ENHANCED PERFORMANCE OF FQPSK-B RECEIVER BASED ON TRELLIS-CODED VITERBI DEMODULATION

Dennis Lee, Marvin Simon, and Tsun-Yee Yan
Jet Propulsion Laboratory

ABSTRACT

Commercial FQPSK-B receivers traditionally use symbol-by-symbol detection and have a 2 dB E_b/N_o loss relative to ideal QPSK at a bit error rate (BER) of 10^{-5}. An enhanced FQPSK-B receiver using a Viterbi algorithm (VA) to perform trellis decoding is simulated and shown to have a 1.2 dB E_b/N_o improvement over symbol-by-symbol detection for 10^{-5} BER at the cost of increased complexity. A simplified Viterbi receiver with a reduced trellis and significantly less complexity is introduced with only a slight BER degradation compared to the full Viterbi receiver. In addition, a theoretical bit error probability expression for the symbol-by-symbol FQPSK-B receiver is derived and compared with simulation results.

KEY WORDS

Feher's quadrature phase shift keying, Trellis coding, Bit error probability

INTRODUCTION

Feher-patented QPSK (FQPSK) is a spectrally efficient form of offset QPSK modulation with pulse shaping to reduce spectral sidelobes and cross-correlation between inphase and quadrature phase baseband signals to maintain a nearly constant envelope. The suppressed sidelobes and quasi-constant envelope makes FQPSK desirable for communications in nonlinear channels when bandwidth is a constraint. Of particular interest is a baseband filtered version of FQPSK called FQPSK-B [1] [2] which has been recommended by the Consultative Committee on Space Data Systems (CCSDS) and IRIG [3]. However, the narrow bandwidth of FQPSK-B comes at the cost of BER degradation, approximately 1.4 dB at 10^{-3} BER using the traditional FQPSK-B receiver.

To improve BER performance, a trellis-coded interpretation of FQPSK was introduced in [4]. Using this interpretation, FQPSK is generated by transmitting one of 16 shaped waveforms (8 unique waveforms shown in Figure 1 and their negatives) based on a 16-state trellis described in [4]. It was shown that using Viterbi demodulation resulted in a significant improvement in bit error performance for unfiltered FQPSK over a conventional symbol-by-symbol receiver at the cost of increased complexity. This paper extends the results of [4] by considering the bit error performance of FQPSK-B with Viterbi demodulation. FQPSK-B is more spectrally efficient than unfiltered FQPSK but has inter-symbol interference (ISI) due to the baseband filtering. A reduced complexity Viterbi receiver is then introduced which has fewer correlators and fewer states in the VA, and its performance is simulated.

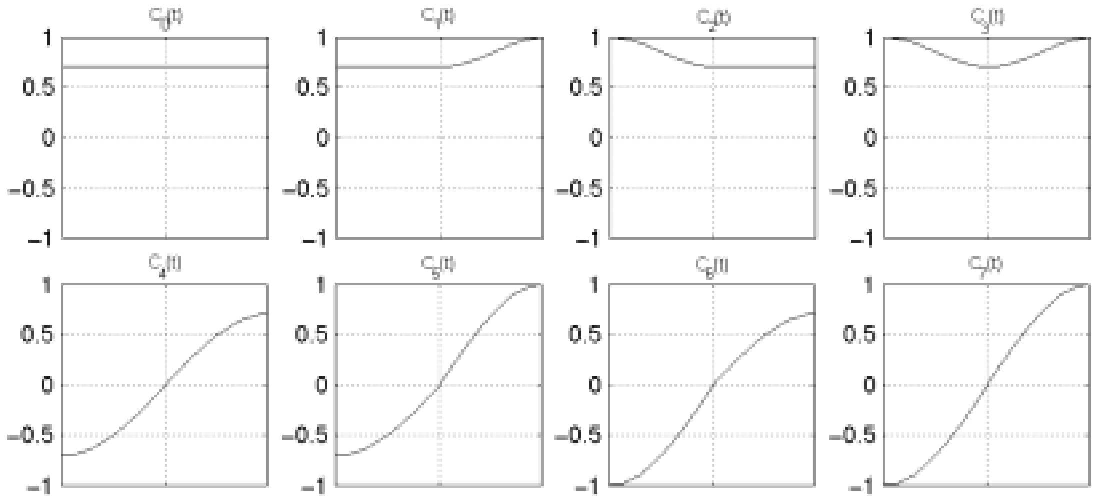

Figure 1: FQPSK waveforms; $C_i(t) = -C_{i-8}(t), i = 8, ..., 15$

TRADITIONAL FQPSK-B RECEIVER

The traditional commercial FQPSK-B receiver is a sample-and-hold (S&H) receiver with symbol-by-symbol detection. The received signal is downconverted to baseband and then filtered using a detection filter whose bandwidth-symbol period (BT_s) is approximately 0.6 as given in [2] (the exact filter type and optimal BT_s are subject to a non-disclosure agreement). The output of the detection filter is sampled at the maximum eye opening, and a decision is made on the symbol.

A theoretical expression for the bit error probability of the S&H FQPSK-B receiver in a linear channel is derived in the Appendix using superposition arguments. Due to ISI, there are a large number of terms in the theoretical expression; however, an approximation with only 32 terms matches closely with simulation results. The approximation is given in Equation 6 of the Appendix. Figure 2 shows a comparison between the 32-term theoretical approximation of the bit error probability and computer simulated results. The bit error probability of ideal QPSK is also included for comparison.

FQPSK-B VITERBI RECEIVER

The FQPSK-B Viterbi receiver correlates the baseband received signal with the FQPSK waveforms and uses a Viterbi algorithm to perform the trellis decoding. The Viterbi algorithm searches through the FQPSK trellis which has sixteen states and four transitions to each state. The VA branch metrics, Z_j, are given as follows:

$$Z_j = R_j - \frac{E_j}{2} \qquad j = 0, ..., 15 \tag{1}$$

where R_j is the correlation of the received signal and the j^{th} waveform and E_j is the energy in the j^{th} waveform. The correlation values R_8 through R_{15} are obtained by taking the negatives of R_0 through R_7 (i.e., $R_0 = -R_8$, $R_1 = -R_9$, $R_2 = -R_{10}$, etc.). A total of sixteen correlators are needed, eight each for the inphase and quadrature phase channels. A block diagram of the full FQPSK-B Viterbi receiver is

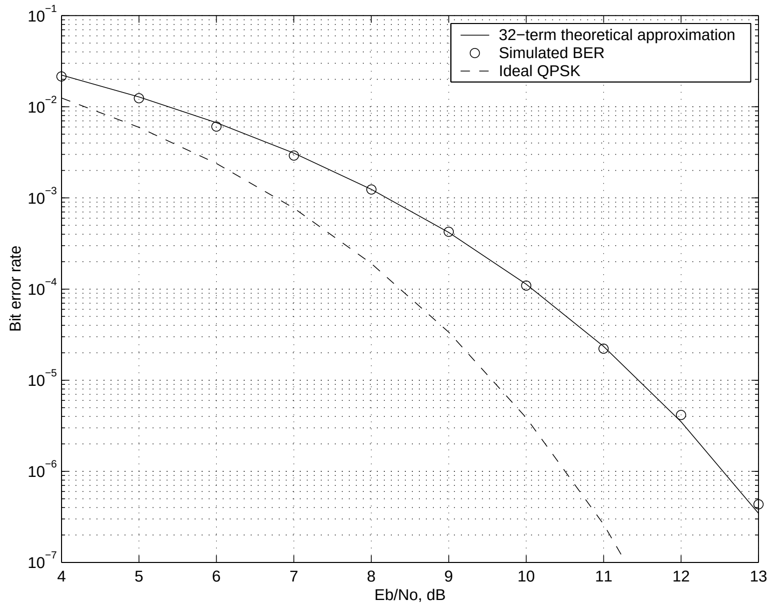

Figure 2: Comparison of theoretical BER and simulated BER for FQPSK-B S&H receiver

shown in Figure 3. Note that in the figure, the subtraction of $E_j/2$ is performed in the FQPSK-B Viterbi algorithm block.

SIMPLIFIED FQPSK-B VITERBI RECEIVER

A simplified FQPSK-B Viterbi receiver can be formed by grouping sets of waveforms together and creating a reduced trellis. In this case, the sixteen FQPSK waveforms are separated into 4 different groups. The first group consists of waveforms C_0 through C_3 (see Figure 1); the second group consists of waveforms C_4 through C_7; the third group consists of waveforms C_8 through C_{11}; and the fourth group consists of waveforms C_{12} through C_{15}. With this grouping, the trellis coded structure of FQPSK splits into two independent inphase (I) and quadrature phase (Q) 2-state trellises. The trellis structure of the simplified Viterbi receiver is identical to that of trellis-coded OQPSK in [5] but with different waveforms.

A block diagram of the simplified FQPSK-B Viterbi receiver is shown in Figure 5. The received FQPSK-B signal is demodulated and then correlated against the average of the waveforms in each group, $q_i(t)$, given below and shown in Figure 4.

$$q_0(t) = \frac{1}{4}\sum_{i=0}^{3} C_i(t) \qquad\qquad q_2(t) = -q_0(t)$$

$$q_1(t) = \frac{1}{4}\sum_{i=4}^{7} C_i(t) \qquad\qquad q_3(t) = -q_1(t)$$

Since $q_2(t)$ and $q_3(t)$ are the negatives of $q_0(t)$ and $q_1(t)$, only two correlators each are needed for the I and Q channels. The VA metrics are again formed as in Eq. (1) except E_j is now the energy of the

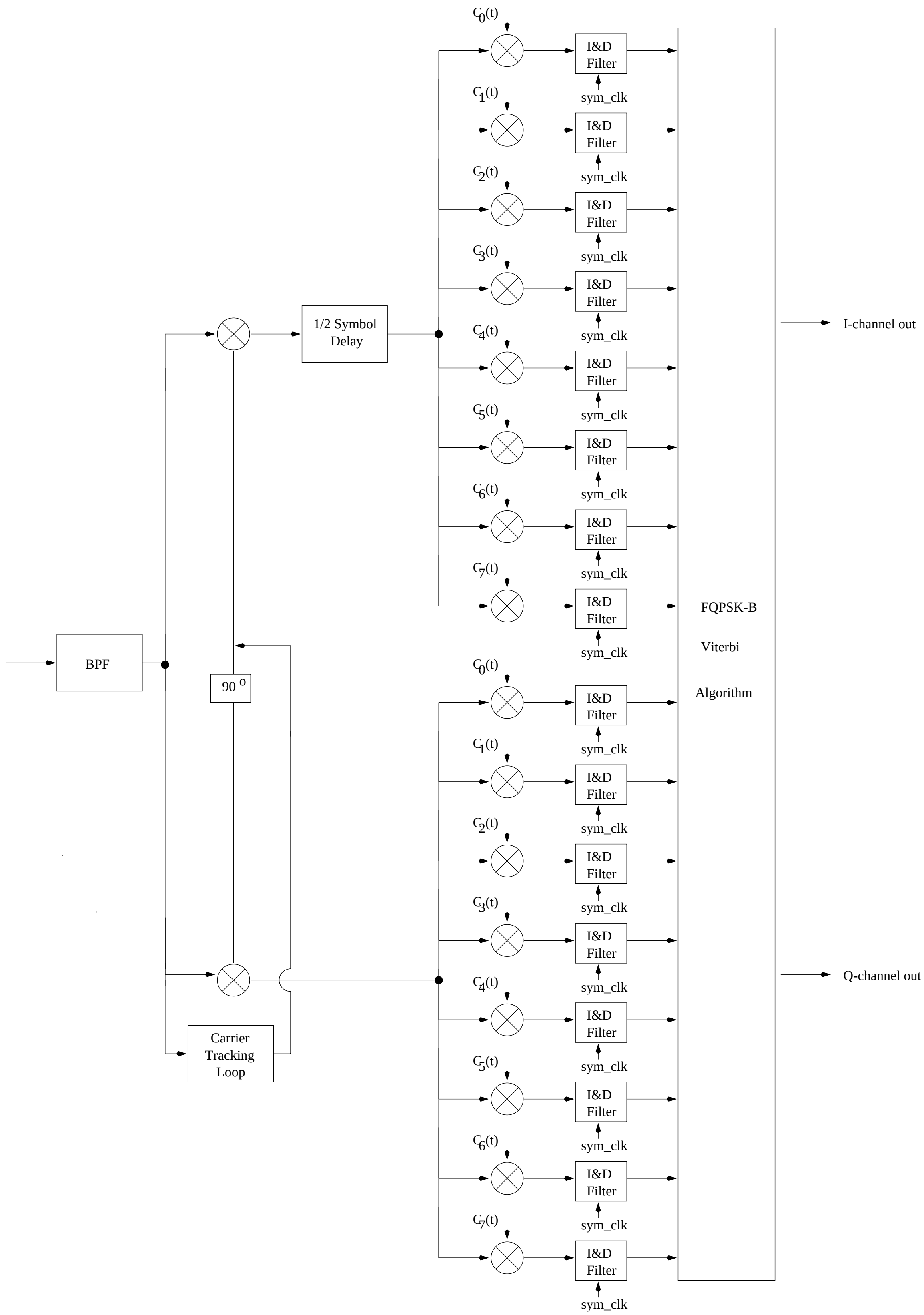

Figure 3: Block diagram of FQPSK-B Viterbi Receiver

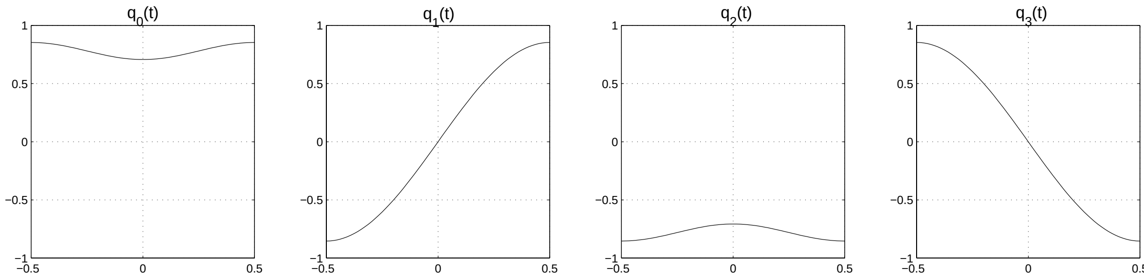

Figure 4: Averaged waveforms for simplified Viterbi receiver

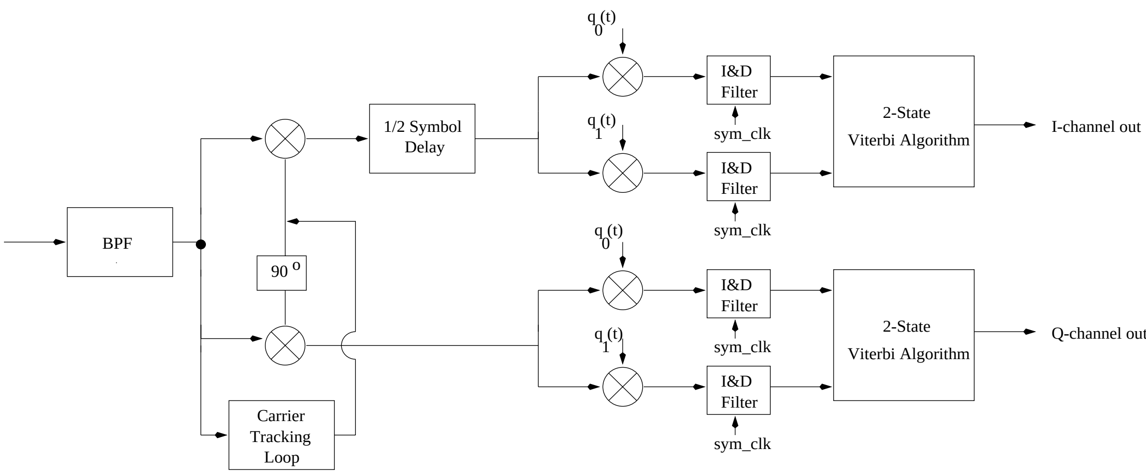

Figure 5: Block diagram of Simplified FQPSK-B Viterbi receiver

group average waveform $q_j(t)$. Figure 6 shows the trellis of the grouped signals which has two states and two transitions to each state. The two Viterbi algorithms for the I and Q channels can be combined into a single 4-state VA. Compared to the full Viterbi receiver, the simplified Viterbi receiver has 12 fewer correlators and an 8-fold reduction in the number of VA computations per decoded bit.

BIT ERROR RATE PERFORMANCE

Using Signal Processing WorkSystem (SPW) computer simulations, the bit error performance of the simplified and full FQPSK-B Viterbi receivers is compared with the traditional FQPSK-B S&H receiver and ideal QPSK. Ideal carrier and symbol synchronization is assumed. The simulated channel includes a non-linear solid-state power amplifier (SSPA) operating in full saturation. For the full Viterbi receiver, a truncation path length (decoding depth) of 50 bits is used in the VA. Due to the short constraint length nature of the reduced trellis, a truncation path length of only 10 bits is needed for the simplified Viterbi receiver.

BER curves are shown in Figure 7 and results are summarized in Table 1. The Viterbi receiver performs 0.8 dB better than the S&H receiver at 10^{-3} which is comparable to the results in [4] for unfiltered

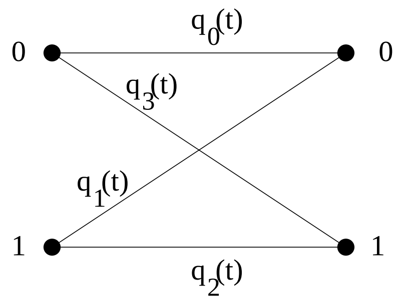

Figure 6: Trellis for simplified FQPSK-B Viterbi receiver

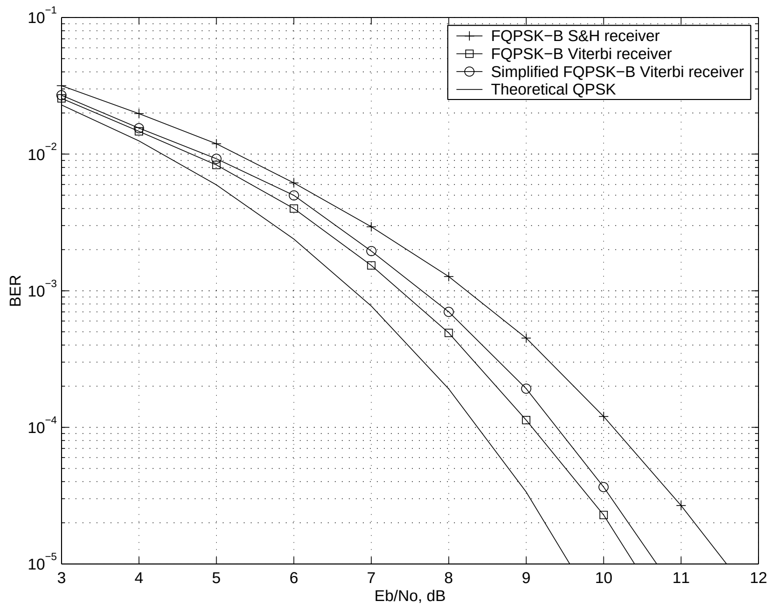

Figure 7: BER performance of FQPSK-B S&H and Viterbi receivers, saturated SSPA

FQPSK. This indicates that the Viterbi receiver works for FQPSK-B almost as well as for unfiltered FQPSK. The simplified FQPSK-B Viterbi receiver suffers a slight degradation with respect to the full Viterbi receiver but is still 0.55 dB better than the S&H FQPSK-B receiver at 10^{-3} BER. At 10^{-5} BER, the full and simplified Viterbi receivers are 1.2 and 0.9 dB better than the S&H receiver, respectively.

CONCLUSION

The performance of a Viterbi receiver for FQPSK-B is simulated and shown to have a 1.2 dB E_b/N_o improvement in bit error rate performance over the traditional sample-and-hold receiver at 10^{-5} BER. A simplified FQPSK-B Viterbi receiver is introduced by appropriately grouping the waveforms to create a 2-state trellis instead of the 16-state trellis needed for the full Viterbi receiver. The reduction in the number of states allows for a factor of eight reduction in computational complexity of the VA and a

Table 1: Comparison of BER performance

	E_b/N_o for 10^{-3} BER (dB)	Loss compared to ideal QPSK at 10^{-3} BER (dB)	E_b/N_o for 10^{-5} BER (dB)	Loss compared to ideal QPSK at 10^{-5} BER (dB)
Viterbi receiver	7.4	0.6	10.4	0.8
Simplified Viterbi receiver	7.65	0.85	10.7	1.1
S&H receiver	8.2	1.4	11.6	2.0

factor of four reduction in the number of correlators required. This simplified Viterbi receiver has only a 0.3 dB degradation compared to the full Viterbi receiver and still provides 0.9 dB improvement over the sample-and-hold FQPSK-B receiver at 10^{-5} BER.

References

[1] Kato, S. and Feher, K., ”Correlated Signal Processor”, U.S. Patent 4,567,602, January 1986.

[2] Feher, K., ”F-Modulation Amplification”, U.S. Patent 5,491,457, February 1996.

[3] ”IRIG Telemetry Standard 106-00”, published by Secretariat of Range Commanders Council, U.S. Army, White Sands, New Mexico, January 2000.

[4] Simon, M.K. and Yan, T.-Y., ”Performance Evaluation and Interpretation of Unfiltered Feher-patented Quadrature Phase Shift Keying (FQPSK)”, JPL TMO Progress Report 42-137, May 1999.

[5] Simon, M.K., Arabshahi, P., and Srinivasan, M., ”Trellis-Coded Quadrature Phase Shift Keying (QPSK) with Variable Overlapped Raised-Cosine Pulse Shaping”, JPL TMO Progress Report 42-136, February 1999.

APPENDIX. THEORETICAL BIT ERROR PROBABILITY OF S&H FQPSK-B RECEIVER IN A LINEAR CHANNEL

The theoretical expression for the bit error probability of FQPSK-B is complicated by ISI introduced by the transmit and receive baseband filters and the correlation between symbols. To derive the probability of error of the sample-and-hold receiver, a method using superposition is applied which averages the probability of error over all possible ISI combinations. The ISI combinations are constrained by the FQPSK trellis. The computational complexity is reduced by considering each filtered waveform separately at the sampling instances only and using superposition. The resulting probability of error expression has a large number of terms, but is well approximated by only 32 terms.

The sixteen FQPSK waveforms can be rearranged as follows to reduce the number of ISI symbols that need to be considered:

$$
\begin{aligned}
s_0(t) &= A, \qquad -\frac{T_s}{2} \le t \le \frac{T_s}{2} \\
s_1(t) &= \begin{cases} A, & -\frac{T_s}{2} \le t \le 0 \\ A\cos\frac{\pi t}{T_s}, & 0 < t \le \frac{T_s}{2} \end{cases} \\
s_2(t) &= 1-(1-A)\sin^2\frac{\pi t}{T_s}, \qquad -\frac{T_s}{2} \le t \le \frac{T_s}{2} \\
s_3(t) &= \begin{cases} 1-(1-A)\sin^2\frac{\pi t}{T_s}, & -\frac{T_s}{2} \le t \le 0 \\ \cos\frac{\pi t}{T_s}, & 0 < t \le \frac{T_s}{2} \end{cases} \\
s_4(t) &= A\cos\frac{\pi t}{T_s}, \qquad -\frac{T_s}{2} \le t \le \frac{T_s}{2} \\
s_5(t) &= \begin{cases} A\cos\frac{\pi t}{T_s}, & -\frac{T_s}{2} \le t \le 0 \\ A, & 0 < t \le \frac{T_s}{2} \end{cases} \\
s_6(t) &= \cos\frac{\pi t}{T_s}, \qquad -\frac{T_s}{2} \le t \le \frac{T_s}{2} \\
s_7(t) &= \begin{cases} \cos\frac{\pi t}{T_s}, & -\frac{T_s}{2} \le t \le 0 \\ 1-(1-A)\sin^2\frac{\pi t}{T_s}, & 0 < t \le \frac{T_s}{2} \end{cases} \\
s_i(t) &= -s_{i-8} \qquad i = 8, 9, ..., 15
\end{aligned} \tag{2}
$$

Let $h(t) = h_t(t) * h_r(t)$ denote the impulse response of the cascaded baseband FQPSK-B transmit filter $h_t(t)$ and receive filter $h_r(t)$. The filtered waveforms after the receive filter, $r_i(t)$, are given by:

$$r_i(t) = h(t) * s_i(t)$$

Denote t_o as the time of the maximum eye opening. Then the sampled values of the filtered waveforms, $r_{i,k}$, are given by:

$$r_{i,k} = r_i(t_o + kT_s)$$

Assuming that only the $-L_1 \le k \le L_2, k \ne 0$ symbols have a significant ISI contribution to the decision made on the $k = 0$ symbol, the decision variable $z_{i,p,q}$ for the $k = 0$ symbol given that $s_i(t)$ was transmitted is:

$$z_{i,p,q} = r_{i,0} + \sum_{k=1}^{L_2} r_{g_k(i,p),-k} + \sum_{m=1}^{L_1} r_{h_m(i,q),m} + n \tag{3}$$

where the g_k's are the possible waveforms k symbols later and the h_m's are the possible waveforms m symbols before, and are defined recursively as follows:

$$
\begin{aligned}
g_k(i,p) &= \mathbf{F}(g_{k-1}(i,p)+1, j_{k-1}+1), \qquad g_0(i,p) = i \\
h_m(i,q) &= \mathbf{B}(h_{m-1}(i,q)+1, c_{m-1}+1), \qquad h_0(i,q) = i
\end{aligned}
$$

where $p = 0, ..., 4^{L_2} - 1$ represent the different forward paths through the trellis starting with the i^{th} waveform and $q = 0, ..., 4^{L_1} - 1$ represent the different backwards paths through the trellis starting with the i^{th} waveform. The j_k's are defined by $p = j_0 + 4j_1 + ... + 4^{L_2-1}j_{L_2-1}$ and the c_k's are defined by $q = c_0 + 4c_1 + ... + 4^{L_1-1}c_{L_1-1}$. $\mathbf{F}(x, y)$ is the element in the x^{th} row and y^{th} column in the forward trellis matrix, **F**, and $\mathbf{B}(x, y)$ is the element in the x^{th} row and y^{th} column in the backwards trellis matrix, **B**,

$$\mathbf{F} = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 12 & 13 & 14 & 15 \\ 0 & 1 & 2 & 3 \\ 12 & 13 & 14 & 15 \\ 12 & 13 & 14 & 15 \\ 0 & 1 & 2 & 3 \\ 12 & 13 & 14 & 15 \\ 0 & 1 & 2 & 3 \\ 8 & 9 & 10 & 11 \\ 4 & 5 & 6 & 7 \\ 8 & 9 & 10 & 11 \\ 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 \\ 8 & 9 & 10 & 11 \\ 4 & 5 & 6 & 7 \\ 8 & 9 & 10 & 11 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 0 & 2 & 5 & 7 \\ 0 & 2 & 5 & 7 \\ 0 & 2 & 5 & 7 \\ 0 & 2 & 5 & 7 \\ 9 & 11 & 12 & 14 \\ 9 & 11 & 12 & 14 \\ 9 & 11 & 12 & 14 \\ 9 & 11 & 12 & 14 \\ 8 & 10 & 13 & 15 \\ 8 & 10 & 13 & 15 \\ 8 & 10 & 13 & 15 \\ 8 & 10 & 13 & 15 \\ 1 & 3 & 4 & 6 \\ 1 & 3 & 4 & 6 \\ 1 & 3 & 4 & 6 \\ 1 & 3 & 4 & 6 \end{bmatrix} \tag{4}$$

The **F** and **B** matrices are constructed so that the row number minus one defines the current waveform (e.g., row one corresponds to $s_0(t)$), and the elements of the row contain the possible waveforms immediately preceding or following that waveform. The preceding waveforms are given in the backwards trellis matrix **B** and the following waveforms are in forward trellis matrix **F**. There are only four possible waveform transitions for each waveform, all which are equiprobable. The random variable n in Equation 3 is filtered Gaussian noise with variance

$$\sigma_n^2 = \frac{N_o}{2}||h||^2$$

where $||h||^2 = \int_{-\infty}^{\infty} |h^2(t)|dt$.

Only the probability of error for waveforms $s_0(t)$ through $s_7(t)$ need to be considered as the probability of error for $s_8(t)$ through $s_{15}(t)$ will be the same. Thus the average probability of a bit error is given by:

$$\begin{aligned} P_e &= \frac{1}{2(4^{L_1+L_2+1})} \sum_{i=0}^{7} \sum_{p=0}^{(4^{L_2})-1} \sum_{q=0}^{(4^{L_1})-1} Q\left(\frac{z_{i,p,q}}{\sigma_n}\right) \\ &= \frac{1}{8(4^{L_1+L_2})} \sum_{i=0}^{7} \sum_{p=0}^{(4^{L_2})-1} \sum_{q=0}^{(4^{L_1})-1} Q\left(\sqrt{\frac{2E_b}{N_o}\frac{(z'_{i,p,q})^2}{||h||^2}}\right) \end{aligned} \tag{5}$$

where $z'_{i,p,q} = z_{i,p,q}/\sqrt{E_b}$. Note that the energy per bit, E_b, is not simply the average of the individual waveform energies because of the ISI. Rather, E_b is given by:

$$E_b = \frac{1}{8(4^{L_1+L_2})} \sum_{i=0}^{7} \sum_{p=0}^{4^{L_2}-1} \sum_{q=0}^{4^{L_1}-1} \int_{-\frac{T_s}{2}}^{\frac{T_s}{2}} [h_t(t) * (s_i(t) + \sum_{k=1}^{L_2} s_{g_k(i,p)}(t - kT_s) + \sum_{m=1}^{L_1} s_{h_m(i,q)}(t + mT_s))]^2 dt$$

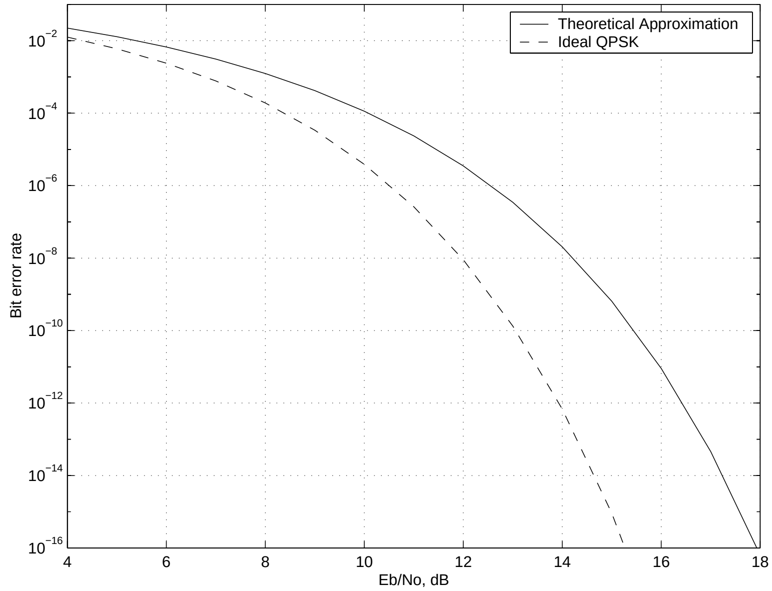

Figure 8: Theoretical bit error probability of FQPSK-B S&H receiver at low BER

The number of terms in Eq. (5) can be quite large for $L_1, L_2 > 2$. However, the $z_{i,p,q}$ terms are sums of different $r_{i,k}$ terms which are just samples of the 16 filtered FQPSK-B waveforms at T_s intervals and only need to be computed once. In addition, for the case of FQPSK-B transmit and receive filters, a good approximation to the bit error probability can be found by setting L_1 = 1 and L_2=0. Using this approximation, the probability of error expression reduces to

$$P_e = \frac{1}{32} \sum_{i=0}^{7} \sum_{q=0}^{3} Q\left(\sqrt{\frac{2E_b}{N_o} \frac{(z'_{i,q})^2}{||h||^2}}\right) \tag{6}$$

where $z'_{i,q} = (r_{i,0} + r_{h_1(i,q),1})/\sqrt{E_b}$. In this case, only 32 terms need to be computed. Figure 2 shows a comparison of Eq. (6) and the simulated BER of the FQPSK-B S&H receiver. As the figure shows, the 32-term approximation matches closely with the simulation results.

The advantage of theoretical expressions is that the bit error performance can be evaluated at extremely low probabilities of error without prohibitively long computer simulations. For example, data containing compressed images may require a bit error rate on the order of 10^{-10} depending on the amount of compression. Figure 8 shows the bit error performance of the FQPSK-B S&H receiver using the 32-term theoretical approximation at low bit error rates. The FQPSK-B S&H receiver requires about 15.4 dB E_b/N_o for a 10^{-10} BER. For very large E_b/N_o, the asymptotic loss of the S&H receiver with respect to ideal QPSK is approximately 3 dB.