A METHOD TO ENHANCE THE BIT RATE OF LINEAR CODE GENERATOR IN SPREAD-SPECTRUM COMMUNICATION SYSTEM

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ABSTRACT

Because of the limits of feedback devices, high-speed pseudo-noise code generators cannot depend simply on the improvement of clock rate. Based on the characteristic equation of linear feedback registers and the m-sequence sampling theory as well , deduction is made to indicate a novel way to improve the speed of pseudo-noise code generators $2^l\ (2^l < n, n \text{ is the length of registers})$ times as fast as the conventional one. Also, we extend our applications to non-reducible and non-primitive polynomials. It could be a good way to generate these linear codes at higher rates.

KEY WORDS

Spread-Spectrum, LSR(Linear Shift Registers) , and m-sequences

INTRODUCTION

There are two important indexes in spread-spectrum system: spread-spectrum processing gain G_t and anti-interference threshold M_j, which can be expressed as below:

$$G_P = R_c / r_b$$

where R_c is spread spectrum code rate in DSSS(direct spreading spectrum system), r_b is base-band information data stream speed. Also

$$M_J = G_p - [L_{sys} + (\frac{S}{N})_{out}]$$

where, L_{sys} is system loss , and $\left(\frac{S}{N}\right)_{out}$ is output ratio of signal to noise . Obviously, for the same base-band data rate, to enhance the spread spectrum code rate directly means that G_p and M_j will be raised , especially when compression potential of high rate data is limited. Therefore, we must adopt high rate spread spectrum code when devising system to ensure anti-interference capability and spread-spectrum gain. However, because feedback devices always restrain high-speed clock, we cannot depend simply on the increased clock rate to speed the rate of spread spectrum code. In the first and second section of this paper, the method to enhanced the rate of spread spectrum code generator is deduced through the characteristic equation of LSR(Linear Shifting Registers) and m-sequence principle. In the third section there is a designing sample and the practical capability of the method is concluded in the final section.

1. THE BASIC PRINCIPLE OF ENHANCING LINEAR CODE-RATE

A typical and Simple LSR matrix description is set up as Figure1. We suppose the length of LSR is n, $x_i(j)$ stands for the status of the ith register-unit after being shifted j times. So we can use vectors to depict the whole LSR, namely $X(j) = [x_1(j) \quad x_2(j) \quad \cdots \quad x_n(j)]^T$, the relationship between two shifts of LSR is

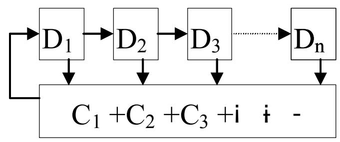

Figure1 Scheme of a Simple Linear Shift Register

$$X(j+1) = AX(j) \tag{1-1}$$

namely,

$$\begin{bmatrix} x_1(j+1) \\ x_2(j+1) \\ \vdots \\ \vdots \\ x_n(j+1) \end{bmatrix} = \begin{bmatrix} c_1 & c_2 & \cdots & \cdots & c_n \\ 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(j) \\ x_2(j) \\ \vdots \\ \vdots \\ x_n(j) \end{bmatrix} \tag{1-2}$$

A , here represents vector changing relationship between two adjacent moments, element $a_{ik} \in GF(2), 1 \le i,k \le n$. Expression in matrix form has such a advantage that for multi-return LSR (without considering feedback return connection) , like (1-1) and (1-2) to denote the matrix, we can still get a succinct form as A only through doing some simple primary matrix row-change to standard matrix. Namely, multi-return LSR always has a equivalent simple LSR. We will not thereafter differentiate two cases and always call them LSRs.

We now look for the LSR's characteristic polynomial. By using $f(l) = \det[A - lI]$, to deduct non-reducible polynomial, we can get $f(l) = \sum_{i=0}^{n} c_i l^{n-i}$, $where\ c_0 = c_n = 1$.

According to Cayley-Hamilton theorem, a full-rank matrix must satisfy its characteristic matrix-equation.

$$F(A) = A^n + c_1 A^{n-1} + c_2 A^{n-2} + \cdots + I = 0$$

Now we discuss the characteristic polynomial of A^2. One method is to compute its characteristic polynomial through $g(l) = \det[A^2 - lI]$, but it is very complicated . We will adopt a method as follows. We can get from formula above:

$$\begin{aligned}[F(A)]^2 &= [A^n + c_1 A^{n-1} + c_2 A^{n-2} + \cdots + I]^2 \\ &= (A^2)^n + c_1 (A^2)^{n-1} + c_2 (A^2)^{n-2} + \cdots + I = 0\end{aligned}$$

It is obvious that

$$[F(A^2)] = [F(A)]^2 = 0 \tag{1-3}$$

So it is proved that the characteristic equation with transferring matrix is A^2 is the identical characteristic equation whose transferring matrix is *A*. A^{2^l} shows the status after being shifted 2^l times of conventional LSRs, but it conforms to the same characteristic equation, and their generating sequence length periods are same.

Although coefficient matrix A^{2^l} doesn't change the characteristic equation of whole LSR, it potentially changes the logic return-connection relationship between any register unit and each other directly. Maybe it turns a simple LSR to a multi-return LSR. This is agreeable to what we have discussed that a multi-return LSR is equal to a simple LSR.

NOTE: Reference[2] gives , when characteristic equation is a primitive polynomial, the relationship among the changes of LSR status. According to that method, we can enhance the rate of m-sequence generator too, but the discussion about adjacent matrix and characteristic polynomial in this proposed paper doesn't require that non-reducible polynomial must be a primitive one. So, it is suitable for some non-reducible polynomial. For example, $f(x) = 1 + x + x^2 + x^3 + x^4$, whose adjacent matrix is *A,* its output is 11110, and when A^2 is the formula's coefficient matrix, the characteristic polynomial would not be changed and we can still get the same output.

2.THE SAMPLING PRINCIPLE TO ENHANCN m-SEQUENCE RATE

Since m-sequences are the most basic and the most important in pseudo-noise code theory and engineering project, the follow discussion will be based on m sequence sampling property.

Suppose that the output of conventional m-sequence generator's *n*th class is a infinite sequence at the zero moment, represented as $\{x_n\}$. Considering the relationship among sampling interval, it can be expressed as:

$$\{x_n(0), x_n(1), \cdots \text{£»} x_n(2^l), x_n(2^l+1), \cdots \text{£»} x_n(h\cdot 2^l), x_n(h\cdot 2^l+1)\text{£»}\cdots\} \tag{2-1}$$

we also suppose that output primitive code wave function is $p(t)(0 \le t < T)$, T is the time duration of code cell, $j(0) = -1, j(1) = 1$ *is a mapping from* $\{0,1\}$ *to* $\{1\text{£¬}1\}$. So the output wave function is :

$$y_o(t) = \sum_{j=0}^{+\infty} j\,[x_n(j)]p(t - jT) \tag{2-2}$$

then, by putting all the elements of (2-1) into a matrix composed of infinite 2^l-column vectors, and satisfying the relationship of each conventional unit ,we can get output matrix as follows:

$$\begin{bmatrix} x_n(0) & x_n(2^l) & x_n(2\cdot 2^l) & \cdots & x_n(h\cdot 2^l) & \cdots \\ x_{n-1}(0) & x_{n-1}(2^l) & x_{n-1}(2\cdot 2^l) & \cdots & x_{n-1}(h\cdot 2^l) & \cdots \\ x_{n-2}(0) & x_{n-2}(2^l) & x_{n-2}(2\cdot 2^l) & \cdots & x_{n-2}(h\cdot 2^l) & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{n-2^l+1}(0) & x_{n-2^l+1}(2^l) & x_{n-2^l+1}(2\cdot 2^l) & \cdots & x_{n-2^l+1}(h\cdot 2^l) & \cdots \end{bmatrix} \quad (2\text{-}3)$$

Pay attention to the row vectors: the *i*th register-unit output of our ameliorated register is surely the sampled sequence of $\{x_{n-i+1}(j)\}$, sampled at 2^l interval from the output of *i*th register-unit of conventional register t; when each *l* value is different, each 2^l belong to one coset of *GF(*2^l-1*)*, so the output line vector must be the identical with the m-sequence(ignorant of the difference of initial phase). we can get (*n-i*)th unit output

$$y_{oi}(t) = \sum_{h=0}^{+\infty} j\,[x_{n-i}(h\cdot 2^l)]p(t-hT)$$

Without considering the difference of sampling m-sequence first phases, we can neglect difference of initial code cells in the formula above. Obviously ,they are the same m-sequence founction waves:

$$y_{oi}(t) = \sum_{j=0}^{+\infty} j\,[x_{n-i}(j)]p(t-jT)$$

If we extract the status of all 2^l registers in turn during unit cell duration time, outputting a column element in the conventional code cell duration time can compress the duration of every code cell to $\frac{1}{2^l}$, so the output is

$$y(t) = \sum_{k=0}^{2^l-1}\sum_{h=0}^{+\infty} j\,[x_{n-k}(h2^l)]p\{2^l[t-(h+\frac{k}{2^l})T]\}$$

$$= \sum_{k=0}^{2^l-1}\sum_{h=0}^{+\infty} j\,[x_n(h2^l+k)]p\{2^l[t-(h+\frac{k}{2^l})T]\}$$

let $j = h2^l + k$, we can simply convert the above to:

$$y(t) = \sum_{j=0}^{+\infty} j\,[x_n(j)]p[2^l(t-\frac{jT}{2^l})] \quad (2\text{-}4)$$

Compared with (2-2), the formula above is reduced 2^l times, so the rate of m-sequence is enhanced 2^l times.

3 A DESIGN OF SPREAD SPETRUM SEQUENCE GENERATOR

We here give a design example. Generally, the calculation of AND gate takes double time as much as the calculation of XOR gate does. The typical use of this method in this text is that when we design TTL or COMS circuit, it overcomes the code sequence generator limitation produced by feedback device. The example as follow is a scheme that m sequence which have 7 register units and whose code length is 127 realizes double rate feedback logic.

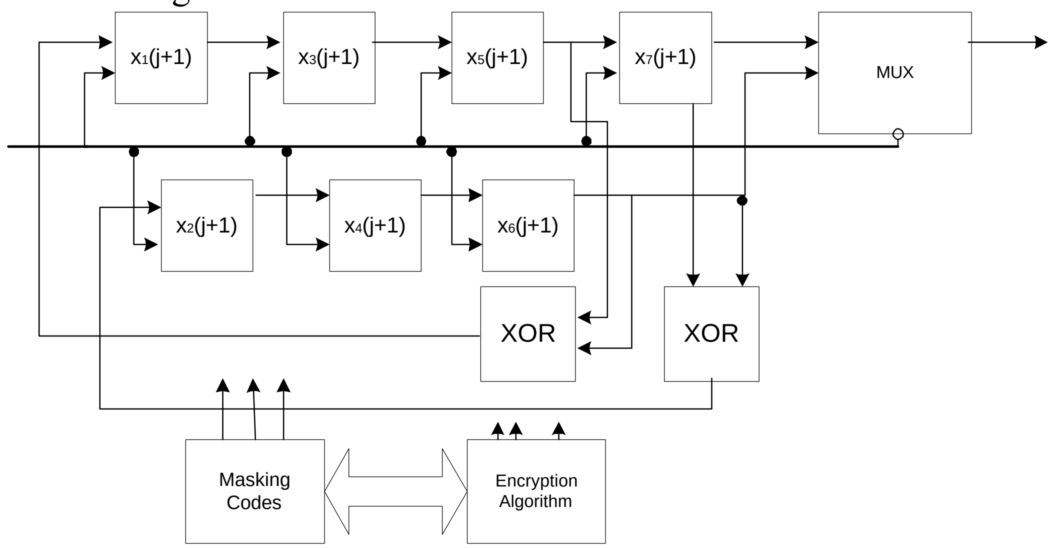

Figure2 A scheme of m-sequence generator at double speed

We use primitive polynomial $f(x)=1+x^6+x^7$ and unit matrix to construct adjacent matrix A , and hence A^2:

$$\begin{bmatrix} x_1(j+1) \\ x_2(j+1) \\ x_3(j+1) \\ x_4(j+1) \\ x_5(j+1) \\ x_6(j+1) \\ x_7(j+1) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(j) \\ x_2(j) \\ x_3(j) \\ x_4(j) \\ x_5(j) \\ x_6(j) \\ x_7(j) \end{bmatrix} \quad \begin{matrix} x_1(j+1)=x_5(j)+x_6(j) \\ x_2(j+1)=x_6(j)+x_7(j) \\ x_3(j+1)=x_1(j) \\ x_4(j+1)=x_2(j) \\ x_5(j+1)=x_3(j) \\ x_6(j+1)=x_4(j) \\ x_7(j+1)=x_5(j) \end{matrix}$$

Then, the concrete feedback juncture logic is showed as above. This is a double-return structure. It returns to $x_1(j+1), x_2(j+1)$ separately. We should add a multiplex to output sequence in design in order to control selective connection of two clock-neighboring unites. So when a traditional LSR exports a code cell , our LSR gets two cells output . The rate of code is enhanced two times. Feedback connecting line doesn't change primitive polynomial of the new sequence. It is the basis of enhancing rate of pseudo-noise code and the key of acceleration method is that it realizes acceleration at multiplex. In Figure 2, we can see how it is realized:

4 CONCLUSION

The method in this paper would enhance the rate of m-sequence spread spectrum generators $2^1 (2^1 < n, n \text{ is the length of registers})$ times as fast as traditional generator. It is realized by making the status of each register unit jump 2^l times in one clock period and at the same time sampling each 2^l register-unit . The reason why we enhance the rate to 2^l times is because it is easy to realize in the digital circuit with 2^l selective connector. And the method in the paper can be extended to non-maximum length sequence, which is based on non- reducible polynomial.

In general, this paper proposes a method to enhance rate of spread spectrum code through two ways: characteristic polynomial and sampling theory of m-sequence. The method can enhance processing gain and anti-interference ability .It offer a method and theory to realize the design of spread spectrum code of broad band, high rate in spread spectrum communication systems.

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