DATA TRANSMISSION OVER CHANNELS WITH NOISY FEEDBACK

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Summary A low-cost error control technique is proposed for bulk data transmission with noisy feedback link.

The scheme is ideally suited for tape-to-tape bulk data transmission as well as the store-and-forward type of data transmission system. By partition data into superblocks, the technique can be used for any feedback retransmission system.

We also show that the scheme can be modified to correct synchronization errors and that noise in the feedback link can be made extremely unlikely to contribute to decoding errors.

Introduction Consider a feedback data transmission system. There are two ways of correcting errors in the forward channel by feedback: decision feedback and information feedback. Decision feedback consists of the following steps:

1. The information is coded and the codeword is sent to the channel.

2. The decoder checks the incoming message, if it is not a codeword, a retransmission request is relayed to the transmitter. Otherwise, an acknowledgement is relayed to the transmitter.

3. The transmitter either transmits the next message or repeats the previous one, depending on the request received from the reverse channel.

Such a system requires the transmission of parity checks over the forward channel; yet little use is made of the reverse channel. Thus the throughput is upper bounded by the code efficiency.

The information feedback system was proposed and analyzed by Chang.[(1)] The system works as follows:

1. Given a set {y} of all q^n n-tuples over a field of q-elements (for binary transmission q = 2), a subset {x} ⊂ {y) is chosen. The set {x} comprises all possible transmitted messages. The transmitter selects a member of x ϵ {x} and sends it through the forward channel to the receiver.

2. The receiver, upon receiving the incoming message say x*, computes the classification of x* according to a prearranged rule known to the transmitter. This information is sent back to transmitter.

3. The transmitter receives the classification information from the reverse channel, compares it with the classification of x; if they agree, the transmitter confirms it by sending another member of {x}, otherwise a denial signal is sent.

The Error Control Scheme As a particular case of an information feedback system, a simplified system can be made as follows: [1]

1. The transmitter computes and saves the parity check, P, from the customer Is data by means of a systematic block code and sends out only the information portion of the codeword through the forward channel to the receiver.

2. The receiver, upon receiving the information block, plus possibly some errors due to channel noise, computes the parity checks P* and sends P* back to the transmitter through the reverse channel.

3. The transmitter compares the P**, the parity checks the receiver computed, plus possibly some error due to channel noise in the reverse channel, with P.

    A. P** = P The transmitter picks up the next block of data from the customer and goes to step 1.

    B. P** ≠ P The transmitter retransmits the old information block and goes to step 2.

The remaining problem for the scheme is, of course, to design a fail-safe scheme, such that the transmitter can tell the receiver which portion of the message is erroneous and should be deleted.

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[1] A rediscovery by J. E. Mazo, the scheme also appears in references 1 and 2.

It is the purpose of this report to show a way of solving this problem.

4. We first add one more operation to step 3, namely, the transmitter keeps a running count of which message blocks have been repeated. This can take, for example, either a form of gap length between incorrect blocks, or absolute addresses of the incorrect blocks.[2] Presumably, such records are kept in a sequential storage such as a tape, a disk, or in the core storage of a computer. This information will compose an additional message called the Diagnostic Message (DM). The other steps required are:

5. When the customer's message runs out, an END symbol followed by the diagnostic message, compiled in step 4, is sent as additional information blocks which are treated exactly the same way' as customer's messages and sent to the forward channel until the DM is completely transmitted. (Note that, during the course of sending the DM, the DM may grow longer due to additional errors detected in the portion of the DM already sent.)

6. The receiver, after having received the last block of message, runs the tape recording of the whole data string backwards. The first block B_O (which actually is the last block transmitted) must be correct, for if it were not, the transmitter, after detecting it, would have sent more blocks of DM.

Block B_O contains information about the last few blocks as to their correctness. Thus the receiver would be able to rewind the tape-and work its way back to delete (or erase) all the wrong information blocks sequentially, [3] namely:

A. Look at the i^{th} block, if it is not on the list of bad blocks, retain it; if it is, erase it and go to the $i+1^{th}$ block.

B. Step A is repeated, starting from B_O block, until the "END" symbol is detected. At this time, the receiver knows all the bad blocks from there on, and those blocks can be erased accordingly.

C. By the time the tape is rewound to its beginning position all the bad blocks would have been erased and the tape contains a version of correct customer data with some spaces scattered that correspond to those blocks removed.

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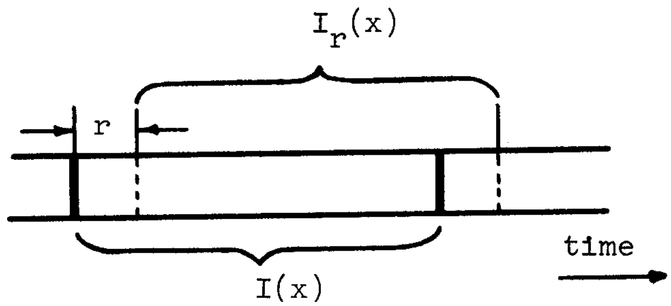
[2] If additional protection is required such information may be further encoded so that later on receiver may be able to check their validity when they are used to delete faulty blocks.

[3] See Appendix for a proof of this statement.

There is one more point that needs clarifying: The receiver must know exactly which one is the last message. The detection of the last message can most easily be done by the modem, if possible, otherwise one may use a “start” symbol at the end of the diagnostic message. The f1start” symbol must, of course, be the end of DM and should not stand alone in the last block. [4] The receiver would expect that the last block has a “start” symbol in association with some information which tells the receiver about the validity of the last few blocks.

Correction of Synchronization Errors The correction of synchronization errors can be achieved very easily. If a synchronization slippage of up to s bits is to be corrected, the transmitter not only computes P but P_r , $1 \leq r \leq$ so where P is the check bits of the information block I(x) and P_r is the check bits for the information block $I_r(x)$ which is shown schematically below:



We modify the step 3. That is, the transmitter not only compares P** with P, but also with P_r , $1 \leq r \leq s$. If $P^{**} = P_i$, $-s \leq i \leq s$ then the synchronization error is corrected by deleting (if $i < 0$) or adding (if $i > 0$) i time pulses at the transmitter. Note that if cyclic codes are used then P_r can be generated rather trivially, hence there is no need to store P_r for comparison, rather one may compute them as needed. Furthermore, coset codes [(3)] can be used to ensure that all P_r are distinct, thus avoiding ambiguity in deciding corrective measures.

Discussion The features of such an error control system are:

1. Throughput is limited by noise, independent of code efficiency. It can be shown that the throughput rate is proportional to $1 - P_B$ where P_B is the block retransmission probability.

2. The code efficiency is lower bounded by the reverse-channel signaling rate.

3. Errors in the reverse channel may reduce throughput but are extremely unlikely to give decoding errors assuming good error detecting codes are utilized.

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[4] One can always avoid this situation by duplicating a part of the information in the second to last block, (i.e., the B_1 block).

4. Mutilating due to reverse channel error is impossible.

5. Synchronization errors are corrected as well.

6. For a full-duplex telephone channel one may use a rate 1/2 code. In this case one may use information feedback thus doing away with parity check calculators in both transmitter and receiver; this results in an extremely simple error control system.

One sees that such an error control technique is not limited to the application of tape-to-tape data transmission. In principle, it can be applied to any other kind of feedback retransmission system where information blocks are grouped into superblocks, and one sends one superblock at a time as if it is a reel of tape. In such a case, it can be interpreted as variable length coding scheme, and the length of the code depends on the errors in the message.

REFERENCES

1. S. S. L. Chang, "Theory of Information Feedback Systems", IEEE PGIT Vol. II-2, pp. 29-40, September, 1956.

2. W. L. Bishop and B. L. Buchanan, "Message Redundancy vs. Feedback for Reducing Message Uncertainty", IRE National Convention Record, Vol. 5, Part 2, pp. 33-39, March 1957.

3. S. Y. Tong, "Synchronization Recovery Techniques for Binary Cyclic Codes," BSTJ Vol. 45, pp. 561-596, April, 1966.

APPENDIX

Justification of Decoding Procedure

To justify the procedure of decoding DM we intend to prove that DM so constructed is always decodable at the receiver. Given

A) The last block (B_O) is located properly

B) All error blocks are detected by the transmitter.

Definition: Let B_i be the i^{th} block of DM counted from the last block which has a start symbol and is called B_O block.

Lemma 1. B_j cannot appear in the list of incorrect blocks specified by the information carried by B_i for all $j \leq i$.

Proof: Information cannot be sent before it is created.

Lemma 2. The list of incorrect blocks specified by B_i cannot contain B_{i+k} , $k = 1, 2... \ell$, if the first good block after B_i is $B_{i+\ell+1}$.

Proof:

1) If B_{s+1} is bad then B_s must be a repetition of B_{s+1} . By lemma 1 B_s cannot carry information about B_{s+1} .

2) Induction on s, since the first good block after B_i is $B_{i+\ell+1}$ thus the lemma is true for $s = i + k$, $k = 1, 2... \ell$.

Lemma 3. Information in a good block reveals at least one more good block.

Proof: From Lemma 1 and 2; one knows that if B_i is good, then it must contain information that reveals at least one good block, namely $B_{i+\ell+1}$, which is the first good block after B_i .

Since B_O must be a good block, by Lemma 3, the DM can be decoded sequentially without premature termination. This is what we intended to prove.