BANDWIDTH EFFICIENT CONCATENATED CODES FOR EARTH OBSERVATION TELEMETRY

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ABSTRACT

Telemetry for Earth-Observation missions is characterized by very high data rates and stringent requirements. Channel codes both power and bandwidth efficient must be used to improve down-link performance and to achieve the very low values of error rates needed at the received side. In this paper, we review and analyzed three codes of possible interest for these applications: turbo codes, serial turbo codes and product codes. These schemes are evaluated and compared both by simulation and analytical techniques. A particular attention is devoted to complexity, a key issue for practical implementation at high data rates.

KEY WORDS

Earth-Observation Telemetry, Channel Coding, Turbo Codes and Product Codes

INTRODUCTION

Space missions for Earth Observation (EO) are becoming more and more important. Synthetic Aperture Radar (SAR) and high resolution/wide swath optical imagers are used to produce always increasing amounts of data. These data are stored on board and down-linked to Earth when the ground stations become visible. The down-link performance are a crucial issue for mission design: if the data flow became a bottle-neck, the duty cycle of the instrument must be limited. In this context, a high telemetry transmission efficiency is essential.

CCSDS (Consultative Committee for Space Data Systems) is currently investigating new channel codes to update its recommendation for telemetry channel coding [1] with the aim of supporting higher data rate EO transmissions. Currently, most of the codes included in [1] are coding schemes with limited code rates, not specially designed for bandwidth efficiency or designed for deep space applications characterized by low data rates and no particular bandwidth problems. As a consequence, these codes were chosen aiming to maximize the coding gain, rather than to satisfy the requirements essential for Earth observation applications.

REQUIREMENTS FOR EARTH OBSERVATION TELEMETRY

The requirements for EO applications are extremely stringent. They are reviewed and discussed in the following.

Data rate: The data rates R for these applications will be very high. They will range from a few Mbps to hundreds of Mbps. Some applications (for example Tracking and Data Relay satellites) are planned for the 500 Mbps region.

Bandwidth efficiency: Currently, CCSDS supports only 2-PSK and 4-PSK modulations (8-PSK is currently being discussed for the future). High data rates and spectral crowding from many satellites will limit the available bandwidth. To obtain a large bandwidth efficiency, binary codes with high code rate, near to the unity, are required. In the following, we will focus on code rates not less than 0.75 = 3/4, i.e., spectral efficiencies not less than 1.5 bps/Hz over a 4-PSK.

Power efficiency: The transmitted power, from Earth observation satellites of small dimension, will be a limited resource. The new codes should be able to achieve large coding gains.

Error Rates: Telemetry data are collected in packets and transmitted in frames. In line of principle, almost any frame length ranging from 0 to 16384 is acceptable. Any transmitted frame has an error control field of 16 bits added by a Cyclic Redundancy Check (CRC) code, used for frame integrity validation at the receiver side. In this context, Frame Error Rates (FER) are more important than Bit Error Rates (BER). The image compression techniques used for near-Earth observation will impose a severe constraint on the FER. In some cases, very low values of FER will be required, even as low as FER $\approx 10^{-8}$.

Complexity: The complexity of the encoder/decoder algorithms must be limited. Their practical implementation will be particularly critical because of the high data rates involved. (This consideration is most of all true for the encoder implementation, that must be allocated on board).

CODES ALREADY INCLUDED IN THE CCSDS TELEMETRY CHANNEL CODING STANDARD

The coding schemes currently standardized in [1] are:

- $\mathbf{C_{RS}}$: a (255,223) Reed-Solomon code with 8-bit symbols and Error Correction Capability ECC = 16 symbols.
- $\mathbf{C_{RS8}}$: a (255,239) Reed-Solomon code with 8-bit symbols and ECC = 8 symbols.
- $\mathbf{C_{CC}}$: a 64-state, rate-1/2 binary convolutional code.
- $\mathbf{C_{CC2}}$ (rate-2/3), $\mathbf{C_{CC3}}$ (3/4), $\mathbf{C_{CC5}}$ (5/6), and $\mathbf{C_{CC7}}$ (7/8): a class of 64-state binary convolutional codes obtained by puncturing the rate-1/2 code $\mathbf{C_{CC}}$.
- All possible serial concatenation of a Reed-Solomon code ($\mathbf{C_{RS}}$ / $\mathbf{C_{RS8}}$) (outer code) and a convolutional code ($\mathbf{C_{CC}}$ / $\mathbf{C_{CC2}}$ / $\mathbf{C_{CC3}}$ / $\mathbf{C_{CC5}}$ / $\mathbf{C_{CC7}}$) (inner code) through an interleaver of ($n \cdot I$) bytes, where n is the Reed-Solomon codeword length and I = 1,2,3,4, or 5.

- **C_{TC}**: a family of turbo codes with nominal rates 1/2, 1/3, 1/4, or 1/6, composed by two 16-state, rate-1/4 convolutional codes and an interleaver with length ranging from 1784 to 16384 bits.

DESCRIPTION OF THE CODING SCHEMES UNDER STUDY

In this Section, the coding schemes considered in the paper are briefly described.

Turbo codes: Turbo codes, invented in [2], has revolutionized channel coding theory. Their growing success and their introduction in many important international standards (UMTS for third generation mobile telephony, DVB for digital video broadcasting, CCSDS for low rate deep space telemetry) has generated an enormous interest for new or reinterpreted codes, able to approach the Shannon limit as never before.

A turbo code is composed by two binary systematic convolutional codes, connected in parallel through a block interleaver of length N equal to the data frame length F [2]. At the receiver side, it is decoded by iterating the BCJR algorithm [3] on the constituent convolutional codes. Turbo codes were invented as binary codes with low rate, usually less than 1/2. Higher rates can be obtained by puncturing. A proper approach to maximize the performance of punctured turbo codes has been presented in [4]. The turbo code encoding algorithm is simple to realize. On the contrary, the decoding complexity must be properly taken in account, because of the high data rates involved for EO applications.

Turbo codes are characterized by coding gains extremely large for low/medium Signal-to-Noise Ratio (SNR), very close to the theoretical Shannon limits. However, it is well known that their performance may be not so good at very high signal-to-noise ratios (very low error rates), where the "error floor" occurs. In this region, the performance of any binary code is dominated by its minimum distance $d_{\min}$ (the minimum Hamming distance between codewords which coincides with the minimum Hamming weight of a nonzero codeword for linear codes) and its multiplicity. In some cases, concatenated codes with interleavers may have very low minimum distances, despite of very large interleaver lengths. This causes their performance curves to flatten according to the slope imposed by $d_{\min}$, after the typical "water-fall" decrease at low signal-to-noise ratios. This behavior must be taken in account when studying solutions for EO applications, which require very low FER values. In this paper, we will consider two different classes of turbo codes: (1) punctured CCSDS turbo codes, with 16-state constituent encoders, obtained by external puncturing of CCSDS turbo codes, and (2) turbo codes with 8-state constituent encoders.

Serial turbo codes: A Serially Concatenated Convolutional Code (SCCC) is composed by two binary systematic convolutional codes, serially connected through an interleaver [5]. SCCC with high rates can be obtained, for example, by puncturing [6]. With comparable decoding complexity, SCCCs have usually slightly worse performance than turbo codes at high/medium error rates. Anyway, SCCCs may potentially outperform turbo codes at low error rates thanks to larger minimum distance values. In this paper, we will consider three classes of SCCCs, that will be described later on.

Product codes: In their simplest form, product codes are obtained by writing the information bits in a square matrix ($k\times k$), and by encoding the rows and the columns with an (n,k) block code that adds $(n-k)$ parity check bits. This way, one obtains an (n^2,k^2) square product code. At the receiver side, soft-output decoding algorithms are used to decode rows and columns. By iterating the decoding process, product code error rate performances improve, similarly to those of turbo codes. Decoding algorithms simpler than the BCJR algorithm have been successfully studied and applied [7].
Usually, product codes achieve slightly worse performance than concatenated codes at low signal-to-noise ratios. Their big advantage stays in their large minimum distance, equal to the square of the constituent code minimum distance. In line of principle, product codes should outperform concatenated codes at very low error rates. However, the role of their multiplicity, that can be very high (and huge in some cases) cannot be undervalued. In this paper, we will consider product codes with constituent Hamming codes.

METHODS OF ANALYSIS

The performance of the considered coding schemes have been analyzed both by simulation and by analytical techniques. At high/medium error rates (e.g., Frame Error Rates higher or equal to 10^{-5}) simulation has been employed. All the simulation points have been evaluated with at least 100 erroneous frames. At low error rates, reliable simulations are too long and practically unfeasible. In this region, analytical techniques can be employed. In fact, at high SNR the performance of any binary code is dominated by its minimum distance $d_{\min}$ and its multiplicity.

Let us consider a $C(n,k)$ code of rate $R = k/n$, with minimum distance $d_{\min}$, multiplicity $A_{\min}$ (defined as the number of codewords with Hamming weight $d_{\min}$) and information bit multiplicity $w_{\min}$ (defined as the sum of the Hamming weights of the $A_{\min}$ information sequences generating the codewords with weight $d_{\min}$). We can relate the BER and FER expressions to the ratio between the energy per information bit E_b and the one-sided noise spectral density N_0. At high signal-to-noise ratios (i.e., at low error rates), we can write:

$$\mathrm{BER} \approx \frac{1}{2}\frac{w_{\min}}{k}\operatorname{erfc}\sqrt{Rd_{\min}\frac{E_b}{N_0}} \qquad , \qquad \mathrm{FER} \approx \frac{1}{2}A_{\min}\operatorname{erfc}\sqrt{Rd_{\min}\frac{E_b}{N_0}} .$$

In this region, in fact, the code performance practically coincides with the expression of the union bound, truncated to the contribution of the first distance. For concatenated codes, a small penalty (usually less than 0.5 dB) must be also taken in account, due to the sub-optimality of the iterative decoding. These BER and FER expressions are often called the code “error floor”. This analytical approach requires the knowledge of the code minimum distance and its multiplicity. An algorithm to compute the minimum distance of turbo codes and serial turbo codes was presented in [8]. Moreover, the exact computation of the minimum distance and the multiplicity of Hamming turbo codes has been provided in [9].

PUNCTURED CCSDS TURBO CODES

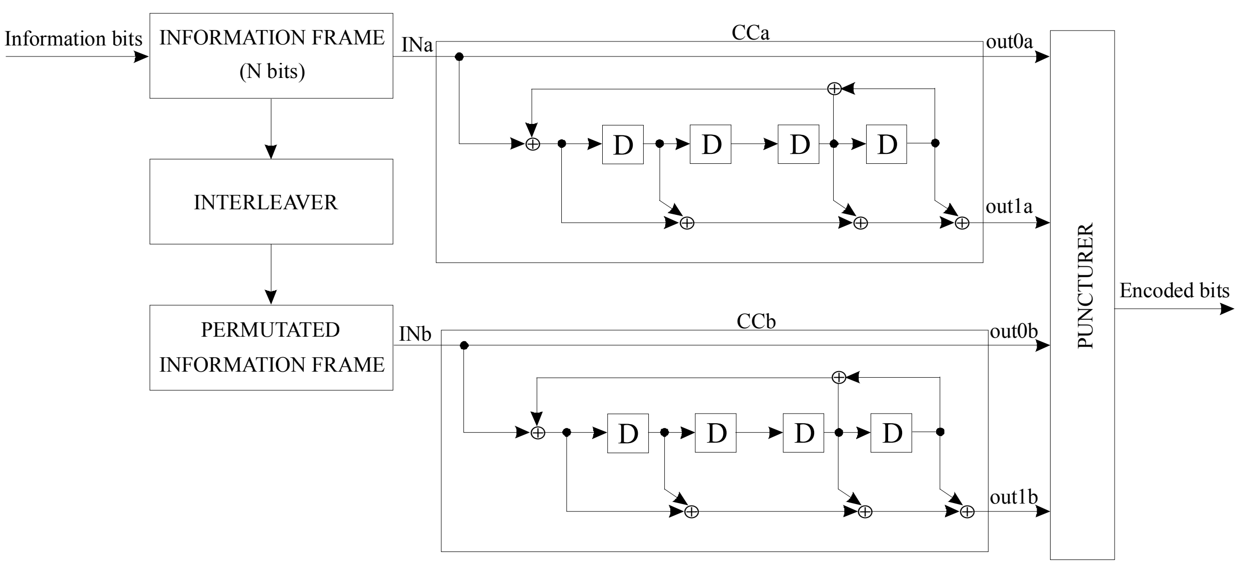
The CCSDS turbo code $\mathbf{C}_{TC}$ of [1] is composed by:

- Constituent codes: two equal 16-state, rate-1/4 convolutional codes with feedback polynomial $(1+D^3+D^4)$, and feedforward polynomials $(1+D+D^3+D^4)$, $(1+D^2+D^4)$ and $(1+D+D^2+D^3+D^4)$.
- Interleaver: Algorithmic, Berrou interleavers with length N = 1784, 3568, 7136, 8920, or 16384 bits (equal to data frame length F).

The nominal rates of $\mathbf{C}_{TC}$ range from 1/6 to 1/2. By applying an external puncturer, higher code rates can be obtained. To do this, we have considered only the first feedforward polynomial. The basic encoder is depicted in Figure 1. By varying the puncturing pattern, we have studied Punctured CCSDS Turbo Codes (PCTC) with:

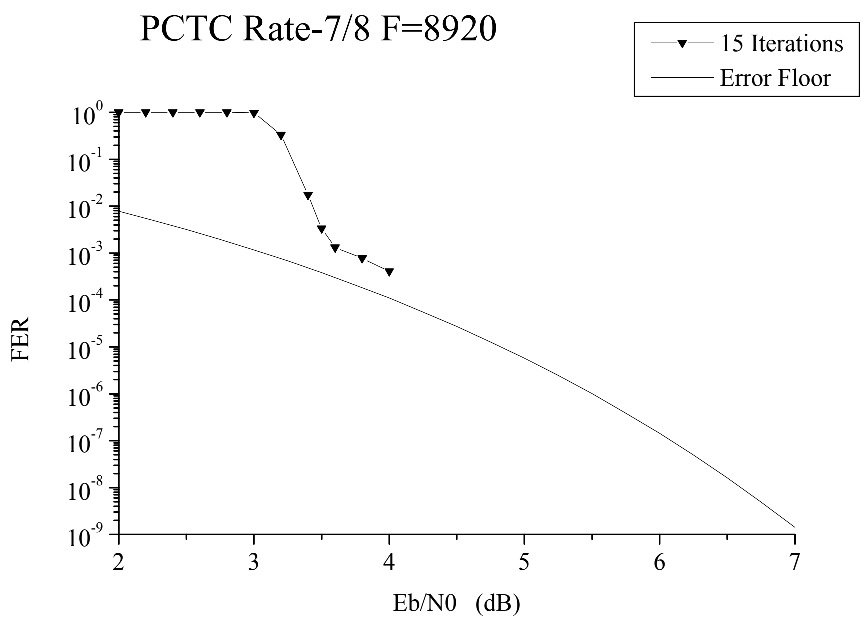
- Nominal code-rates: 3/4, 7/8, 8/9, 11/12, and 15/16.
- Data frame length: F = 1784, 7136, or 8920 bits.

Figure 1: Punctured CCSDS turbo encoder



These schemes have been analyzed both by simulation and by evaluating their error floors. As an example, we report here the performances of the rate-7/8 PCTC with F = 8920. For this code, we have computed $(d_{\min}/A_{\min}/w_{\min})$ = (5/79/316). In Figure 2 its error floor is compared against a simulation curve corresponding to 15 iterations of the BCJR algorithm. The slope change is evident, together with the match between the simulation curve and the error floor at low FER. An enormous time amount would be necessary to analyze the code performance at low error rates (e.g., FER less than 10^{-6}) by simulation. Instead, they are clearly delineated by the error floor slope.

Figure 2: Simulated performance vs. error floor for rate-7/8 PCTC



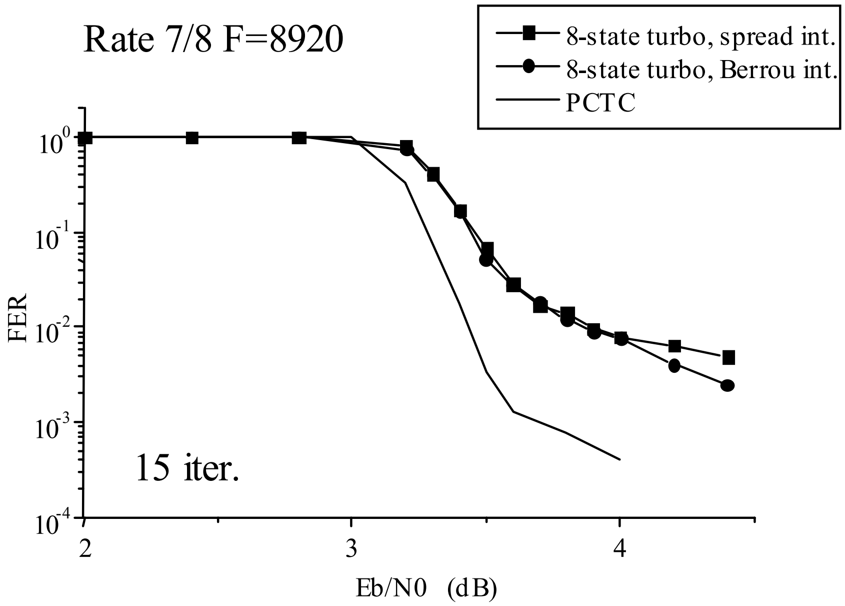
8-STATE TURBO CODES

In this Section, the performance of PCTC schemes are compared against turbo codes composed by 8-state (instead of 16-state) convolutional codes. Two different interleavers have been considered: spread interleavers and the same Berrou interleavers employed by PCTC schemes.

Decoding of 8-state turbo codes is less complex than PCTC decoding. However, as expected, their performance are inferior to those of PCTC, especially at low error rates. Let us consider the same rate (7/8) and data frame length (8920 bits) considered in the previous Section. By computing the minimum distances and the multiplicity of the 8-state turbo codes we have obtained ($d_{\min}/A_{\min}/w_{\min}$) = (2/4/8) for the spread interleaver and ($d_{\min}/A_{\min}/w_{\min}$) = (4/269/1076) for the Berrou interleaver. Both these distances are lower than PCTC minimum distance. Then the asymptotic performance of PCTC are better than those of 8-state turbo codes, as confirmed by the simulation results depicted in Figure 3, referred to 15 iterations.

Note that 8-state turbo codes were chosen as channel codes of UMTS third generation mobile telephony standard. A more complete comparison between PCTC and 8-state turbo codes can be found in [10].

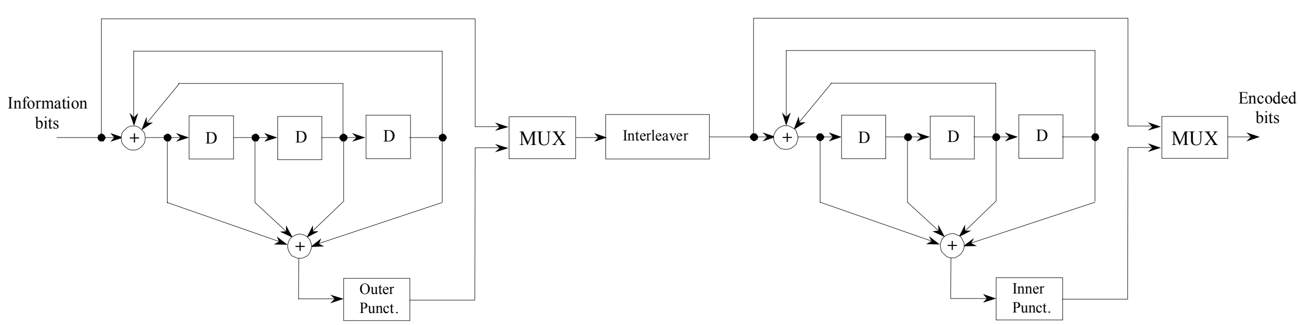
Figure 3: Simulated performance for rate-7/8 codes: 8-state turbo codes vs. PCTC



SERIAL TURBO CODES

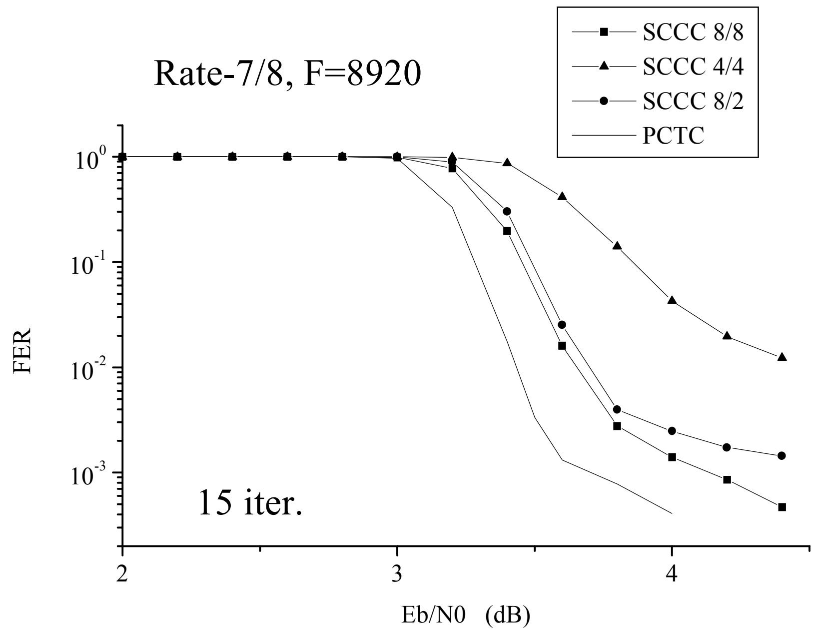
In this Section, we consider three different SCCC schemes. The first one, called "SCCC 8/8" employs equal 8-state inner and outer rate-1/2 convolutional codes. The second one, denoted by "SCCC 4/4" has equal 4-state inner and outer rate-1/2 convolutional codes. The last one, called "SCCC 8/2" employs an outer 8-state rate-1/2 convolutional code and an inner rate-1 differential encoder. As an example, the SCCC 8/8 encoder is reported in Figure 4. Two puncturer, acting on the parity sequences of the two encoders, are used to increase the code rate. As for the decoding, SCCC 8/8 schemes have decoding complexity comparable to PCTC, while SCCC 4/4 and SCCC 8/2 complexity is lower.

Figure 4: SCCC 8/8 encoder



The considered punctured serial turbo codes behave worse than PCTC both at low and high error rates. As an example, the performance of the analyzed serial turbo codes are reported in Figure 5, and compared with PCTC performance, for the same rate-7/8 and data frame length 8920 bits considered before.

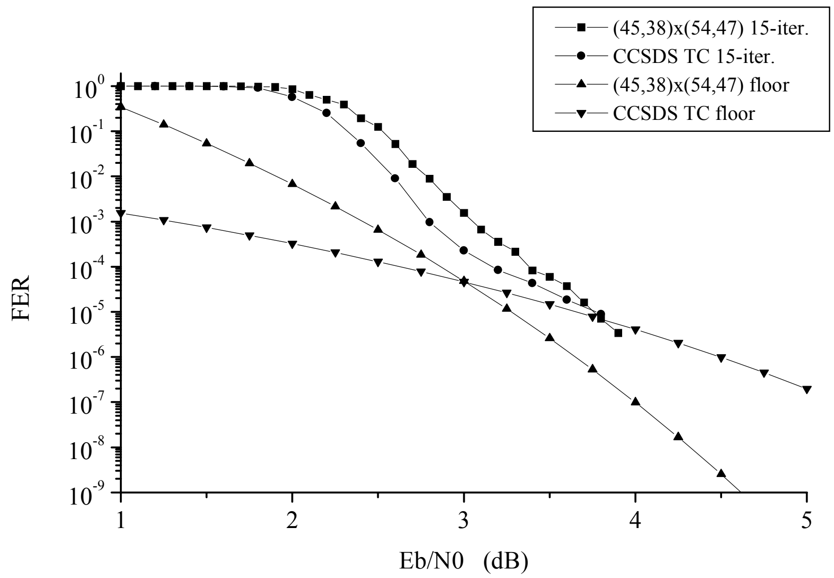
Figure 5: Simulated performance for rate-7/8 codes: SCCC vs. PCTC



PRODUCT CODES

In this Section, we consider product codes employing equal extended Hamming codes. The minimum distance and the multiplicity of extended Hamming codes can be computed by applying the procedure described in [9]. Given a Hamming product code, it is possible to adapt the data frame length by shortening the column code, the row code, or both. As an example, starting from the Hamming product code $(\mathbf{EH_6})^2 = (64,57)^2 = (4096,3249)$, we have designed a shortened product code able to realize a data frame length of about 1784 bits and a code rate near to 3/4. The code (45,38)x(54,47) = (2430,1786), obtained by shortening the column and the row code of 19 and 10 bits, respectively, achieves a code rate equal to 0.735. The minimum distance and the multiplicity of this code are equal to $(d_{\min}/A_{\min}/w_{\min})$ = (16/13,073,136/156,551,688). For comparison, a punctured CCSDS turbo code with data frame length 1784 bits and nominal rate 3/4, can be considered. The simulated curves (15-iterations in the decoding process) and the error floors of the two codes are compared in Figure 6. We can observe that PCTC is better than the considered product code down to FER $\approx 10^{-5}$. For lower error rates, product code performances are better. This behaviour is supported by the error floor curves, that also cross at approximately the same value.

Figure 6: Simulated performance for rate-3/4 codes: Product code vs. PCTC



Product coding looks a very powerful solution, competitive with PCTC, especially at very low error rates. As for the decoding complexity, it is worthwhile to mention that some commercial implementations are already available, working up to some hundreds of Mbps.

CONCLUSIONS

In this paper, we have analyzed three coding schemes for possible applications to EO missions. First, we have verified that punctured turbo codes require at least 16 states to be competitive with other solutions. Punctured CCSDS turbo codes, obtained by external puncturing of the CCSDS 16-state turbo code $\mathbf{C}_{\mathbf{TC}}$, represent a pragmatic and powerful solution. In line of principle, a single component with external puncturing could range from very low (1/6) to very high code rate (15/16). So, these codes look particularly interesting as a short/medium term solution for missions with not too high data rates (for example R <10 Mbps). As for serial codes, puncturing may seriously reduce their performance. The scheme employing an inner differential encoder was the most interesting one within this class.

Product codes outperform punctured CCSDS turbo codes at extremely low error rates (FER less than 10^{-7}). At the contrary, CCSDS turbo codes win at high/medium error rates (FER higher than 10^{-5}). The required error frames will then be a key issue for the comparison between the two schemes. Solutions able to further improve the performance of both PCTC and product codes are currently under study.

ACKNOWLEDGEMENTS

The authors are grateful to Dr. Garik Markarian, Dr. Bill Thomson and Dr. Carly Lister of Advanced Hardware Architectures Inc., for useful support and for providing the product code software evaluation tool AHA Galaxy software.

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