COMPARSION OF PHASE TRACKING SCHEMES FOR PSK

CHARLES R. CAHN
Magnavox Research Laboratories
Torrance, California

Summary Based on straightforward approximations to evaluate the error rate with phase jitter in the reconstituted carrier and phase slips in the tracking loop, comparison is made of three schemes for biphase and quadriphase data transmission:

1. Track phase on carrier component,

2. Track on suppressed carrier - use differentially encoded PSK,

3. Track on suppressed carrier - retain a carrier component to resolve the inherent ambiguity.

For operation at relatively high P_E, as with error correction, schemes 1 and 3 both avoid the inherent degradation of differential encoding; however, both require a relatively narrow band phase tracking loop. Scheme 3, suppressed carrier tracking with ambiguity resolution, achieves the minimum E_b/N_o degradation at high P_E when the loop bandwidth parameter is held fixed by system requirements.

Introduction When transmitting data by symmetrical phase shift keying (PSK), the receiver must reconstitute the carrier which has been suppressed by the modulation. This can be done from the PSK signal itself, in which case there is an inherent ambiguity to be resolved, or alternatively an unmodulated carrier component can be transmitted, thereby degrading the achieveable E_b/N_o performance. It is presumed that carrier phase is derived from a tracking loop of finite bandwidth set by system dynamics; hence, the reference carrier reconstituted in the presence of additive noise is noisy. In addition, phase ambiguity due to slips in the tracking loop must be accounted for.

As candidate schemes for deriving a phase reference for PSK demodulation, one can consider the following basic choices:1

1. Track phase on carrier component only;

2. Track on suppressed carrier - use differentially encoded data;

3. Track on suppressed carrier - retain a carrier component to resolve the inherent phase ambiguity associated with suppressed carrier tracking.

For comparison purposes, the error rate performance in white Gaussian noise is desired for each approach, based on straightforward approximations to the distributions of phase error and phase slips. Both biphase and quadriphase are considered, and the problem of operation at high error rates is of particular interest for error-correction coding.

Tracking on Carrier Component With an unmodulated carrier component assume a slowly changing phase error over the duration of one binary digit. To summarize the analysis[2], let the steady state probability density of phase error be $p(\emptyset;\alpha)$, where α is the loop signal-to-noise ratio defined by

$$\alpha = \frac{C}{N_o B_L} \qquad (1)$$

In (1), C is the power in the unmodulated carrier component, B_L is the (one-sided) loop noise bandwidth, and N_o is the received noise density (one-sided). Then, the average probability of bit error for biphase is

$$P_E(\emptyset) = \int_{-\pi}^{\pi} p(\emptyset;\alpha) \left[1 - \Phi\left(\sqrt{2ST_b/N_o}\cos\emptyset\right)\right] d\emptyset \qquad (2)$$

where S is the power in the data modulated component, T_b is the bit duration, and $\Phi(x)$ is the Gaussian probability distribution function.

The phase lock loop is presumed to have the steady state distribution[3]

$$p(\emptyset;\alpha) = \frac{\exp(\alpha\cos\emptyset)}{2\pi I_o(\alpha)}\,;\ \left|\emptyset\right| \leq \pi \qquad (3)$$

Then (2) may be numerically evaluated after substitution of (3) with the result given in Figure 1. It may be observed that the question of loss of lock does not arise.

Figure 1 can be applied to give the performance degradation for any specified power sharing between the carrier and the data component with $B_L T_b$ as a parameter, by defining the effective E_b/N_o

$$\frac{E_b}{N_o} = \frac{(C+S)T_b}{N_o} = \alpha\,(B_L T_b) + \frac{S\,T_b}{N_o} \qquad (4)$$

At relatively high values of P_E, in the vicinity of 0.05 as would be encountered with error correction coding, Figure 1 shows that a is required to be at least 10 db.

With quadriphase, the signal with power $S = 2\ S_b$ in the data component can be viewed simply as the combination of two independent biphase modulations each with bit duration T_b. The quadriphase digit duration, thus, is T_b also. The average probability of bit error, in similarity with (2), is

$$P_E = \int_{-\pi}^{\pi} p(\emptyset;\alpha) \left[1 - \frac{\Phi\left(\sqrt{2ST_b/N_o}\ \cos\left(\frac{\pi}{4} + \emptyset\right)\right) + \Phi\left(\sqrt{2ST_b/N_o}\ \cos\left(\frac{\pi}{4} - \emptyset\right)\right)}{2}\right] d\emptyset \qquad (5)$$

where $p(\emptyset;\alpha)$ is given by (3), and by (1).

Some results of numerical integration of (5) are shown in Figure 2. The conclusions are basically similar to those found for biphase; namely, if α exceeds a given value, the performance is close to coherent PSK. For operation at relatively low values of S_bT_b/N_o, a should exceed 16 db, and even higher α would be needed with larger S_bT_b/N_o.

Tracking on Suppressed Carrier With Differentially Encoded PSK A Costas loop, or its equivalent the squaring loop, can be utilized to track a completely suppressed carrier with symmetrical sidebands as with biphase. The phase error can be assumed to be the same as for an ordinary carrier lock loop provided that an adjustment is made in signal-to-noise ratio.[4] Let W denote the low pass bandwidth due to modulation which must be passed by the filter in the Costas or squaring loop. Taking the filter as a rectangular pass band, the loop signal-to-noise ratio

$$\alpha = \frac{S}{4\ N_o B_L\ [1 + N_o W/S]} \qquad (6)$$

and (6) may be introduced into (3) to give the distribution of phase error in the second harmonic of the carrier. When the second harmonic is divided to recover the original carrier, a phase ambiguity of 180° is introduced in the reconstituted carrier, and the effect of this ambiguity must be recognized. For differentially encoded PSK, the conditional probability of error is

$$P_E(\emptyset)\Big]_{\substack{\text{Diff.}\\ \text{Encode}}} = 2\ \Phi\left(\sqrt{2S\ T_b/N_o}\ \cos\ \frac{\emptyset}{2}\right)\left[1 - \Phi\left(\sqrt{2S\ T_b/N_o}\ \cos\ \frac{\emptyset}{2}\right)\right] \qquad (7)$$

where $\emptyset$ is the phase error in the second harmonic, and this expression is invariant to the 180° phase ambiguity. The average probability of error due to phase jitter becomes

$$P_E\Big]_{\text{phase jitter}} = \int_{-\pi}^{\pi} p(\emptyset;\alpha)\ P_E(\emptyset)\Big]_{\substack{\text{Diff.}\\ \text{Encode}}} d\emptyset \qquad (8)$$

where α is given by (6) and $p(\emptyset;\alpha)$ by (3).

Figure 3 is a numerical evaluation of (8) with $W = 1/2T_b$ in (6), presuming a matched filter prior to the squaring process. It is seen that with $B_LT_b \leq 0.1$, performance is very close to that with perfect tracking even at high probability of error values such as $P_E = .05$. Because of the differential encoding, however, there is a decibel gap relative to coherent PSK, which represents an inherent loss even though no power is wasted in a carrier component.

Equation (8) does not include errors that must arise during a phase slip because of the change of polarity at some point during the phase transient. As a reasonable approximation to obtain the total error rate, we may assume one bit error whenever a phase slip occurs, to be added to (8) which expresses the probability of error due to phase jitter. Applying the rate of phase slips computed for a first order phase lock loop[3] to the squaring loop so that a is given by (6), the average probability of error due to phase slips is

$$P_E\Big]_{\text{phase slips}} = \frac{2}{\pi^2} (B_LT_b) / \alpha I_o^2(\alpha) \tag{9\_}$$

Equation (9) is plotted in Figure 3 with $W = 1/2T_b$ to show the relative importance of the errors caused by phase slips with biphase data.

It can be seen that with $B_LT_b<0.25$, the effect of phase slips is negligible. Since $B_LT_b < 0.1$ has already been specified to approach perfect tracking, the additional requirement of having a negligible probability of phase slips does not impose a more stringent restriction on the tracking bandwidth despite the fact that (6) displays a minimum of 6 db degradation with respect to rate of phase slips compared with a standard phase lock loop receiving the same power.

Suppressed carrier tracking for quadriphase PSK may be viewed in concept as a "quadrupling" loop that tracks the fourth harmonic of the signal. (Decision feedback implementations similar to the Costas for biphase are possible.) There is an inherent phase ambiguity of 0, 90, 180°, or 270° in this method of tracking. As a good approximation, we may assume that the loop signal-to-noise ratio is

$$\alpha \cong \frac{S}{16\, N_oB_L} = \frac{S_b}{8\, N_oB_L} \tag{10}$$

and simply ignore the further suppression of quadrupling at low ST_b/N_o . In similarity with biphase, performance approaching that for perfect tracking is achieved by reducing the B_LT_b product. Although computations corresponding to these previously carried out for the biphase case have not been made, similar conclusions are anticipated on the maximum allowed B_L,T_b product, except that the values will have to be smaller to keep down the rate of-phase slips. As the phase error becomes negligible, the same inherent degradation due to the differential encoding arises with quadriphase as previously found for biphase.

Comparison of (10) with (6) shows that when biphase and quadriphase are to be compared at equal total bit rates, the requisite loop bandwidth of the latter must be narrowed by a factor of about 4 to yield equal rates of phase slips. Equation (10) indicates that with respect to phase slips, the quadrupling loop is approximately 12 db degraded relative to a standard phase lock loop receiving the same power.

Tracking on Suppressed Carrier With Ambiguity Resolution by a Carrier Component A third alternative is to track the suppressed carrier, but in addition provide a small amount of unmodulated carrier component for resolution of the phase ambiguity using the resultant polarity of the detected amplitude. In order that the polarity decision be essentially error free, the integration duration must be sufficiently long. As a consequence, the duration of ambiguity resolution necessarily extends over a finite number of digits, and an ambiguity resolution error causes a burst of errors.

The analysis of this tracking scheme is not straightforward because it requires introducing the effects of the error extension produced by the finite duration of the ambiguity resolution process and the consequent recovery time after a phase slip. As an approximation, one can identify three essentially independent contributions to bit errors:

1. Decision errors due to noise,
2. Incorrect ambiguity resolution due to noise,
3. Incorrect ambiguity resolution during recovery time after a phase slip in the tracking loop.

Of these contributions, the second presumably can be made negligible by lengthening the integration duration, at the cost of greater error extension due to a longer recovery time after a phase slip has occurred. The contribution of decision errors for biphase is

$$P_E\Big]_{\substack{\text{No}\\ \text{Ambiguity}\\ \text{Errors}}} = \int_{-\pi}^{\pi} p(\emptyset;\alpha)\left[1 - \Phi\left(\sqrt{2ST_b/N_o}\cos\frac{\emptyset}{2}\right)\right]d\emptyset \qquad (11)$$

where $p(\emptyset;\alpha)$ is given by (3) and a by (6). (Note the difference between (11) and (2)). Figure 4 plots (11) for various values of B_LT_b with $W = 1/2T_b$. It is observed that performance close to coherent PSK is obtained for $B_L T_b < 0.1$. However, this does not include errors due to incorrect ambiguity resolution.* Thus, for $B_L T_b < 0.1$, performance degradation will be due almost entirely to errors introduced by incorrect ambiguity resolution immediately following each phase slip.

An approximate evaluation of the additional errors resulting from incorrect ambiguity resolution after phase slips can be made by assuming a simple numerical factor for the error extension of the ambiguity resolution process. For instance, if the amplitude integration extends over 30 digits, the error extension numerical factor will be roughly 30, to be applied to the frequency of phase slip occurrences given by (9). Let T_{amb} equal the average duration of incorrect ambiguity resolution after a phase slip. Then, the average probability of error due to phase slips may be approximated as

$$P_E\Big]_{\text{phase slips}} \cong \frac{2}{\pi^2} (B_L T_b)(T_{amb}/T_b)/\alpha I_o^2(\alpha) \tag{12}$$

Equation (12) is plotted in Figure 4 (again with $W = 1/2T_b$).

In view of the approximate nature of the analysis, only qualitative conclusions will be drawn. To avoid excessive phase jitter effects, $B_L T_b$ must be less than 0.1. Then, at some point the probability of error will suddenly start to increase sharply above the coherent PSK curve as E_b/N_o is decreased. The value of E_b/N_o at which this phenomenon takes place depends on the loop bandwidth and the error extension characteristics of the ambiguity resolution process. For biphase, a reasonable requirement is $B_L T_b <. 075$ in order to guarantee that operation will be close to coherent PSK at $P_E = .05$ provided that the method of ambiguity resolution does not extend the error over more than several hundred digits.

It may be noted that a carrier component of 5 percent will degrade E_b/N_o by 0.2 db due to the wasted power. For this fraction the ambiguity resolution error will be in the vicinity of 10^{-3} at $E_b/N_o = 0$ db if $T_{amb}/T_b = 100$, since the E_b/N_o value for the polarity decision is then 0 db + 10 $\log_{10}$ (0.05 x 100) = 7 db. Thus, the contribution of incorrect ambiguity resolution is negligible, as desired.

With suppressed carrier tracking of quadriphase, similar considerations apply as with biphase except that (10) is now to be utilized for loop signal-to-noise ratio instead of (6). This indicates need to reduce $B_L T_b$ by roughly a factor of 2 to maintain the same probability of error due to ptase slips as shown in Figure 4; otherwise, the behavior is

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* This behavior may be viewed as the limiting performance of PSK with suppressed carrier tracking when a “genie” correctly resolves the ambiguity for each digit.

similar. (This means a factor of 4 if biphase and quadriphase are compared at equal total bit rates.)

In similarity with (11), the probability of bit error ignoring ambiguity resolution errors may be expressed as

$$P_{E_{\substack{\text{No}\\ \text{Ambiguity}\\ \text{Errors}}}} = \int_{-\pi}^{\pi} p(\emptyset;\alpha)\left\{1 - \frac{\Phi\left(\sqrt{2S\,T_b/N_o}\,\cos\left(\frac{\pi}{4}+\frac{\emptyset}{4}\right)\right) + \Phi\left(\sqrt{2S\,T_b/N_o}\,\cos\left(\frac{\pi}{4}-\frac{\emptyset}{4}\right)\right)}{2}\right\} d\emptyset \qquad (13)$$

where $p(\emptyset;\alpha)$ is given by (3) and α by (10). The contribution to probability of error due to phase slips is given by (12) with a from (10). Computed behavior is presented in Figure 5. It is seen that values of $B_L T_b < .05$ are necessary to approach coherent quadriphase performance closely, considering both the steady state distribution of phase error in the loop and thin effect of phase slips with error extension typically over a duration of several hundred digits.

Conclusions An analytical approach towards comparison of carrier tracking and suppressed carrier tracking for PSK systems has been described, including effects of phase slips in tracking. The broad goal may be described as achieving E_b/N_o performance close to theoretical coherent PSK, still keeping tracking loop bandwidths as wide as possible.

At high E_b/N_o, the simple expedient of differential encoding may be applied to remove the phase ambiguity of suppressed carrier tracking, with a relatively small degradation. The advantage is that loss due to extra power in an unmodulated carrier component is avoided. However, at low E_b/N_o, or high probability of error where a data transmission system must operate with error correction, differential encoding has the disadvantage that the inherent E_b/N_o degradation is substantial. Furthermore, additional degradation is incurred unless the tracking loop is sufficiently narrow banded; for example, $B_L T_b < 0\text{- }1$ is required for biphase, and in the order of $B_L T_b < 0.\ 05$ for quadriphase. To avoid the inherent degradation of differential encoded PSK, consideration of an uninodulated carrier component to resolve the phase ambiguity of suppressed carrier tracking is appropriate at low E_b/N_o operations

The comparison between tracking on the carrier component versus suppressed carrier tracking with the carrier component used for ambiguity resolution leads to the conclusion that a small degradation in E_b/N^o can be achieved only with a narrow tracking bandwidth for either scheme. For tracking on the carrier component, the behavior is simple to understand. The loop signal-to-noise ratio must exceed a value of approximately 10 db for biphase and 16 db for quadriphase; hence, the maximum allowed loop bandwidth is proportional to the fraction of power allocated to the carrier component. This fraction must be low to have a small degradation. For suppressed carrier tracking with ambiguity resolution, the new problem is introduced of bursts of errors whenever a phase slip occurs in the tracking loop. Hence, the loop bandwidth must be narrowed to satisfy both; (1) small degradation in E_bN_o due to phase jitter in the tracking loop; and (2) sufficiently low rate of phase slips so that the number of errors arising from phase slips is smaller than due to noise in the demodulation process itself. For $P_E < .05$, it appears that $B_LT_b < 0.075$ should suffice for biphase and $B_LT_b < .05$ for quadriphase unless the error bursts due to phase slips extend over Durations greater than several hundred digits. This duration is set by the further requirement that the ambiguity resolution has a low probability of being incorrect. At equal total bit rates, quadriphase thus requires a tracking bandwidth roughly a third of the value for biphase.

A quantitative comparison of carrier tracking and suppressed carrier tracking depends on the probability of error. As an illustration, assume $E_b/N_o = 0$ db is the design operating point. Based on the minimum loop signal-to-noise ratios required with carrier tracking and assuming the loop bandwidths derived for suppressed carrier tracking are used for both schemes, it is found that carrier tracking demands a carrier component which is at least 75 percent of the data component with biphase and equal to the data component for quadriphase. Thus, the superiority of suppressed carrier tracking is apparent when the system design calls for a small degradation in E_b/N_o.

On the other hand, it may be noted that phase slips can introduce effects other than bit errors in a given system. One example is where carrier phase is being used for a Doppler measurement and it is important not to lose count of carrier cycles due to a phase slip. In view of the inherent 6 db degradation for biphase and 12 db for quadriphase, it is seen that when a carrier component is provided which exceeds 25 percent for biphase and 6.25 percent for quadriphase, the rate of phase slips actually will be lower for tracking on the unmodulated carrier component than for suppressed carrier tracking assuming the same loop noise bandwidth and the same total signal power (carrier component plus data modulation component). This has not generally been appreciated, and is a direct consequence of the phase ambiguity of suppressed carrier tracking, rather than a noise penalty due to a squaring or quadrupling.

References

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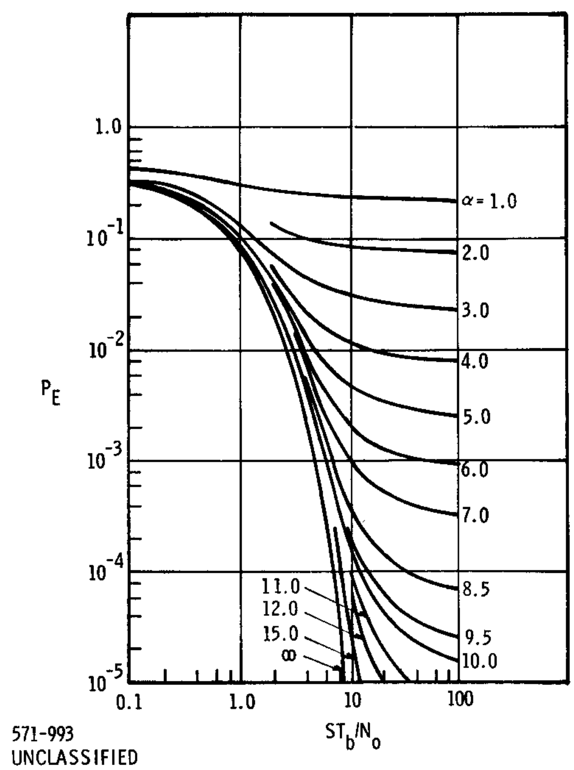

Figure 1. Biphase Performance With Carrier Tracking

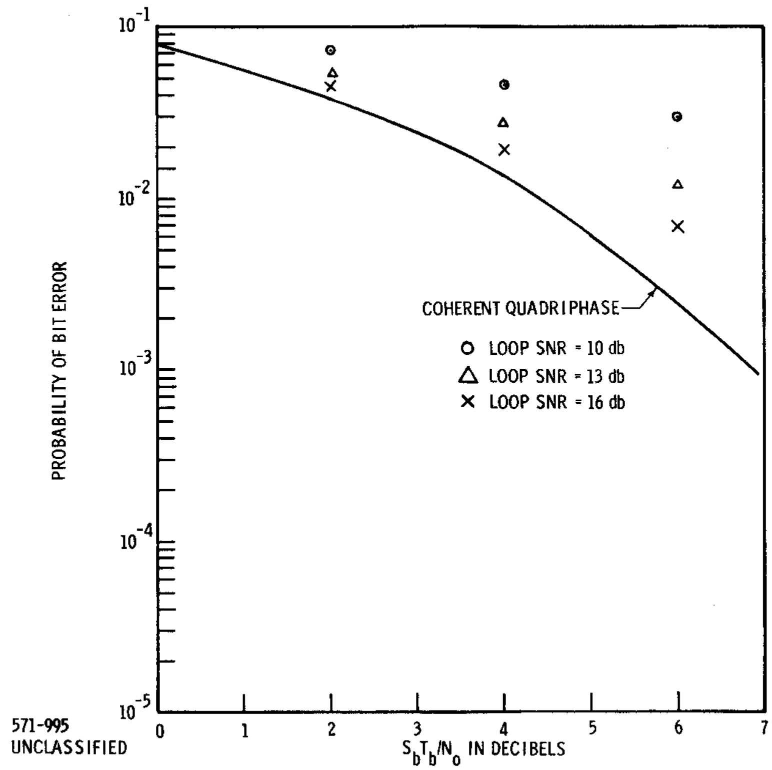

Figure 2. Quadriphase Performance With Carrier Tracking

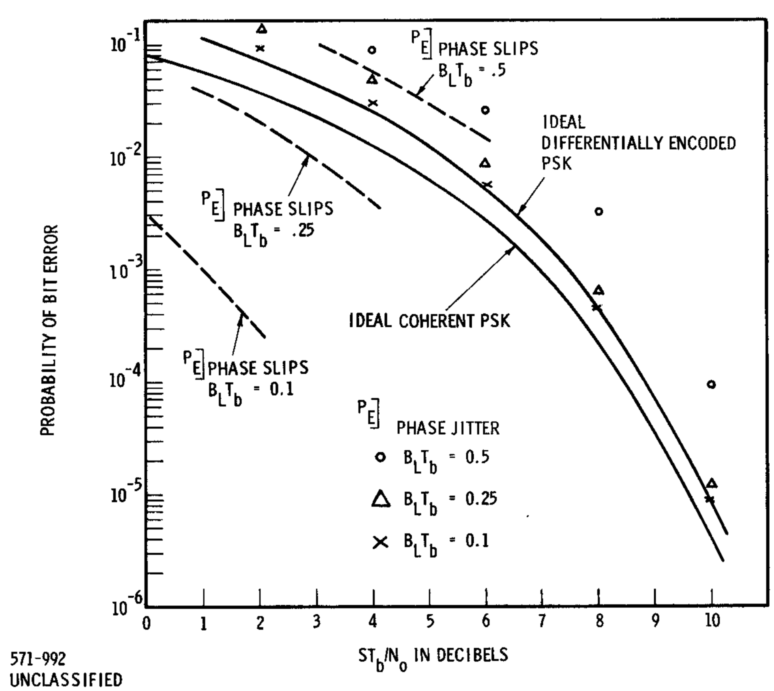

Figure 3. Differentially Encoded Biphase Performance with Suppressed Carrier Trackin

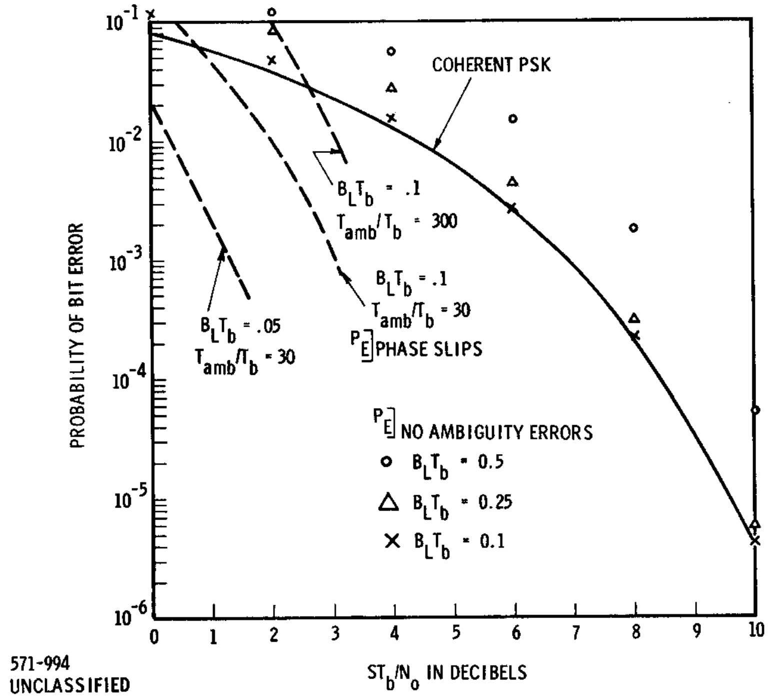

Figure 4. Biphase Performance with Suppressed Carrier Tracking - Ambiguity Resolution on Carrier Component

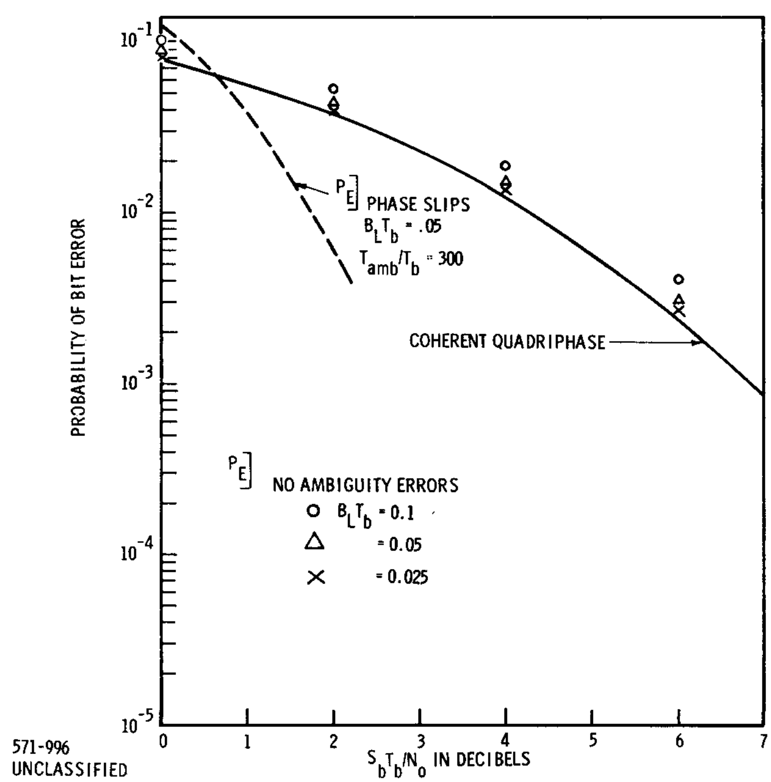

Figure 5. Quadriphase Performance With Suppressed Carrier Tracking - Ambiguity Resolution on Carrier Component