Coded Orthogonal Frequency Division Multiplexing for the Multipath Fading Channel

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ABSTRACT

This paper presents a mathematical model for Coded Orthogonal Frequency Division Multiplexing (COFDM) in frequency selective multipath encountered in aeronautical telemetry. The use of the fast Fourier transform (FFT) for modulation and demodulation is reviewed. Error control coding with interleaving in frequency is able to provide reliable data communications during frequency selective multipath fade events. Simulations demonstrate QPSK mapped COFDM performs well in a multipath fading environment with parameters typically encountered in aeronautical telemetry.

KEY WORDS

COFDM, OFDM, Multicarrier Modulation, MCM, Modulation, Error Control Coding, Multipath fading.

INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a method of data modulation that has gained more attention with the development of faster and more efficient signal processing technologies and components. OFDM is used in the European digital audio broadcasting (DAB) standard, and several DAB systems proposed for North America are also based on OFDM [1]. OFDM is in class of modulation techniques known as Multicarrier Modulation (MCM), and refers specifically to transmission in a wireless environment. In a wired environment such as Asymmetric Digital Subscriber Lines (ASDL), Multicarrier Modulation is usually referred to as Discrete Multitone (DMT).

OFDM has been developing for more than thirty years. As its name suggests, it is a method of Frequency Division Multiplexing, but the orthogonality of the carriers allows their individual spectra to overlap, resulting in efficient bandwidth usage along with some other desirable properties such as multipath and echo immunity. Immunity to multipath fading make it very interesting to military telemetry. Virtually any modulation scheme can be used along with OFDM, depending on the desired number of bits per symbol. Two common mappings are differential phase shift keying (DPSK) which is appropriate for low data rates and requires no channel estimation, and quadrature amplitude modulation (QAM) which is more spectrally efficient but requires estimation and tracking of the fading channel [1].

This paper presents an overview of OFDM, a discussion of the principle of orthogonality and how the guard band is implemented to achieve subcarrier orthogonality, and explains how the discrete Fourier transform (DFT) can be used for OFDM modulation and demodulation. The two-ray multipath fading model is presented, the error control coding and equalization for the system are described, and the results of simulations to determine COFDM performance are presented and discussed, followed by the conclusions.

OFDM OVERVIEW

OFDM is a wideband modulation method that divides the available bandwidth into multiple subchannels to take advantage of the benefits possessed by a wideband signal

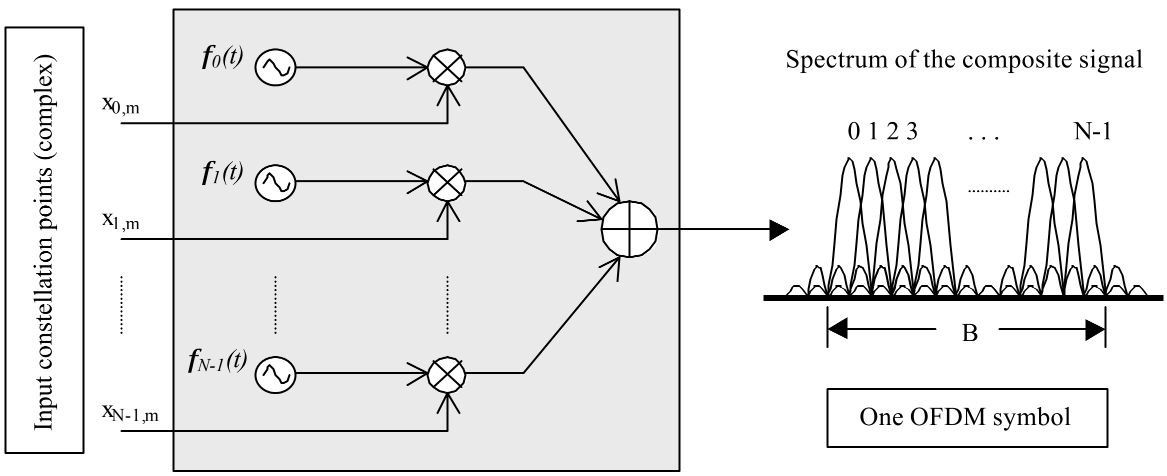

Figure (1) Conceptual method of OFDM symbol creation. All of the operations in the shaded box can be replaced with the inverse DFT.

under multipath fading. Figure (1) presents a diagram of a conceptual method for generating OFDM. The carriers $f_n(t)$ can be thought of as sinusoids or exponentials of the form

$$f_n(t) = \begin{cases} \frac{1}{\sqrt{T}} e^{j2p\frac{W}{N}kt} & if \quad t \in [0,T] \\ 0 & otherwise \end{cases} \quad (1)$$

spaced W/N Hz apart, where W is the available bandwidth. Each carrier is scaled by a complex constellation value $x_{n,m}$ from the input data; the subscript n corresponds to the index number of the carrier, and m is the index of the entire OFDM symbol, or frame. For a continuous transmission, m is an integer, $m \in (-\infty,\infty)$. The constellation points most commonly come from M-ary Quadrature Phase Shift Keying (QPSK) or M-ary Quadrature Amplitude Modulation (M-QAM), but almost any mapping will work. The scaled carriers are then summed to yield the time waveform to be transmitted over the channel,

$$s_m(t) = \sum_{n=0}^{N-1} x_{n,m} f_n(t - mT) \ . \tag{2}$$

An infinite sequence of OFDM symbols or frames is a juxtaposition of all the individual OFDM symbols, and we drop the index *m* to give

$$\begin{aligned} s(t) &= \sum_{m=-\infty}^{\infty} s_m(t) \\ &= \sum_{m=-\infty}^{\infty} \sum_{n=0}^{N-1} x_{n,m} f_n(t - mT). \end{aligned} \tag{3}$$

Since $f_n(t)$ is a rectangular pulse modulated on the carrier frequency kW/N, OFDM is commonly viewed as having N subcarriers, each carrying the lower symbol rate $R_{OFDM}=R_s/N$. Note the symbol rate of each subchannel is the rate at which OFDM symbols or frames are sent.

ORTHOGONALITY AND THE GUARD BAND

The key to bandwidth efficiency is the orthogonality of the carrier waveforms. In a normal frequency division multiplexing (FDM) system the carriers are separated by a guard band to allow them to be received and demodulated through conventional filtering procedures. These guard bands result in a lower spectral efficiency. If the carriers are mathematically orthogonal, they can be arranged so that their sidebands overlap while still allowing reception without interference from adjacent carriers (or intercarrier interference, ICI). Guard bands turn out to be essential in maintaining carrier orthogonality in OFDM, but the guard band is implemented in a manner different from normal FDM.

The OFDM receiver is a bank of demodulators, translating each carrier down to baseband and integrating over one symbol period to recover the raw data. If carriers other than the desired carrier all mix down to frequencies that have a whole number of cycles in the symbol period (τ), then they will integrate to zero over the symbol period. Thus, the carriers will be linearly independent, or orthogonal, if the carrier spacing is a multiple of $1/\tau$. Mathematically, a set of functions will be linearly independent, or orthogonal if [12]

$$\int_a^b \Psi_p \Psi_q^*(t)dt = \begin{cases} K & for\ p = q \\ 0 & for\ p \neq q \end{cases} \tag{4}$$

where * denotes the complex conjugate. There are many sets of orthogonal function, the most famous of which are the complex exponentials which form the basis of the Fourier transform [5]:

$$\Psi_k(t) = e^{jw_k t}$$

$$\text{with} \quad w_k = w_0 + 2p\frac{k}{t} \ . \tag{5}$$

Applying the orthogonality principle in Equation (4) to these functions, we can use the fact that *p* and *q* are integers to show that they are indeed linearly independent. Their orthogonality makes them good candidates for OFDM transmission and suggests the discrete Fourier transform (DFT) as a viable method for generating the transmission waveform.

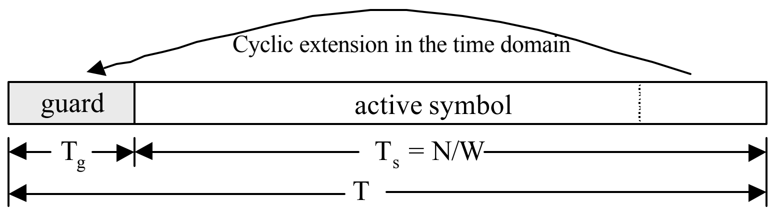

Figure (2) Method for implementing the guard band, creating a cyclic extension of the OFDM symbol.

The only obstacle to using the DFT for OFDM waveform generation is the non-periodic nature of the time domain signal, which can be overcome by adding a guard interval [6] of length T_g, comprised of a copy of the last T_g seconds of the active symbol prefixed to the symbol [7], as seen in Figure (2). This is often called a cyclic prefix because it makes the OFDM symbol appear periodic to the receiver. The received signal then approximates the transmitted signal cyclically (circularly) convolved with the channel impulse response [5], [1].

The length of the guard band must be limited to realize bandwidth efficiency, yet it must be longer than the impulse response of the channel to ensure subcarrier orthogonality and eliminate Inter-Symbol Interference (ISI) and Inter-Carrier Interference (ICI). These benefits generally outweigh the loss in spectral efficiency and SNR due to guard band insertion. To illustrate this, it can be observed that the transmitted energy increases with the length of the cyclic prefix T_g, while the received and sampled signal remains the same. The transmitted energy per subcarrier is

$$\int \left| f_k(t) \right|^2 dt = \frac{T}{T - T_g},$$

and according to Edfors [1], the SNR loss due to the discarded cyclic prefix in the receiver is

$$SNR_{loss} = -10 \log_{10}\left(1 - \frac{T_g}{T}\right). \quad (6)$$

The longer the prefix, the larger the SNR loss. Typically the relative length of the cyclic prefix is kept small, and the ICI- and ISI-free transmission motivates the SNR loss (less than 1dB for $T_g/T<0.2$).

THE DISCRETE FOURIER TRANSFORM

Three primary methods for separating carriers in OFDM have been evaluated during its development [3]. The earliest two used actual filters to separate the bands, and suffered from the difficulty of implementing filters with sharp band edges. The third and most promising method, as first proposed by Weinstein and Ebert [6] uses baseband processing, wherein both transmitter and receiver can be implemented using the discrete Fourier transform (DFT) [9], [5].

Each carrier in the OFDM system can be written in the form

$$s_{n,m}(t) = x_{n,m} e^{j2p f_c t}, \tag{7}$$

where $x_{n,m}$ is the complex magnitude corresponding to the nth subcarrier in the mth OFDM symbol and is nonzero over the time period $(m-1)\mathbf{t}<t<m\mathbf{t}$, where $\mathbf{t}$ is the symbol period. This allows us to rewrite Equation (2) as the complex continuous time average of the carriers in Equation (7) for a given m:

$$s_m(t) = \frac{1}{N}\sum_{n=0}^{N-1} x_{n,m} e^{j2p f_n t} \tag{8}$$

$$\text{where } f_n = f_0 + n\Delta f ,$$

with f_0 as the base frequency and Df as the subcarrier spacing. Without loss of generality, let f_0=0. Substituting for f_n and sampling Equation (8) at a frequency of $1/T$ results in

$$s_m(kT) = \frac{1}{N}\sum_{n=0}^{N-1} x_{n,m} e^{j(2pn\Delta f)kT} . \tag{9}$$

It is convenient at this point to take N samples over the period of one data symbol, yielding the relationship $\mathbf{t}=NT$. Comparing Equation (9) to the general form of the inverse DFT

$$g(kT) = \frac{1}{N}\sum_{n=0}^{N-1} G\left(\frac{n}{NT}\right) e^{j2pnk/N} , \tag{10}$$

we see that the complex function $x_{n,m}$ of n is no more than a definition of the signal in the sampled frequency domain, and $s(kT)$ is the time domain representation. Due to the relationship between the Fourier transform and the discrete Fourier transform

$$\tilde{G}[n] = G(e^{jw})\Big|_{w=\frac{2p}{N}n} ,$$

Equation (10) and Equation (11) are equivalent if

$$\Delta f = \frac{1}{NT} = \frac{1}{\mathbf{t}},$$

which is the same condition as required for orthogonality in Equation (5). Thus, by maintaining orthogonality the OFDM signal can be defined using the Fourier Transform.

The ability to use the DFT is important for two reasons. First, the DFT is a variant on the normal Fourier transform where the signals are sampled and thus periodic in both the time and the frequency domain. This avoids storage and aliasing problems that result from signals with an infinite spectrum or that are not time-limited. This transform conveniently goes along with our guard band insertion which makes each OFDM symbol appear periodic, allowing us to assume circular convolution with the channel transfer function. The second benefit from using the DFT is that it can be calculated cheaply and easily using the fast Fourier transform (FFT).

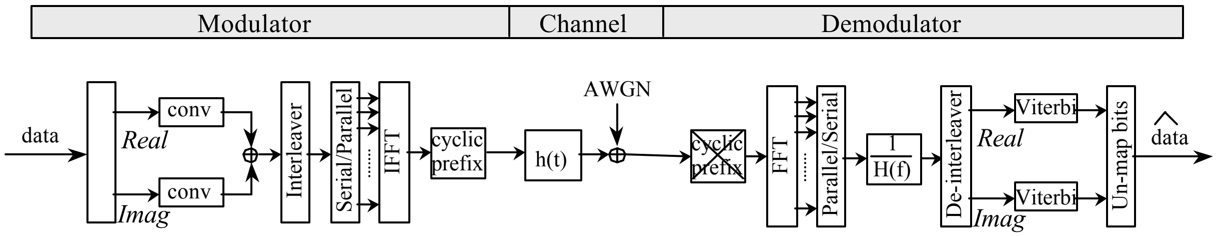

Figure (3) Block diagram for OFDM modulation and demodulation using the FFT.

A block diagram of the OFDM system evaluated in this paper is shown in Figure (3), with the IFFT performing the operations in the shaded area of Figure (1). Input data is mapped using virtually any common M-ary modulation scheme, after which the real and imaginary components of the signal are split and separately encoded with a rate ½ convolutional code. A block interleaver [10] functionally performs an interleaving of the carriers (known as interleaving in frequency), and the IFFT block creates a time domain waveform with the exact frequency content specified by the carrier weights. After the cyclic prefix is added to the OFDM symbol, it is transmitted through the two-ray multipath fading channel and Additive White Gaussian Noise (AWGN) is added. After the cyclic prefix is removed from the OFDM symbol, equalization and the FFT are applied. The bins of the FFT of the time domain signal correspond to the carrier weights, which are subsequently de-interleaved, decoded with the Viterbi algorithm, and un-mapped to give an estimate of the original data stream. A two-ray model of the multipath fading channel, and the encoding and equalization are explained in more detail in the next two sections.

TWO-RAY MULTIPATH FADING MODEL

Multipath propagation is the primary cause of fading for normal radio signals. Multipath propagation results when delayed copies of the desired signal arrive at the receiver; fading occurs when the delayed copies arrive out-of-phase with the main signal component. These delayed copies are typically reflections from fixed terrain features such as trees, mountains or buildings, and from mobile objects such as people and vehicles, and even the ionosphere. Since the delayed copies have an angle of arrival different from that of the main component, directional antenna gain patterns reduce the interference from reflections [8], but this is not always enough.

The typical impulse response for a channel with a single reflection will have the form

$$h(t) = d(t) - \Gamma d(t - t) \qquad (11)$$

where G is the (complex) attenuation of the reflection and t is the delay relative to the main component. The Fourier transform of the impulse response is

$$H(f) = 1 + \Gamma e^{j2pft} \quad . \tag{12}$$

The amplitude and phase of the frequency response of a null at baseband is plotted in Figure (4), where the phase *G* of the reflection varies with the geometry of the channel environment. Notice that some frequencies will be enhanced, while others will be attenuated. If receiver and all other objects affecting the system are stationary, and if the desired signal is relatively narrowband and falls into the portion of the band with significant attenuation, then there will be flat fading and the reception will be degraded. When there is movement, the distortion for a narrowband signal is usually minimized if the bandwidth is less than the correlation bandwidth of the channel, yet there is still a significant chance of severe attenuation. On the other hand, if the desired signal occupies a wider bandwidth, wider than the correlation bandwidth of the channel, it will be subject to more distortion. But because of the wider bandwidth, the variation in total received power will be less, even for significant levels of multipath interference [5].

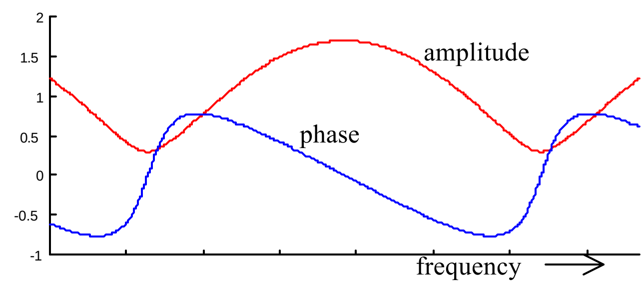

Figure (4) Frequency response of null with τ = 50 ns, Γ=0.7e$^{j\pi/2}$.

CODING AND EQUALIZATION

Using OFDM on the two-ray multipath fading channel model creates a scenario where a null will wipe out only a subset of the transmitted symbols, provided the number of channels *N* is sufficiently high and the null is sufficiently narrow. Applying error control coding, the *C* in COFDM, allows for the possibility of recovering all of the symbols. The optimal rate ½ convolutional codes with constraint lengths *K* = 3 and *K* = 4 [10] were implemented in the COFDM system. The use of a rate *k/n* convolutional code means that *k* input bits produce *n* output bits; in our case there are two output bits for every one input bit. In order to maintain the same information rate over the channel without changing the transmitter power, the input bit rate must increase by a factor of *n/k*. This decreases the power per symbol, i.e. the signal to noise ratio (SNR), effectively decreasing the performance by the same factor. The benefits from making the transmitted symbols co-dependent more than makes up for the loss. By coupling convolutional coding with interleaving in frequency, adjacent symbols that are smeared by a null are co-dependent with symbols that may have had a high SNR, and the effect of the null is spread across many symbols. Following convolutional coding and interleaving with the Viterbi algorithm with soft-decision decoding in the demodulator allows the system to use the symbols with high SNR to help recover the symbols with low SNR. As we shall see, the resulting bit error rates from this system can be very acceptable.

Channel estimation for the system is very simple. A single known OFDM symbol is transmitted, whereupon the receiver calculates the inverse channel filter by subtracting the phase of the received frequency response (signal plus AWGN) from the known OFDM symbol frequency response (estimation and equalization can be performed entirely within the frequency domain). The accuracy of the estimate is dependent on the SNR of the channel. The magnitude of the null can also be estimated using this method, but incorporating the magnitude information into the equalizer destroys the channel state information (CSI) used by the soft-decision Viterbi decoder. This greatly decreases system performance.

The upper bound on the bit-error-rate performance of the Viterbi decoder over the AWGN channel comes from [10]

$$\begin{aligned} P_b &\le \frac{1}{k} \sum_{d=d_{free}}^{\infty} b_d P_d \\ &= \frac{1}{k} \sum_{d=d_{free}}^{\infty} b_d Q\left(\sqrt{2d \frac{E_b}{N_0}} \right) \end{aligned} \tag{13}$$

where b_d is the total number of nonzero information bits associated with the code words of weight d. These values are enumerated by Odenwalder [11] for eight values of d. This is the equation for the theoretical bound plotted along with the performance curves from the simulations (Figure (5)).

RESULTS

The simulations in this paper are for a bit rate $R_b = 10^7$ bits/sec; QPSK mapping creates the constellation points yielding symbol rate $R_s = R_b/2$. This gives an OFDM symbol period $T_O = 1/R_O = 51.2$ µsec for N=256 carriers, and $T_O = 204$ µsec for N=1024 carriers. For the multipath model, Rice, *et al.* [13] found that a typical delay on the multipath fading channel is 50 ns with a magnitude of $G = 0.7$. A 50 ns delay over the channel is equivalent to a channel impulse response 50 ns long, so the guard band is a very small fraction of the OFDM symbol in both cases. The cyclic prefix allows us to assume ISI-free transmission in exchange for a negligible SNR_{loss} of less than -0.005 dB as calculated from Equation (6).

The performance of the channel is based on the location of the null within the frequency band. In a typical telemetry environment the null will 'sweep' across the spectrum. The simulations have assumed the null was stationary, eliminating the need for adaptive equalization, yet still giving us a worst-case scenario for the performance of OFDM on the multipath fading channel. The plot in the upper left hand corner of Figure (5) illustrates the dependence of channel performance on null phase g. This plot also depicts for proper placement of the null, a reflection would arrive at the receiver in phase with the direct component resulting in enhanced performance.

The remainder of the performance plots in Figure (5) illustrate that regardless of the location of the null, error-free transmission is attainable. The *Theoretic* curve, calculated from Equation (13), gives the bound on the performance of coded QPSK on the AWGN channel without frequency selective fading. Since there are only eight values of d used in its calculation, this is a truncated bound, explaining its relationship to the data in the graph for $g = p/2$. The downward slopes of the performance curves implying the possibility of reliable communication in the presence of multipath fading is viewed in stark contrast with the performance of a single QPSK modulated carrier in the presence of multipath fading. Without the aid of COFDM, multipath fading will cause QPSK to give a bit error probability of approximately ½, causing the receiver to lose sync and lose even more data. COFDM allows the transmission to continue uninterrupted through the null. This increased performance comes because the effect of the null is spread across multiple subcarriers/symbols from the combination of frequency interleaving, convolutional coding, and soft-decision Viterbi decoding.

CONCLUSIONS

The two-ray multipath fading model portrays a situation that plagues aeronautical telemetry. Simple QPSK modulation performs very poorly in the presence of multipath fading. COFDM provides a viable solution to this performance problem. Advancements fast Fourier transform (FFT) technology have made OFDM easy to generate, and the insertion of a cyclic prefix in the guard band permits ICI- and ISI-free transmission. The combined efforts of frequency interleaving, convolutional coding and soft-decision Viterbi decoding form subcarrier interdepencies which function to spread the effects of the null across multiple subcarriers, allowing each symbol to be recovered. Simulations for various null locations verify that it is possible to get good transmission over the multipath fading channel.

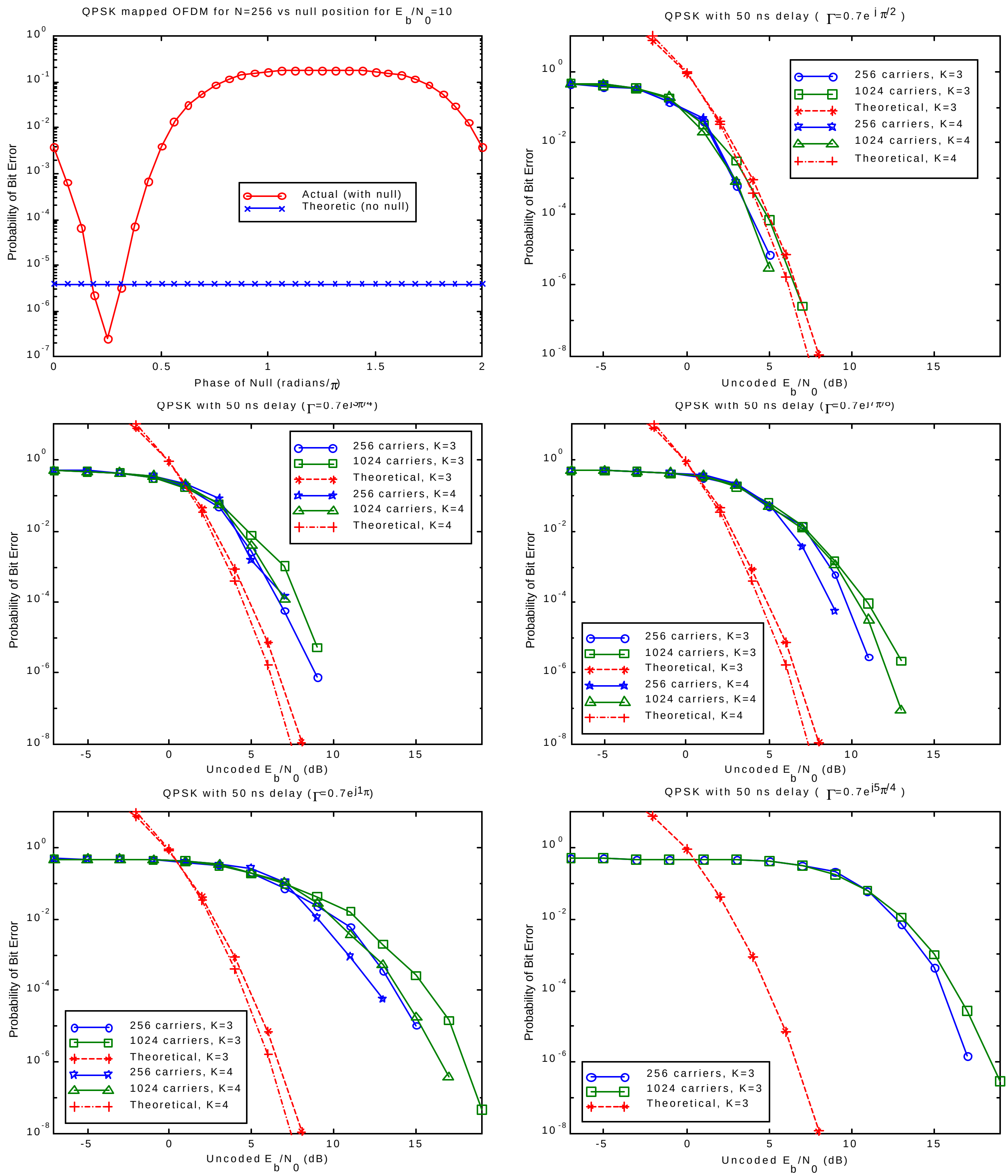

Figure (5) The effects of the phase of the null on telemetry transmission. UL: Probability of bit error as a function of null phase g for fixed E_b/N_0 = 10 dB and no coding. Clockwise from upper right: Performance of theoretical rate ½ convolutionally encoded QPSK in the AWGN channel without fading vs. COFDM on multipath fading channel with null phase $g = \pi/2$, $7\pi/8$, $5\pi/4$, π, $3\pi/4$.

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