NOTCH NOISE LOADING DATA ON BASEBAND TAPE RECORDING

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Summary Notch power ratio tests were performed on a magnetic tape recorder/ reproducer, using direct recording in the baseband. For the equipment tested, it is concluded that the IRIG method of setting the record power level as that which produces 1% third harmonic on a single tone, does not necessarily result in an optimum record/reproduce cycle. It is concluded that the input and output levels should be set with reference to notch noise test data to optimize baseband tape recording performance for baseband recording of frequency division multiplexed systems. In order to interpret the notch noise data, it was necessary to assume two non-linear processes, one acting in conjunction with the record process and one in conjunction with the playback process.

Test Data A "Notch Power Ratio" test of an HP 3950 recorder/ reproducer and Scotch 888 tape was performed. The test set up is shown in Figure 1. The recorder/reproducer was equalized, the bias current set, and the record level set to IRIG 106-69 standards. Record gain was set so that a single tone input at 1 volt rms at a frequency corresponding to 10% of the upper edge of the baseband produced 10% third harmonic. The record gain was not changed thereafter.

The input noise level from the Marconi test set was monitored with an rms voltmeter. The Marconi 204 kHz low pass filter was used to restrict out-of -band signals at the input. In order to fully utilize the available baseband with the 204 kHz filter the test was conducted at 15 ips. NPR was determined for four notches: 14, 34, 70 and 185 kHz. These data are shown in Figure 2. Third harmonic output as a function of input level is included in Table 1. A single tone at 20 kHz was used for this test. In addition, the power in the notch with signal absent (attenuated 40db) was measured in order to determine the background noise due to the playback system. If:

S = "signal" power in a notch,
N_o = background noise in a notch,
N_i = intermodulation noise in a notch,

$$NPR = \frac{S + N_o + N_i}{N_o + N_i} \equiv A \qquad (1)$$

With signal absent (attenuated 40db):

$$B \equiv \frac{N_o + N_i}{N_o}, \qquad (2)$$

Where B is the observed relative change in the notch ratio.

A significant quantity is:

$$NPRI \equiv \frac{S + N_i}{N_i} = \frac{AB-1}{B-1} \qquad (3)$$

which is a measure of intermodulation alone. If$(S + N_o) >> N_i$ then:

$$NPRO \equiv \frac{S + N_o}{N_o} \approx AB \qquad (4)$$

which is a measure of additive noise alone.

Figure 3 shows a plot of NPRO as a function of notch frequency with input level as parameter. One volt rms corresponds to the zero db input level. Figure 4 shows a plot of NPRI as a function of input level for the various notches. For all input levels the output level of the reproduce amplifier was held constant at a level well below the overload level of the output stage. The intermodulation characteristic of the output amplifier was determined by holding the input level at 0 db (1 vrms) and measuring the NPR at the output amplifier. This is plotted in Figure 5 as a function of output level along with the third harmonic distortion obtained from the 20 kHz test tone. Upon overload, the characteristic of the amplifier used is a hard limiter at 3 volts peak. Figure 5 shows that the output level must be adjusted to accommodate the crests of the baseband signal to be recorded. If the output level is adjusted to the maximum consistent with the 1% permissible third harmonic distortion to a 20 kHz test tone in accordance with IRIG 106-69, then the notch noise data indicate considerable intermodulation.

All test data presented in Figures 2, 3, and 4 were taken at an output level of approximately 0. 3 volts rms or - 8db on Figure 5 so that the output amplifier contributed no appreciable distortion.

Analysis - The NPRI result from power measurements, so phase shift across the baseband is unimportant. Therefore, it is assumed that only static type non-linearity generates the observed intermodulation. We assume that the static non-linear characteristic can be represented by a power series

$$\underset{\sim}{v}(t) = a_1 \underset{\sim}{u}(t) + a_2 \underset{\sim}{u}^2(t) + a_3 \underset{\sim}{u}^3(t) + \cdots \tag{5}$$

where $\underset{\sim}{u}(t)$ is the input random process and $\underset{\sim}{v}(t)$ is the output process. The a_i's are constants.

As seen in Table 1, the observed distortion produces mainly third harmonic so only the cube term will be considered quantitatively. Since the data in Figure 5 show that the output amplifier, if not overloaded, does not contribute appreciably to the NPRI, the non-linearity must be attributed to the record and playback process excluding the output amplifier. The block diagram of Figure 6 shows two possible locations for non-linear effects. Since the maximum baseband frequency is 204 kHz and the HP 3950 is a 1.5 mHz machine at 120 ips, it may be assumed that the input amplifier delivers constant magnetizing current across the baseband so the gain is a constant G_1 across the band.

Consider first the effect of non-linear element number 1 on the NPRI at the tape machine output assuming that non-linear element number 2 is shorted out. Assume that equation (5) with appropriate constants a_i, describes non-linear element number 1. For the notch noise test the input signal is a gaussian random process (thermal noise). It can be shown* that the spectrum of $\underset{\sim}{v}(t)$ in equation (5) is given by

$$S_{vv}(f) = \sum_{n=1}^{\infty} n! \; S_{uu}^{(n)}(f) \left[\sum_{k=0}^{\infty} \frac{(n+2k)!}{2^k k! n!} \; a_{n+2k} \; R_{uu}^{k}(0) \right]^2 \text{Volts}^2/\text{Hz} \tag{6}$$

where $S_{uu}(f)$ is the spectrum of the input random process $\underset{\sim}{u}(t)$ and S(n)(f) the (n-1) convolution of S(f) - i.e. for example, $S^{(3)}(f) = S(f) * S(f) * S(f)$ - and $R_{uu}(\tau)$ is the autocorrelation function of $\underset{\sim}{u}(t)$.

If the distortion is not too great so that the intermodulation is small compared to the signal spectrum $S_{vv}(f)$, then to a good approximation the NPRI in the i^{th} notch due to non-linear element number 1 is:

$$\text{NPRI}\Big]_1 \cong \frac{a_1^2 G_1^2 S_{xx}(f_i)}{\sum_{n=2}^{\infty} n! G_1^{2n} S_{xx}^{(n)}(f_i) \left[\sum \frac{(n+2k)!}{2^k k! n!} \; a_{n+2k} G_1^{2k} \; R_{xx}^{k}(0) \right]^2} \tag{7}$$

since the gain $H_o(j2\pi f)$ and Ha $(2\pi f)$ are common to both the signal $S_{uu}(f)$ and intermodulation products and $S_{uu}(f) = G_1^2 \; S_{xx}(f)$.

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* See for example Ahmad F. Ghais, et.al, ITC '67 Proceedings Page 26

We assume that the input amplifier is flat and the input test signal is flat from 12 kHz to 204 kHz. Therefore, the convolutions peak up near zero frequency and the NPRI of the 185 kHz notch should be the highest and the NPRI of the 14 kHz notch the lowest. This is not the case in Figure 4 especially at the lower input levels. At the higher levels the trend is toward this condition.

We now consider the effects of non-linear element number 2 under the condition that non-linear element number 1 is shorted out. We assume

$$\underset{\sim}{z}(t) = b_1\underset{\sim}{y}(t) + b_2\underset{\sim}{y}^2(t) + b_3\underset{\sim}{y}^3(t) + \text{----} \tag{8}$$

where the bi are constants and $\underset{\sim}{y}(t)$ is a stationary gaussian random process of zero mean because of the assumptions.

We have

$$NPRI]_2 \cong \frac{b_1^2 G_1^2 H_a^2 (2\pi f_i)\, S_{xx}(f_i)}{\sum_{n=2}^{\infty} n!\, G^{2n} \left[H_a(2\pi f) S_{xx}(f)\right]_{f_i}^{(n)} \left[\sum_{k=0}^{\infty} \frac{(n+2k)!}{2^k k!\, n!}\, b_{n+2k}\, R_{yy}^{k}(0)\right]^2} \tag{9}$$

since the gain H_o ($j\pi f$) is common to both signal and intermodulation components.

Measured data on the roll off of the record-playback process given in Figure 7 were obtained by using a flat output amplifier. Figure 8 is the convolution $[H_a(2\pi f)S_{xx}(f)]^{(3)}$ which was computed numerically.

Thus, the convolutions in denominator of equation (9) are less sensitive to the roll off in $H_a(2\pi f)$ than is the numerator. On this basis, the behavior of Figure (4) in which the NPRI for the 185 kHz notch is lowest is predicted. In the next section, this prediction is evaluated quantitatively.

COMPARISON WITH TESTS From Table 1 the third harmonic at 0 db input level is down 40db in accordance with IRIG set up procedure. The output level was 0.78 Vrms which is well below the distortion level of the output amplifier. Notice from Table 1 that in the region of interest the distortion is nearly all third harmonic, i.e., the cube term in equations (7) and (9) predominate. If only third harmonic distortion is assumed to be present, then equation (9) becomes

$$NPRI]_2 = \frac{b_1^2\, G_1^2\, H_a^2\, (2\pi f_i)\, S_{xx}\, (f_i)}{3!\, G_1^6 \left[H_a\, (2\pi f)\, S_{xx}(f)\right]_{f_i}^{(3)} b_3^2} \tag{10}$$

In order to determine the constant $b_1^2/b_3^2\ G_1^4$ we assume that all the third harmonic distortion is in non-linear element number 2. Let f_c = frequency of test tone. Since $A^3 \cos^3 2\pi f_c t = A^3 (\cos 6\pi f_c t + 3 \cos 2\pi f_c t)/4$, the ratio, β, of third harmonic power to fundamental power is given by

$$\beta = \frac{A^6 G_1^6\ H_a^6\ (2\pi f_c)\ b_3^2\ \overline{\cos^2\ 6\pi f_c t}\ \left| H_o\ (j6\pi f_c)\right|^2}{16A^2\ G_1^2\ b_1^2\ \overline{\cos^2\ 2\pi f_c t}\ H_a^2\ (2\pi f_c)\ \left| H_o(j2\pi f_c)\right|^2} \tag{11}$$

From Figure (7), $H_a^2\ (2\pi f_c) \cong 1$ for f_c = 20 kHz, the test tone frequency, and $H_a(6\pi f_c) \cong H_a\ (2\pi f_c)/2$ or down 6db for the third harmonic.

Because of the equalization, $H_a^2\ (\cdot)\ |H_o\ (\cdot)|^2$ = constant so $|H_o\ (j6\pi f_c)|^2 = 4\ |\ H_o\ (j2\pi f_c)|^2$. Since $\beta\ = 10^{-4}$ (-40db) as per IRIG with zero db input (1 volt rm s) and $A = \sqrt{2}$we have

$$\frac{G_1^4\ b_3^2}{b_1^2} = \beta = 10^{-4} \tag{12}$$

Using this value and the values in Table 1, Figure (9) is calculated from equation (10) and the fact that $S_{xx}(f) \cdot (2f_c) = 1$ since the input is 1 volt rms (0 db). Here, f_c = 204 kHz, the upper edge of the baseband test signal. Note that this model predicts that the NPRI for the 185 kHz notch falls well below the NPRI for the lower frequency channels. This is Wiat happens in the test data of Figure 4.

If only non-linearity number I in Figure 6 is present, equation 11 becomes for this case for 0 db input

$$\frac{G_1^4\ a_3^2}{4a_1^2} = \beta = 10^{-4} \tag{13}$$

Substituting this value into equation (7) under the assumption that only third harmonic is present, the dotted lines in Figure (9) are calculated. Note that in this case the 185 kHz notch shows a little larger NPRI than the other notches which is not in agreement with Figure 4.

Discussion Comparison of Figures 9 and 4 indicates that the actual process involves both types of nonlinearity (possibly others) because intermodulation from nonlinearity number 1 predominates at the low frequency notches and the intermodulation from nonlinearity number 2 predominates for the 185 kHz notch. Suppose, for example, that the nonlinearity is divided equally between the two non-linear elements. We may assume that for the frequency range used in this test and the model assumed for analysis,the intermodulation power adds coherently. Thus, all lines in Figure 9 would be moved up

6db. This would bring the calculated results into agreement within a db or so of the test results of Figure 4.

Among other things, the test data show that the IRIG criterion of 1% third harmonic distortion for setting tape input level does not necessarily lead to consistent NPRI performance under all conditions and for all machines. The writers believe that setting the input level (and output level) in reference to notch noise test data is more reliable in terms of baseband tape recording performance.

TABLE I
HARMONIC GENERATION MEASUREMENTS

		HARMONIC			
	20KHz	2nd	3rd	4th	5th
TEST SIGNAL	0	-62	-72	-58	-58
INPUT LEVEL -					
-6db	0	-56	-50	-54	-54
-4db	0	-57	-47	-55	-56
-2db	0	-59	-43	-56	-57
(IRIG) 0db	0	-60	-40	-56	-58
+2db	0	-60	-37	-57	-59
+4db	0	-61	-34	-58	-60

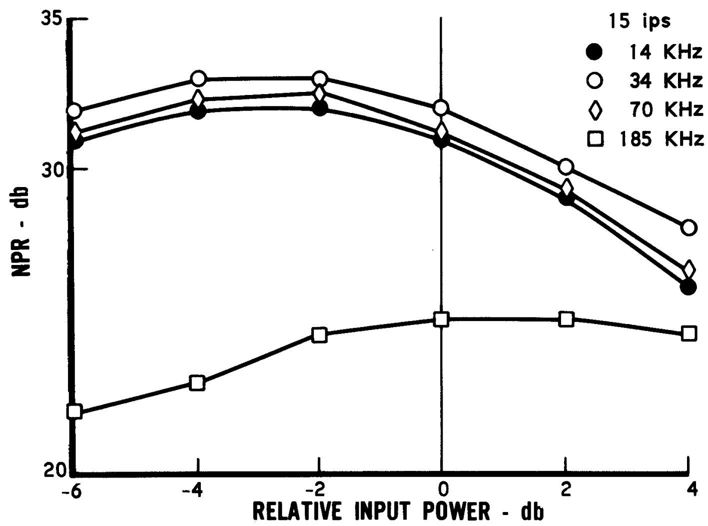

FIGURE 1 NPR TEST SET UP

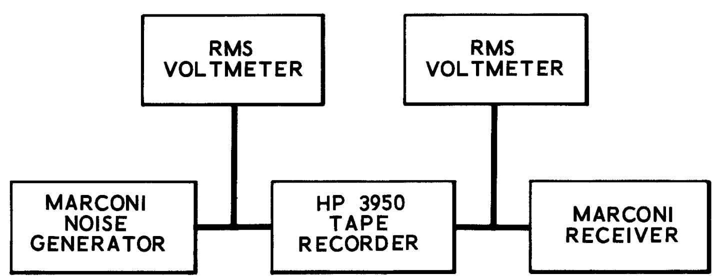

FIGURE 2 NPR TEST DATA

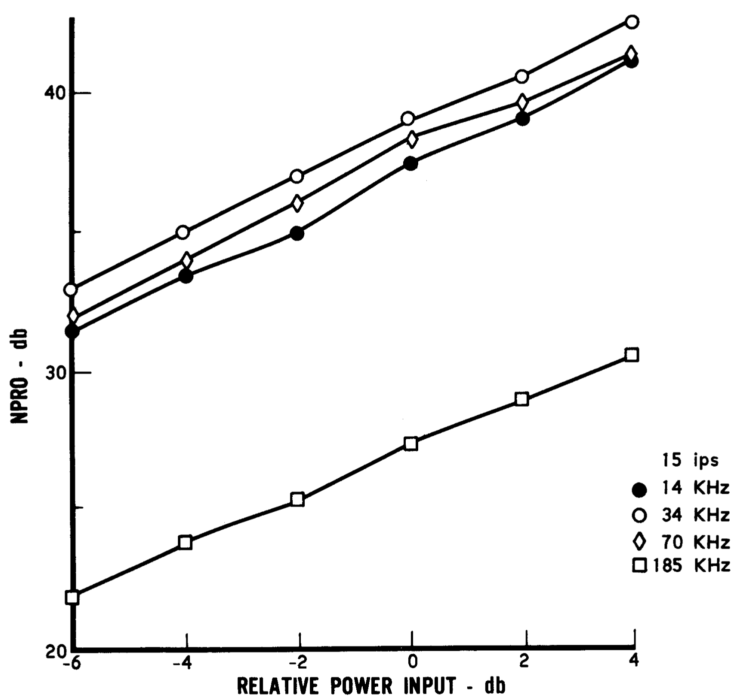

FIGURE 3 NPRO DATA

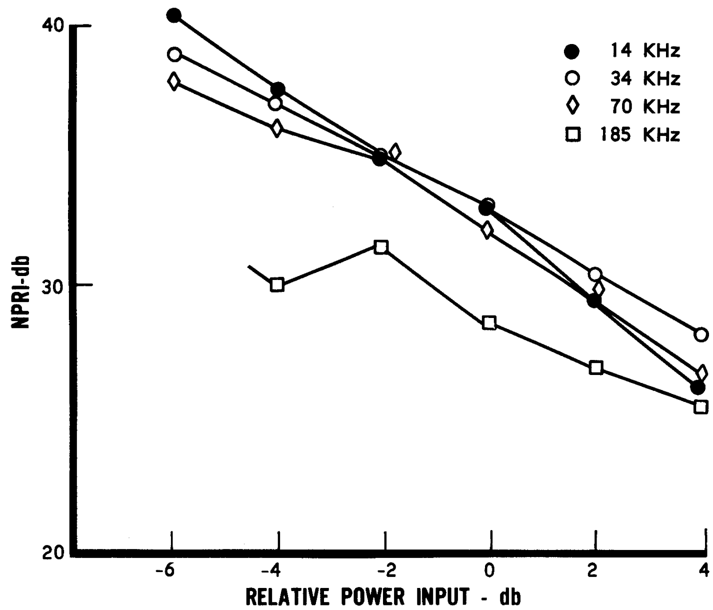

FIGURE 4 NPRI DATA

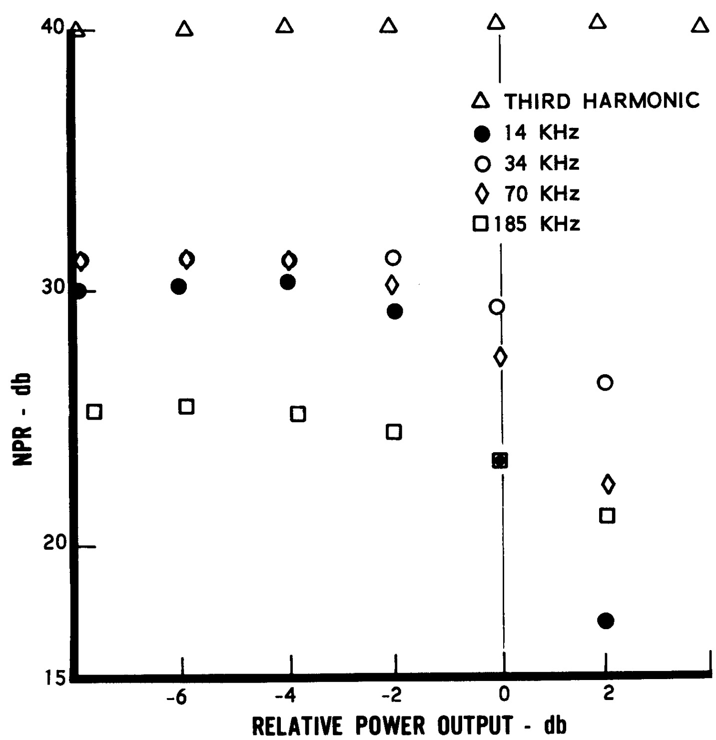

FIGURE 5 NPR TEST DATA AS A FUNCTION OF OUTPUT POWER WITH INPUT POWER CONSTANT

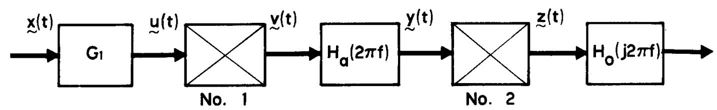

FIGURE 6 PROPOSED BLOCK DIAGRAM FOR EXPLAINING THE DATA OF FIGURE 4

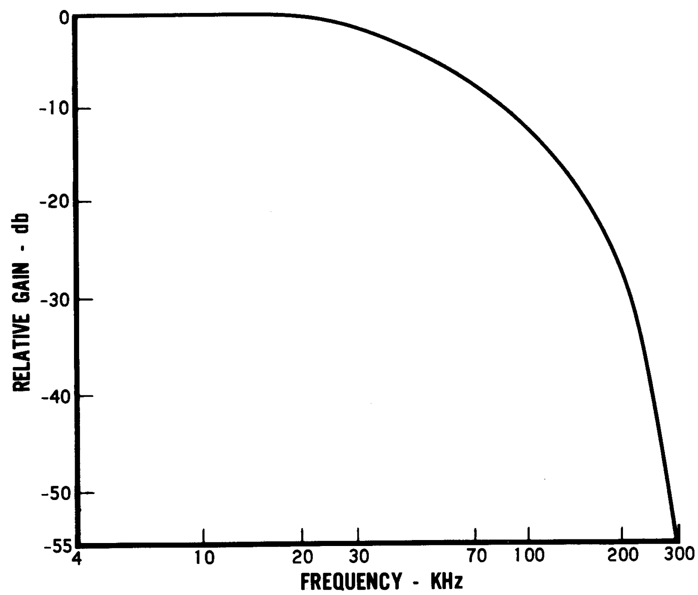

FIGURE 7 ROLL-OFF CHARACTERISTIC OF RECORD/REPRCDUCE PROCESS

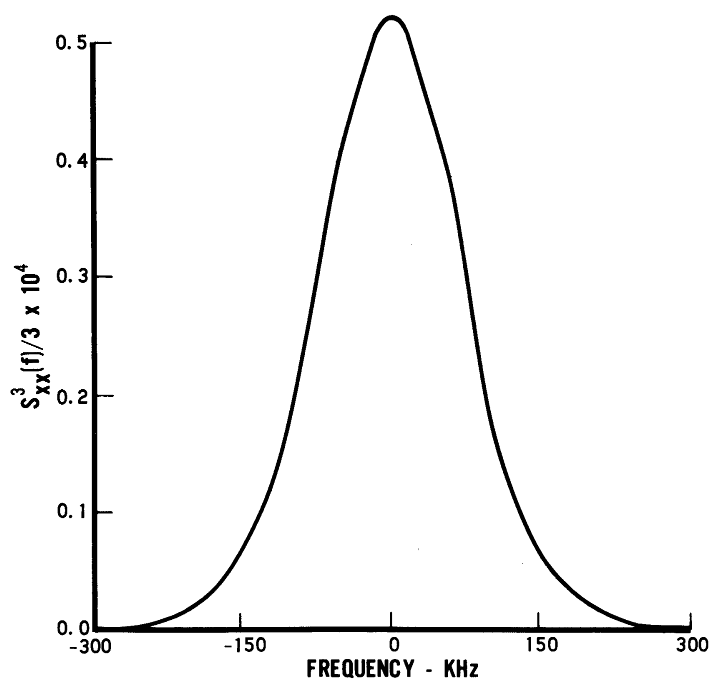

FIGURE 8 SECOND CONVOLUTION OF $S_{xx}(f)$

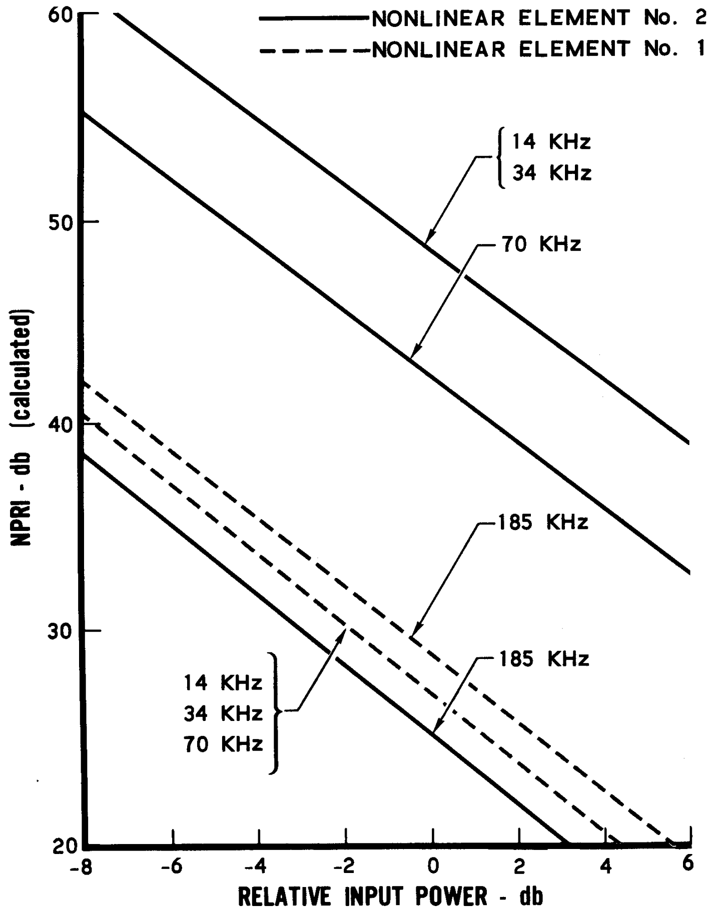

FIGURE 9 CALCULATED NPRI