L-/S-BAND CALIBRATION ERROR ANALYSIS

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Summary A statistical error analysis is performed to determine the degree of uncertainty encountered when calibrating steerable receiving antennas with the solar calibration method. The analysis considers the propagation of precision error indexes.

It is shown that a worst-case one-sigma (1σ) uncertainty of ±0.8 dB in system noise temperature occurs for a solar calibration at L-band. Somewhat better precision can be achieved by monitoring the antenna gain-to-noise temperature (G/T) ratio at a station; a worst-case uncertainty of ±0.5 dB (1σ) can be realized.

An error analysis is made of a method to determine absolute antenna gain based upon solar flux density. The uncertainty in this type measurement is ±0.7 dB (1σ) at L- and S-band frequencies.

Introduction It is desirable to determine the basic antenna calibration accuracy that can be achieved for the solar calibration method. The solar calibration method, described by Hedeman[1] , references receiving system noise temperature at the IRIG L- and S-band telemetering frequencies to solar flux density. A daily prediction during "quiet-sun" periods insures a constant solar flux density necessary for accurate calibration. On the other hand, the stellar-calibration method[2,3] references effective antenna gain, or system noise temperature, to the constant flux density from a radio star such as Cassiopeia A or Cygnus A. Since stellar flux density is known accurately from numerous radio astronomy observations, no predictions are needed.

A brief comparison of the solar and stellar calibration methods will first be made; a statistical error analysis of the solar calibration method then follows. Worst-case one-sigma (1σ) values have been assigned to parameters.

A method is described for determining absolute antenna gain, using solar flux density, at L- and S-band. A statistical error analysis is also made for this method. Finally, an error analysis determines the 1σ uncertainty for both receiving system noise temperature and antenna gain-to-system noise temperature (G/T) ratio.

Comparison of Solar and Stellar Calibration Methods

The noise flux density from the sun is suitable for determining system noise temperature in IRIG L- and S-band receiving-systems using parabolic dish antennas ranging from 6-ft (1.8-m) to 60-ft (18-m) in diameter (Ref. 1). On the other hand, the noise flux density from Cassiopeia A or Cygnus A is sufficient to calibrate L- and S-band receiving systems employing dish antennas as small as 20-ft (6-m) in diameter (Ref. 2). There is no fundamental limit on maximum antenna size, except that the Ko[4] beamwidth correction factor must be applied to stellar calibration for antennas larger than 85-ft (26-m) in diameter, operating at S-band and above. When using the sun, a beamwidth correction factor is required for antennas larger than 30-ft (9-m) in diameter, operating at 1440 MHz and above.

The stellar calibration method has a decided advantage at 136 MHz because the sun is not a stable source at VHF since the solar flux density can vary as much as 25 dB. Antennas operating at 136 MHz, with isotropic gains as low as 16 dBi (dB above isotropic), have been calibrated in the NASA[3] space tracking and data acquisition network (STADAN) using Cas A and Cyg A.

The solar-noise flux density provides higher “on-source” receiver IF signal power than the weaker stellar-noise flux density from a radio star. Shifting the antenna mainlobe onto the sun during a solar calibration results in a typical receiver IF power change of 10 dB, whereas less than a 1 dB change in IF power can result for stellar calibration. Consequently, it is necessary to amplify the weaker signal from the stellar source with 25 dB to 50 dB of post detection gain followed immediately by a smoothing filter with a large RC time constant of $RC \geq 0.3$s.

Finally, the required signal dynamic range for the RF receiver is different for the solar and stellar calibration methods. For an accurate solar calibration, the receiver’s dynamic range should be linear within ±0.5 dB for an input RF signal change that typically ranges from 0 dB up to 10 or 20 dB. On the other hand, the stellar calibration method needs a receiver dynamic range that is linear only from 0 dB up to about 3 dB. In the latter instance, linearity is defined as a linear relationship existing between the input RF signal power and the output dc voltage from the square-law detector.

Statistical Error Analysis of Solar Calibration Method

It is desirable to determine the basic uncertainty in the receiving system noise temperature for a solar calibration. The system noise temperature, T_{sys} , for an L- or S-band IRIG receiving system (see Figure 1), is defined by Hedeman[1] as

$$T_{sys} = \frac{F G \lambda^2}{8 \pi k L} \cdot \frac{1}{\left(\frac{P_2}{P_1} - 1\right)}, \ K \tag{1}$$

where

F = “quiet-sun” solar flux densit , randomly polarized, for a single receiving polarization, w m^{-2} T Hz^{-1}.

G = absolute antenna power gain, above isotropic, referenced to antenna output terminals (see Figure 1).

λ = free-space wavelength at same frequency G is measured, m

k = Boltzmann’s constant = 1.38 X 10^{-23} J/K

L = antenna mainlobe correction factor $\cong 0.38\,(\theta_s/\theta_{HP})^2$ for
$(\theta_s/\theta_{HP}) < 1$,
θ_s = radio diameter of sun $\cong 0.5°$,
θ_{HP} = half-power bearnwidth of mainlobe

P_2 = receiver IF output power for mainlobe “on sun” (see Figure 1), watt, w

P_1 = receiver IF output power with mainlobe on “cold sky” for the same system gain as P_2, w.

All of the variables in (1) influence to a degree the standard-deviation (1σ) error in the value of T_{sys} . For this analysis, the following assumptions are made:

- Cosmic, atmospheric, and environmental noise are constant between “’off-sun” and “on-sun” observations
- Receiving system gain remains constant for above two observations
- Solar flux density is known within ± 7 percent (1σ)
- RF attenuation of atmosphere is negligible; although clear weather is assumed, solar observations are essentially independent of weather conditions at L- and S-band frequencies.

- Multipath and ionospheric effects are negligible; lower limb of solar disk is at an elevation angle $\geq 30°$
- Degree of linear polarization in solar flux density is negligible
- Assumed mean and worst-case 1σ values of F, and P_2/P_1 are:

  $F = 100 \times 10^{-22}$ w m^{-2} Hz^{-1} $\sigma_F = \pm 7$ percent

  $P_2/P_1 = P = 10$ dB, $\sigma_p = \pm 10$ percent (in power).

Redefining (1) as

$$T_{sys} = \frac{FG}{aL} \cdot \frac{1}{(P-1)}, \ K \qquad (2)$$

where $a = 8\pi^{\lambda-2}$ = constant, and $P = P_2/P_1$.

When the component quantities are independent, the rnean-squared standard deviation in system noise temperature, $\sigma^2_{T_{sys}}$, can be obtained from the general expression in Worthing and Gaffner[5] as

$$\sigma^2_{T_{sys}} = \left(\frac{\partial T_{sys}}{\partial F}\right)^2 \sigma^2_F + \left(\frac{\partial T_{sys}}{\partial G}\right)^2 \sigma^2_G + \left(\frac{\partial T_{sys}}{\partial L}\right)^2 \sigma^2_L + \left(\frac{\partial T_{sys}}{\partial P}\right)^2 \sigma^2_P . \qquad (3)$$

where σ represents the one-sigma uncertainty in each variable, and F, G, L and P are mean values.

Taking partial derivations in (3) gives,

$$\frac{\partial T_{sys}}{\partial F} = \frac{T_{sys}}{F}, \ \frac{\partial T_{sys}}{\partial G} = \frac{T_{sys}}{G}, \ \frac{\partial T_{sys}}{\partial L} = -\frac{T_{sys}}{L},$$

and

$$\frac{\partial T_{sys}}{\partial P} = -\frac{T_{sys}}{P-1}.$$

The minus signs can be ignored since the above terms will be squared.

Substituting the above partial derivatives in (3) gives

$$\left[\frac{\sigma_{T_{sys}}}{T_{sys}}\right]^2 = \left(\frac{\sigma_F}{F}\right)^2 + \left(\frac{\sigma_G}{G}\right)^2 + \left(\frac{\sigma_L}{L}\right)^2 + \left(\frac{\sigma_p}{P-1}\right)^2 . \qquad (4)$$

The significance of (4) is described as follows by means of a sample calculation. Typical values are assumed.

L-Band Case (Solar Calibration):

Type antenna	20-ft (6-m) diameter parabolic dish
Frequency	1440 MHz (λ = 0.208 M)
Solar flux density (quiet sun)	$F \pm \sigma_F = (100 \pm 7) \times 10^{-22}$ w m^{-2} Hz^{-1}
Antenna gain	$G \pm \sigma_G$ = 5000 ± 700 power gain = 37 dBi ± 0.6 dB.
Mainlobe correction factor	= 1.0. Ignore $\sigma_L < \pm 2\%$, for $(\theta_s/\theta_{HP}$ 16HP) <1.
Measured receiver IF power ratio; "on-sun" to "off-sun"	$P \pm \sigma_p$ = 10.0 ± 1.0 power ratio.

Substituting the above values in (4):

$$\left[\frac{\sigma_{T_{sys}}}{T_{sys}}\right]^2 = \left(\frac{7}{100}\right)^2 + \left(\frac{700}{5000}\right)^2 + \left(\frac{1.0}{10-1}\right)^2 .$$

$$= \underbrace{(0.07)^2}_{7\%} + \underbrace{(0.14)^2}_{14\%} + \underbrace{(0.11)^2}_{11\%}$$

$$(\sigma_{T_{sys}}/T_{sys}) = \pm\sqrt{0.037} = \pm 0.19 = \pm 19 \text{ percent}.$$

or

$$\sigma_{T_{sys}} = \pm 0.19\, T_{sys} = T_{sys} \pm 0.8 \text{ dB } (1\sigma \text{ worst case}).$$

Antenna Gain Determination Using Solar Flux Density

By combining certain features of the solar and stellar calibration methods, it is possible to determine accurately absolute antenna gain using solar flux density at the L- and S-band frequencies. Such a means of determining antenna gain in a receiving system augments the stellar 2 , 3 method for determining absolute antenna gain.

By rearranging (1), the absolute antenna gain, G, referenced to the antenna output terminals, is defined as

$$G = \frac{aL}{F} \cdot (P - 1)\ [T_{sys}]. \tag{5}$$

where

$$a = 8\pi k \lambda^{-2} = \text{constant}$$

and

$$P = P_2/P_1 .$$

Normally, during a solar calibration, the receiver's IF output power ratio, P, is measured which is in turn used for computing the receiving system noise temperature, T_{sys} , from (1). However, if T_{sys} could be determined by an independent means, a determination of absolute antenna gain G, can then be made directly from (5).

It can be shown that the receiving system temperature, T_{sys} , for the off-sun" position of the mainlobe, is

$$T_{sys} = \epsilon(T_A - T_0) + T_0 + T_R,\ K$$

where

$T_A = \sum_{1}^{n} T_{ant}$ = total antenna noise temperature for n noise sources, K

ϵ = antenna-to-preamplifier transmission line power loss, $0 < \epsilon \lesssim 1$

T_0 = physical temperature of antenna-to-preamplifier transmission line, K

and

$T_R = \sum_{1}^{N} \frac{T_i}{\pi_1^i G_j}$ = receiver noise temperature, referenced to preamplifier input terminals, for N cascaded networks, K.

Assuming that the atmospheric noise temperature and atmospheric RF absorption are both negligible at a sun elevation angle $\theta \geq 30°$, the largest single contribution to the system noise temperature in (6), at L-band and above, is generally T_R, the overall preamplifier-receiver noise temperature.

The parameters in (6) can be obtained as follows. The required value for T_R can be obtained by means of a noise figure measurement referenced directly to the preamplifier input terminals (see Figure 1). Both T_R and the transmission-line cable loss, ϵ , could be measured whenever the receiving system is maintained, repaired or overhauled. The ambient physical temperature, T_0 , of the transmission line can be measured with a bulb thermometer or thermocouple. A measurement precision of ±10 percent (1σ) for the mean values of T_R , T_0 and ϵ is sufficiently accurate. Having thus obtained measured values of T_R , T_0 and ϵ , it remains to make a determination of the total antenna noise temperature, T_A , before the total receiving system noise temperature can be computed from (6).

Since the antenna mainlobe is pointing toward a "cold sky" region in the "off-sun" position, the background brightness temperature within the mainlobe is low, ranging from 3K to 5K at the L- and S-band frequencies. The major contribution to T_A therefore results from the antenna's side lobes and back lobe.

In the ITC-71 paper: "VHF/UHF Stellar Calibration Error Analysis," it is shown that T_A can be expressed for an antenna feed with a Gaussian power pattern, including contributions from both the mainlobe and secondar lobes, as

$$T_A \simeq 0.82\, T_{sky} + 0.13\, (\overline{T}_{sky} + T_E),\ K \qquad (7\_$$

where

T_{sky} = Mean value of sky-brightness temperature, within half-power beamwidth of mainlobe, for "off-sun" position, K

$\overline{T}_{sky}$ = mean value of sky-brightness temperature within secondary antenna lobes, K

and

T_E = effective noise temperature of the earth, K.

Appropriate values for T_{sky} and $\overline{T}_{sky}$ can be obtained directly from a radio-sky map of galactic noise brightness temperatures; normally, T_E is in the vicinity of $T_E \simeq 290K$, over land areas. The value of T_E is less over water, and in general, T_E has been defined by Blake[6] as

$$T_E = T_p\,(1 - R) \qquad (7a)$$

where

T_p = thermal temperature of earth, K

and

R = reflectivity of the terrain viewed by the antenna.

Substituting the mean values of T_{sky} = 5K, $\overline{T}_{sky}$ = 4K, and T_E = 290K, the 1440 MHz antenna noise temperature, T_A , becomes from (7)

$$T_A \simeq 0.82(5) + 0.13\,(4 + 290) \simeq 40\ \mathrm{K}.$$

The uncertainty in the above value of T_A is due primarily to the uncertainty in the value of T_E ; Blake [6,7] indicates that the ground noise contribution to antenna temperature for minor-lobe levels varies from about 20 to 60K. This value reduces to 5 to 10K for Cassegrainian feeds, or antennas over water. The value σ_{T_A} = ±20K (±50 percent) is arbitrarily defined as a one-sigma value for the following statistical error analysis. Therefore, at L-band (1440 MHz)

$$T_A = 40 \pm \sigma_{T_A} = 40 \pm 20\ \mathrm{K}.$$

Combining (5) and (6) gives an overall expression for antenna gain as

$$G = \frac{8\pi kL}{\lambda^2 F}\left(\frac{P_2}{P_1} - 1\right)[\epsilon\,(T_A - T_0) + T_0 + T_R],\ \mathrm{K}. \qquad (8)$$

The significance of (8) is that absolute antenna gain can be determine accurately from a solar calibration. It is good practice to make at least ten such determinations, and then average the observations to obtain a mean value for G.

Applying the propagation of precision indexes to (8) results in the following expression that defines the uncertainty in the antenna gain as

$$\left(\frac{\sigma_G}{G}\right)^2 = \left(\frac{\sigma_F}{F}\right)^2 + \left(\frac{\sigma_L}{L}\right)^2 + \left(\frac{\sigma_P}{P-1}\right)^2 + \left(\frac{\sigma_{T\,sys}}{T_{sys}}\right)^2 \qquad (9)$$

where

$$\left(\frac{\sigma_{Tsys}}{T_{sys}}\right)^2 = \frac{1}{T_{sys}^2}\left[(T_A - T_0)^2\,\sigma_\epsilon^2 + (\epsilon\sigma_{T_A})^2 + (1-\epsilon)^2\sigma_{T_0}^2 + \sigma_{T_R}^2\right]$$

From the sample L-band calculation for a solar calibration, expressed earlier in this report, and again assuming unity mainlobe correction factor, L,

$$\frac{\sigma_F}{F} = \frac{7 \times 10^{-22}\ \mathrm{w\,m^{-2}\,Hz^{-1}}}{100 \times 10^{-22}\ \mathrm{w\,m^{-2}\,Hz^{-1}}} = 0.07\ (7\%)$$

and,

$$\frac{\sigma_P}{P-1} = \frac{1.0}{10 - 1} = 0.11\ (11\%).$$

From the ITC-71 paper: “VHF/UHF Stellar Calibration Error Analysis”:

$$\left(\frac{\sigma_{T_{sys}}}{T_{sys}}\right) = 0.100\ (10\%)$$

which assumes T_{sys} = 400 K.

Substituting the above values in (9) gives

$$\left(\frac{\sigma_G}{G}\right)^2 = (0.07)^2 + (0.11)^2 + (0.100)^2 = 0.027$$

$$\left(\frac{\sigma_G}{G}\right) = \pm 0.165 = \pm 16.5 \text{ per cent}$$

or

$$\sigma_G = \pm 0.165\ G.$$

For example, a 20-ft (6-m) diameter parabolic dish would have a mean value of antenna gain of about 5000, power gain above isotropic, at 1440 MHz. Expressed in dB above isotropic (dBi),

$$G \pm \sigma_G = 5000 \pm 0.165G = 5000 \pm 825 \text{ power ratio},$$

or

$$G = 37 \text{ dBi } \pm 0.7 \text{ dB } (1\sigma).$$

The assumed value of T_{sys} = 400 K is not critical, but the lower value of T_{sys} = 200K results in (σ_{Tsys}/T_{sys}) = 0.165 (16.5%),which makes a σ_G = ± 0. 8 dB.

An advantage is that (8) may be employed for gain-calibrating parabolic dish antennas, as small as 6-ft (1-8-m) in diameter, operating at Land S-band frequencies. By way of comparison, the minimum dish size is about 20-ft (6-m) for a comparable stellar-gain calibration.

Statistical Error Analysis of G/T_{sys} Measurement (Solar Calibration Method)

Having thus defined the basic error in receiving system noise temperature for the solar calibration method, it is of practical interest to determine the basic error in an antenna gain-to-system noise temperature (G/T_{sys}) ratio measurement. Such a G/T_{sys} determination does not require a knowledge of absolute antenna gain; in this sense, the G/T_{sys} ratio is a quality figure-of-merit that can be monitored daily at a station. Rearranging (5) gives G/T_{sys} as

$$\frac{G}{T_{sys}} = \frac{aL}{F} \cdot (P - 1). \tag{10}$$

where

$a = 8\pi k\lambda^{-2}$ = constant

k = Boltzmann's constant = 1.38×10^{-23} J/K

λ = free-space wavelength, m

$$P = \frac{P_2}{P_1} = \frac{\text{"on-sun" receiver IF power, w}}{\text{"off-sun" receiver IF power, w}}$$

L = antenna mainlobe correction factor.

Applying the propagation of precision indexes to (10) gives

$$\left[\frac{\sigma_{G/Tsys}}{G/T_{sys}}\right]^2 = \left(\frac{\sigma_F}{F}\right)^2 + \left(\frac{\sigma_L}{L}\right)^2 + \left(\frac{\sigma_P}{P-1}\right)^2 \tag{11}$$

for solar calibration.

The quantities on the right-hand side of (11) were determined earlier for the L-band case as:

$$\frac{\sigma_P}{P-1} = \frac{1.0}{10-1} = 0.11 \ (11\%)$$

again assuming unity mainlobe correction factor.

Substituting the above quantities in (11) gives

$$\left[\frac{\sigma_{G/Tsys}}{G/T_{sys}}\right] = \pm\left[(0.07)^2 + (0.11)^2\right]^{1/2}$$

$$= \pm\sqrt{0.0170} = \pm 0.13 = \pm 13 \text{ per cent } (1\sigma \text{ worst case}).$$

or

$$(\sigma_{G/Tsys}) = \pm 0.13 \ (G/T_{sys}) = \left(\frac{G}{T_{sys}}\right) \pm 0.5 \text{ dB } (1\sigma \text{ worst case}).$$

Since an uncertainty in antenna gain is not present in (11), assuming unity mainlobe correction factor, it is noteworthy that a determination of the G/T_{sys} ratio from (10) is more accurate basically than a comparable determination of the receiving system noise temperature, T_{sys} , from (1). For example, in the L-band case, a G/T_{sys} determination is six percent more accurate than a receiving noise temperature determination (e.g., 13% compared to 19% in power, respectively). Finally, it should be pointed out that a knowledge of the true design value of G/T_{sys} should be available with which to compare the station measurements.

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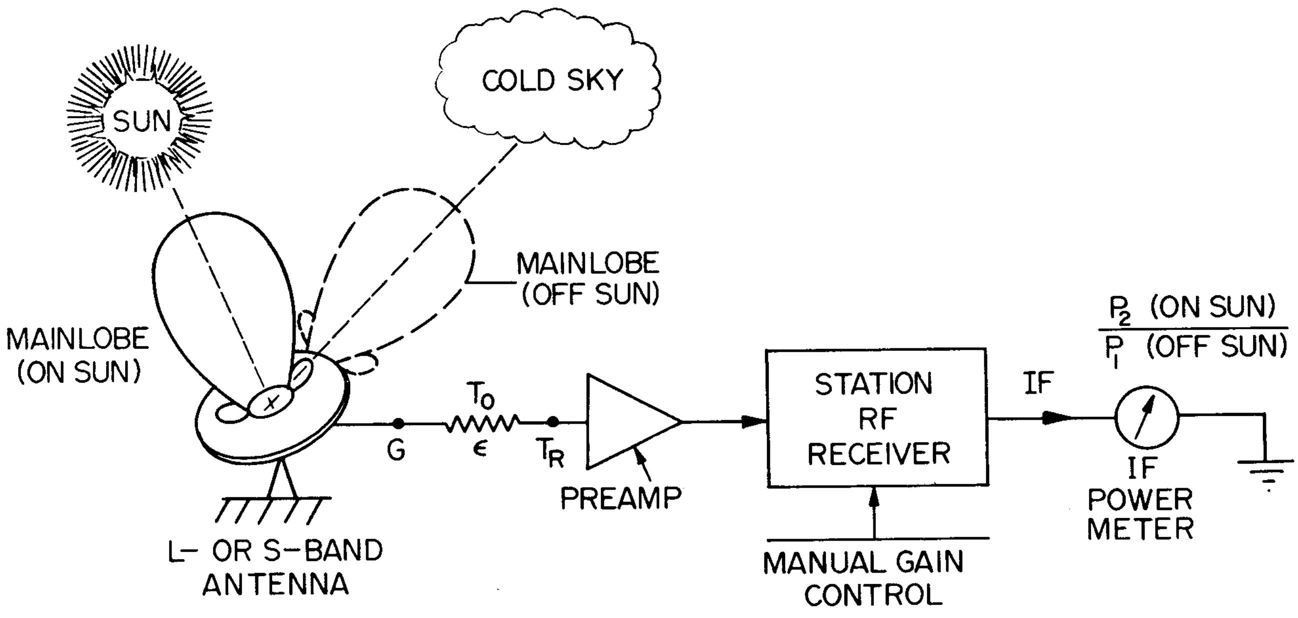

$$T_{sys} = \frac{FG\lambda^2}{8\pi kL} \cdot \frac{1}{\frac{P_2}{P_1} - 1}, K$$

Figure 1. System Noise Temperature Determination Using Solar Flux Density