An Analysis of Error Tolerance Property of Spread Spectrum Sequence

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Abstract

This paper proposes a problem that the error tolerance property of spread spectrum sequence influences the performance of spread spectrum system. Then the relation is analyzed between the error tolerance property and the correlation property of binary sequence when correlation detection is proceeded, and the theoretical limitation of error tolerance is given. Finally, we investigate the relationship between the determination of the output decision threshold of correlation, the probability of correlation peak detection and the error tolerance of the spread spectrum sequence.

Key Words: Spread spectrum code Fault tolerance Correlation detection

1 INTRODUCTION

Spread spectrum code (sequence) plays a very important role in spectrum system. The spread of spectrum is produced by the modulation of spread spectrum code , much merit of Direct-Sequence spectrum system , such as strong anti-interference, low interception probability, anti-multipath-fading, CDMA, timing and distance measurement , has a very close relationship with spread spectrum code .therefor the study of spread spectrum code is a hotspot in the recent research field of Spread spectrum .In the spread spectrum system , the strong anti-interference capacity , strong error tolerance property by other words, is obtained by the pseudorandom code which spreads the spectrum of the transmitted signal and by the correlative detection at receive ending. The communication capacity of spread system, such as the probability of correlation peak detection in system synchronize building period and output signal to noise ratio, is decided directly by the error tolerance ability of spread spectrum code. Error tolerance property is expressed as the tolerable number of occurred error code during a sequence period under the correlative receive method and correct decision condition .the proportion of error tolerance property is closely related to the correlation property (include self-correlation and complementary-correlation). In this paper we analyze the tolerance and correlation property of binary sequence in spectrum receiving. We also analyze the tolerance property of m-sequence used as spread spectrum code in details, the relationship between error tolerance of spread sequence and the probability of correlation peak detection is discussed in details.

2 THE RELATIONSHIP BETWEEN ERROR TORLENCE AND CORRELATIVE PROPERTY

In the theory of code error detection, the distance of the code decides the error corrective property. We assigned c1 and c2 as two codes, d (c1, c2) as the hamming distance of these two codes, then

$$d(c1,c2)=D \tag{1}$$

Where D is the digits of diverse bits within the two codes' correspond bits ,dmin is the minimal distance of all probably hamming distance in the code sets.

$$dmin=mind(c1,c2) \tag{2}$$

The error corrective property of the code can be expressed as:

$$t \le \left[\frac{d_{\min}-1}{2}\right] \tag{3}$$

In the spread spectrum communication systems, the error tolerance capability of spread spectrum sequence has great impact on the system performance, i.e., is related to the accurate detection of synchronous sequence and interference between users.

In a single address spread spectrum system with only one sender and one receiver, the receiver's despread correlator does correlation operation on each shift sequence (equivalent shift sequence) of spread spectrum sequences when detecting correlation of sequence (in synchronization searching process). So by utilizing code sets composed of all shift equivalent sequences, spread spectrum sequence is provided with error tolerance capability due to its autocorrelation properties of spread spectrum sequences.

In spread spectrum multi-access communication system with multi-users, the receiver's despread correlator does correlation operation on spread spectrum sequences of every user. The system's error tolerance capability is depended on the intercorrelation property of code sets composed of every user's unique sequence.

In spread spectrum systems, binary sequence is received after intercorrelation operation. The resemblance between sequences is indicated by correlation property of sequences.

Let U and V denote for two binary sequences with a period of N. The intercorrelation function $q_{U,V}$ is

$$q_{U,V} = \frac{A-D}{N} = \frac{N-2D}{N} \tag{4}$$

Here A denotes for the number of equivalent corresponding elements in U and V. D has the same meaning in equation (1).

Obviously the definition of intercorrelation function is accordant to the one of code distance. Compare equation (1) and equation (4), we can see, the larger D is, the larger $d(C_1, C_2)$ is, which implies difference between U and V becomes greater. So when referred to error correction capability, $q_{U.V}$ and $d(C_1, C_2)$ have same effect. Equation (4) applies to autocorrelation also, this time we regard U as one of shift sequences of V.

The correlation property is one metric for error tolerance capability. From equation (1) and (2), we have

$$d_{\min} = \min d(C_1, C_2) = \min D \qquad (5)$$

From equation (4) we have

$$q_{C_1.C_2} = \frac{N-2D}{N} \qquad (6)$$

Then

$$\max q_{C_1.C_2} = 1 - 2\frac{\min D}{N} = 1 - 2\frac{d_{\min}}{N} \qquad (7)$$

When estimating the error tolerance capability of one code, we can see the larger $d_{\min}$ is, the more errors it can tolerant. From equation (7), this means $\max q_{C_1.C_2}$ must be minimum, but the correlation property needs not to be uniform.

Assume the correlation property of one code set is like figure 1,the maximum value of side lobe is $q_{\max}$. In the code set, when one bit error occurs in code word, its autocorrelation value will be reduced to 1-2/N by 2/N.Meanwhile the worst situation is its maximum side lobe value of intercorrelation peak values set will be increased to 1+2/N by 2/N.Now assume t bits error occur, to make accurate correlation, the underlying equation must be fulfilled.

$$1 - \frac{2}{N}t > q_{\max} + \frac{2}{N}t$$

$$t < \frac{N(1-q_{\max})}{4} \qquad (8)$$

The equation above is the general expression for error tolerant capability of binary sequence set with a period of N. We can see from equation (8), when N is fixed, the smaller $q_{\max}$ is, the larger t is.

3.using m-sequence as spread spectrum code

There are m, L, TP and H sequences as traditionally pseudorandom sequence is concerned. Among those sequences, the highest efficiency and also the longest linear-shift-register sequence is m-sequence.

According to equation (8), those sequence's error tolerance property are:

$$t < \frac{N(1-q\max)}{4} = \frac{N(1-\frac{-1}{N})}{4} = \frac{N+1}{4} \qquad (9)$$

For N is the period of m-sequence, and $N=2^m-1$; m is the number of shift register; and the total number of m sequence' shifting-equivalence sequence are N, equation (9) can be:

$$t < 2^{m-2} \qquad (10)$$

As 2^m-2 is an integer. So equation (10) is equivalence to:

$$t = 2^{m-2} - 1 \qquad (11)$$

When m=3,4,5. t=1,3,7, from here we can see the excellent error-correcting performance of m-sequence.

In the mono-address spread spectrum system, if using m-sequence as its spread spectrum code; the error tolerance property would be the best.

In the binary spread spectrum system, the actually transmitted binary sequence is the code sets of the total shifting equivalence sequence of spread spectrum sequence and the complementary sequence of spread spectrum.

Assume m sequence A's complementary sequence is $\overline{A}$, according to [1], $\overline{A}$ sequence has the following double value self-correlation property:

$$r_A(t) = \begin{cases} 1 & t = 0 \pmod N \\ -\frac{1}{N} & t \neq 0 \pmod N \end{cases} \qquad (12)$$

The cross correlation function of A and $\overline{A}$ is:

$$r_{\overline{A}A}(t) = \begin{cases} -1 & t = 0 \pmod N \\ \frac{1}{N} & t \neq 0 \pmod N \end{cases} \qquad (13)$$

The number of code word in code set which is constituted by m sequence and its 互补 sequence is 2^{m+1}-2. Extend code set ,its $J = 1/n$. From (8), we can get that the error tolerance is:

$$t < \frac{N(1-1/N)}{4} = \frac{N-1}{4} = \frac{2^m-2}{4} = 2^{m-2} - 1/2 \qquad \dots\dots\dots\dots\dots\dots(14)$$

and the $t=2^{m-2}-2$.

It is equal to the result of formula (11). So when both A and $\overline{A}$ constitute a code set, the max value of inter correlation is 1/N, this is one limitation factor to this system's error tolerance of spread spectrum. If the 1/N is too large, some error tolerances of the code set maybe lose. We can prove that, to a common binary sequence, if only the positive and negative peak value of its intent inter correlation is small, we can use this method to extend code set, such as L sequence, TP sequence, H sequence.

4.The relationship between error tolerance and inter correlation peak value detection probability

In DS/SSMA system, the local spread spectrum do correlated computation to received spread spectrum (it is a mix of many spread spectrum sequence in spread spectrum multi address system) and make judge with correlation peak value .The peak value detection probability will directly affect the self-synchronism capability of spread spectrum and the BER of the m-ary spread spectrum communication system such as similar orthogonal spread spectrum, collateral assemble spread spectrum communication. Because in this kind system, the receiving judge is made with a group of values of correlater output's correlation peak value

The reasonable ascertain of judge threshold is important to get a high accurate detection probability of correlation peak value.

We suppose that A_0 is the judge threshold of correlation peak, $J_{max} < A_0 < 1$, J_{max} is the max self correlation value of spread spectrum (using single spread spectrum sequence) or the max value of inter correlation (using many spread spectrum sequences) .We can get from analyze above that spread spectrum has error tolerance when it use inter correlation receiving. So we should ascertain judge threshold of correlation peak according to error toleration of spread spectrum sequence .We suppose that N is the length of sequence period, according to the formula (8), the judge threshold is:

$$A_0 = q_{max} + t \times \frac{2}{N} = \frac{(1 + q_{max})}{2} \quad \text{...(15)}$$

When the number of error code in code word is less than t, We can get the right judge of correlation peak. Suppose A is the output of correlater, so the probability of right detection of correlation peak is:

$$\begin{aligned} p_c &= p(A \geq A_0) \\ &= (1 - p_e)^N - C_N^1 P_e (1 - P_e)^{N-1} + \cdots + C_N^t P_e^t (1 - P_e)^{N-t} \end{aligned} \quad \text{..............(16)}$$

Among the formula, P_e is BER of the demodulated receiver carrier. For BPSK digital channel, the relationship between BER of digital demodulation and signal noise ratio is shown as follow:

$$p_e = erfc\left(\sqrt{(S/N)_e}\right) \quad \text{...(17)}$$

In user unit system, the output signals noise ratio of demodulated carrier is:…….

$$(S/N)_{out} = \frac{(PT_c)^2}{\frac{PT_c N_0}{2}}$$

$$= \frac{2PT_c}{N_0}$$

$$= \frac{2E_c}{N_0} \qquad (18)$$

P denotes the signal power; Tc is the code-width of spread spectrum. Ec is the energy of signal within Tc. N0/2 is the twin-power spectrum's density.

In the multi-user system, assume each user's signal arrive the receiver with equal power, the signal-to-noise ratio is:

$$(S/N)_c = \frac{(PT_c)^2}{\frac{PT_c N_0}{2} + (K-1)(PT_c)^2}$$

$$= \frac{PT_c}{\frac{N_0}{2} + (K-1)PT_c}$$

$$\frac{2E_c / N_0}{1 + 2(K-1)E_c / N_0} \qquad (19)$$

In equation (19), $(K-1)*(PTc)^2$ is the signal power of other (K-1) user, e.i multi-address interference.

Fig.2 denotes the correlation peak detection probability differences of m-sequence, balance GOLD sequence and orthogonal GOLD sequence with the sequence' length N=127 and working in mono-address spread spectrum system.

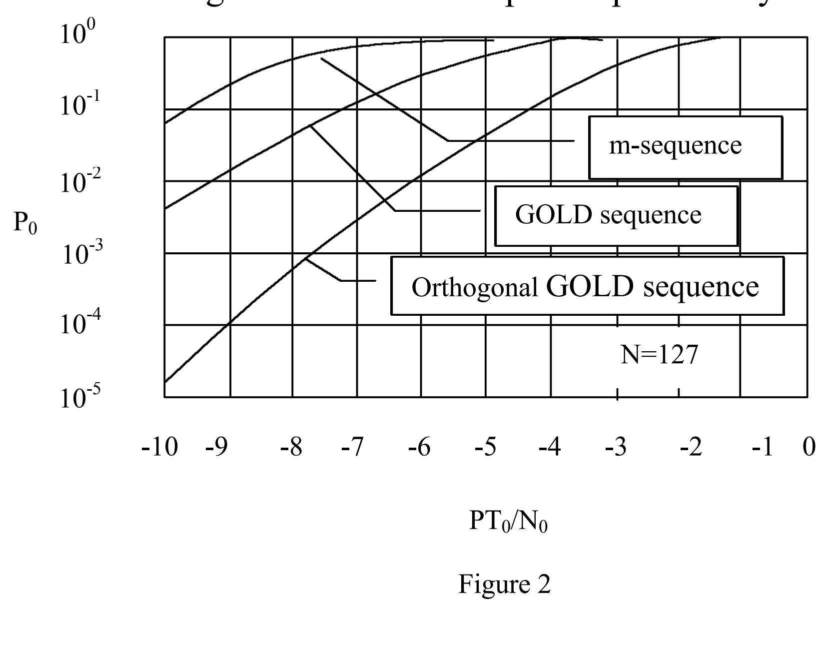

Figure 2

Clearly, the correlation peak detection probability is higher when using m sequence as the spread spectrum code.

5. Conclusion

In this paper, we propose a problem that the error tolerance property of spread spectrum sequence influences the performance of spread spectrum system. Through analysis we draw the conclusion: the error tolerance property of spread spectrum sequence is decided by the self-correlation and complementary property of the spread spectrum binary sequence's sets; the system's error tolerance property will definitely influence some of the system's performance, for example, the detection ability and the degree of multi-address interference. Because of m-sequence good self-correlation property, the error tolerance property would be desirable if the shifting equivalence sequence of m-sequence is used as spread spectrum.

Reference

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