking performance. A smaller bandwidth correlation loop may give a smaller tracking error but likely to consume more time to get into lock and vice versa. Since the mathematical analysis is difficult, computer simulations were used to estimate a reasonable value. Bandwidth of these devices were measured by using a sinusoidal signal for the channel delay, τ,

$$r(t) = \sqrt{P}s(t - A\sin(\omega_m t)) + n(t) \tag{17}$$

The amplitude of the modulating term, A, was stepped over a wide range of values. At each amplitude, the time delay estimate was decomposed as

$$\hat{\tau}(t) = \hat{A}\sin(\omega_m t + \theta) + \zeta(t) \tag{18}$$

where $\zeta(t)$ is a noise term due to non-linearities and self noise. By plotting $\hat{A}/A$ as a function of ω_m it is possible to determine a "frequency response" for each value of A. For conventional ELDLL it is possible to develop a linearized model, and calculate the 3-dB bandwidth of this model [9]. As A increases, it will reach a point, A_o, where the ELDLL performance begins to deviate from the linear model. We assume that for normal tracking applications $A < A_o$. For the purpose of comparison, correlation loops with arbitrary VCC filters were adjusted to exhibit equal or higher bandwidths for the input amplitudes that are within the linear region of the corresponding ELDLL (i.e. for all $A < A_o$). Using this definition of bandwidth, the MSTE of the correlation loop with an arbitrary VCC filter will be compared to a conventional ELDLL.

B. Optimization Procedure

The optimum VCC filter for a given bandwidth and a given input signal-to-noise ratio is determined by using numerical optimization algorithms. The application of optimization algorithms for search using simulation is discussed in detail in [10]. The procedure for using an optimization algorithm in simulation can be depicted as shown in Figure 4 and the overall methodology can be summarized as follows:

1. Assuming that the design performance is sufficiently quantitative such that the performance can be derived from a simulation model, define the performance of a design $\mathbf{x}_i = \{x_{0_i}, x_{1_i}, \ldots, x_{N_i}\}$, in terms of some function $f(\mathbf{x}_i)$, of the results of the simulation. The terms $x_{0_i}, x_{1_i}, \ldots, x_{N_i}$ are called the design parameters of the design $\mathbf{x}_i$.

2. The simulation takes the trial design $\mathbf{x}_i$ as the input to the system and determine the performance of the trial design $f(\mathbf{x}_i)$. This performance is fed back to the search (optimization algorithm) which will pick the next trial design $\mathbf{x}_{i+1}$. This process repeats until the optimal value, $\mathbf{x}_O$, (i.e. $f(\mathbf{x}_O) \geq f(\mathbf{x})$, for all $\mathbf{x} \in \mathbf{S}$ where $\mathbf{S}$ is the complete global search space) is found within some predefined numerical accuracy or the resources allocated to this procedure are expended.

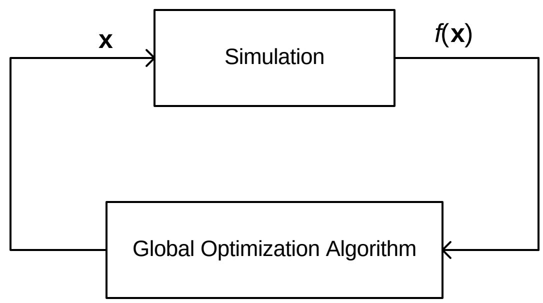

Figure 4: Global Optimization Procedure

To design a simulation to determine tracking performance as a function of an arbitrary VCC filter, it is necessary to define the VCC filter by a large number of design parameters. Some of the widely used optimization algorithms for systems with a large number of design parameters are genetic algorithms [11], simulated annealing [12], and dynamic hill climbing [13]. In this paper, the most of the results are determined by the application of dynamic hill climbing in the global optimization procedure.

C. Simulation Procedure

The simulation determines the MSTE of the tracking process for a given VCC filter, a bandwidth, and input SNR. Mean-square tracking error is defined as the variance of the tracking error, $E[(t - \hat{t})^2]$. The simulation model first computes the cross-correlation function, $R_{XL}(t)$ using the given VCC filter amplitude response. The $R_{XL}(t)$ thus found is used to determine the loop gain, K, of the correlation loop to satisfy the bandwidth requirement. Using these quantities and the estimated equivalent noise spectral density, MSTE is determined by evaluating the p.d.f. of the timing error process by numerically solving (9).

RESULTS OF NUMERICAL OPTIMIZATION

The amplitude response of an optimum VCC filter determined using the optimization procedure discussed in Section III is illustrated in Figures 5 and 6. The phase response of the VCC filter is equivalent to that of a Hilbert transformer as explained in Section II. The bandwidth of the correlation loop was set to a value that is comparable to a 1.0-chip ELDLL as explained in Section III. The cross-correlation functions or S-curves corresponding to optimum VCC filter responses are shown in Figure 7.

The most important observation is that the VCC filter is a function of input signal-to-noise, and not a differentiator as commonly believed [4]. At high SNR, the optimum VCC filter resembles a differentiator i.e. the filter amplitude response has the form $H(f) = Kf$. At low SNR, the optimum VCC filter amplitude response has the form $1/f^{g}$ where the value of γ approaches 3 for very low SNR. This is consistent with the results obtained using square waves as transmitted signals as shown in [3].

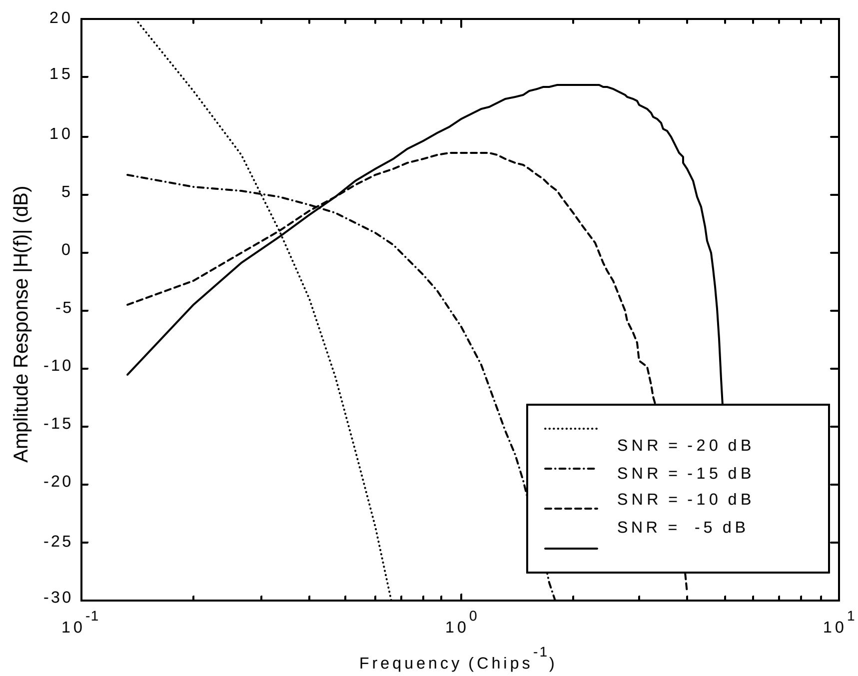

Figure 5: Filter Response Obtained for a Correlation Loop with Comparable Bandwidth as of a 1.0-Chip ELDLL for Input SNR=-20 dB, -15dB, -10dB, and -5dB

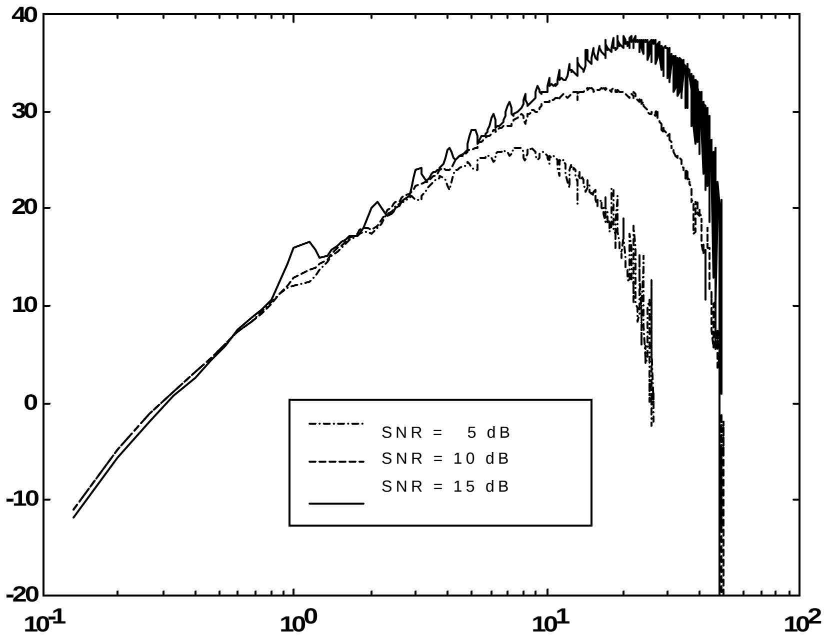

Figure 6: Filter Response Obtained for a Correlation Loop with Comparable Bandwidth as of a 1.0-Chip ELDLL for Input SNR=5 dB, 10 dB, and 15dB

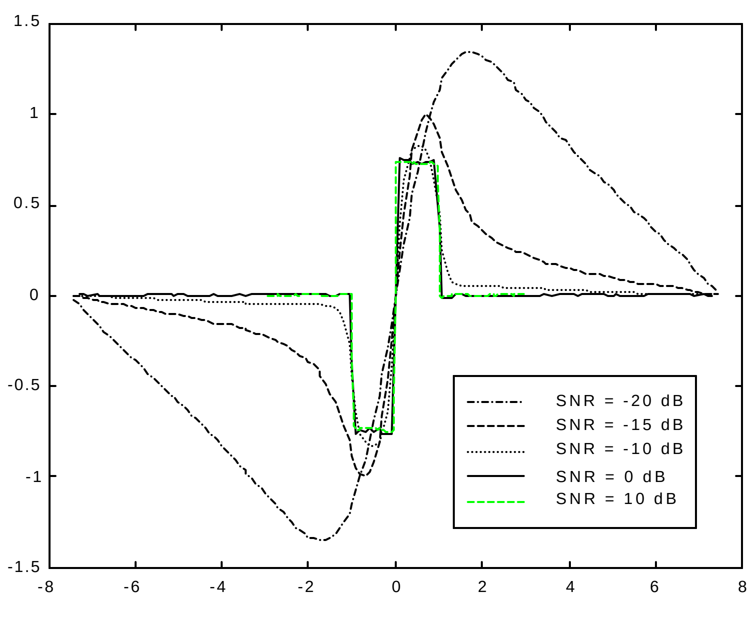

Figure 7: Cross-Correlation Functions Corresponding to Filter Responses Obtained for a Correlation Loop with Comparable Bandwidth as of a 1.0-Chip ELDLL for Input SNR=-20 dB, -15 dB, -10 dB, 0 dB, and 10 dB

From Figure 7 it can be seen that, at low SNR, the s-curves corresponding to the optimum tracking systems exhibit a wider lock range allowing the correlation loop to be in lock with a high probability. At high SNR, the characteristic function corresponding to the optimum tracking system has a narrow (± 1 chips) lock range. However, at high SNR probability of loosing lock is low. This implies that the optimal filter characteristics and the knowledge of the input SNR can be combined to produce a device that has very low probability of loosing lock.

CONCLUSION

In this paper results obtained for optimum reference waveform filters or VCC filters using numerical optimization algorithms were presented. It was shown that the optimum VCC filter should impose a 90-degree phase shift on all frequency components of the output of the local reference waveform generator. The optimum VCC filter amplitude response is highly dependent on the input SNR. At high SNR, the optimum VCC filter resembles a differentiator while at low SNR, the optimum VCC filter exhibits wider lock range. A tracking device that has a smaller mean-square tracking error and a very low probability of loosing lock can be produced by using the optimal VCC filter characteristics and the knowledge of the input signal-to-noise ratio.

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