Bandwidth Efficient Signaling Using Multiscale Wavelet Functions and its Performance in a Rician Fast Fading Channel Employing Differential Detection

Todd K. Moon
Utah State University, Logan, Utah

Chet Lo
Utah State University, Logan, Utah

ABSTRACT

In this paper, orthogonal wavelets are employed to produce multiscale signaling. It is shown that signaling using these functions is bandwidth efficient compared other signaling schemes, including SFSK and GMSK. For signaling in Rician fast fading channel, it is also shown that scaling functions is superior in term of achieving low level of probability of error. Even for multiscale signaling, the level probability of error achieved by using wavelet is lower than conventional flat-top signaling. The benefits are largest for channels with small $B_D T$, in which the degradation due to fading is most severe.

KEY WORDS

Differential phase shift keying, Fast Rician fading channels, Wavelet, Bandwidth efficient

INTRODUCTION

There has been recent interest in digital signaling waveforms employing orthogonal wavelet and scaling functions [1,2,3,4,5]. These functions can be used to send information on several overlapping scales simultaneously. Because of the orthogonality of the signals, time-overlapped signals are non-interfering at the output of an appropriately designed matched filter in the absence of signal corruption. Furthermore, transmission using these functions may be very spectrally efficient. However, because of the multiple scales, an increase in inter-symbol interference (ISI) is unavoidable in the Rician fast fading channel. This increase in ISI can increase in probability of error in comparison to other signaling schemes. Given the desirability of bandwidth efficient signaling, obtainable using wavelet type signaling, it is of intent to see how multiscale transmission fares in the Rician fading channel. In a Rician fast fading channel with Doppler bandwidth B_D, it has been shown [6] that for BPSK, as the product $B_D T$ increases (the Doppler bandwidth times the signal time), the probability of error generally decreases. This trend is preserved for multiscale signaling. Interestingly, we shall show that when the channel has small $B_D T$, multiscale signaling with wavelet functions having long support (and hence good spectral efficiency) has essentially the same probability of error as conventional signaling.

This paper begins by establishing a compact notation for presenting multiscale signaling. The spectral efficiency of the multiscale transmission using Daubechies' wavelet and scaling functions is then presented. The model for the fading channel is presented and followed by the derivation of expressions for the probability of error.

MULTISCALE TRANSMISSION AND FADING CHANNEL MODEL

There are two basic functions in the time-scale family of wavelets. They are the *scaling function* $f(\cdot)$ and the *wavelet function* $y(\cdot)$. Let $f_{j,k}(t) = 2^{j/2} f(2^j t - k)$ and $y_{j,k}(t) = 2^{j/2} y(2^j t - k)$, by the multiresolution analysis [7], they have the following orthogonalities:

$$\langle f_{j,k}, f_{l,m} \rangle = d_{k,l}, \ \langle f_{j,k}, y_{l,m} \rangle = 0, \quad \text{for } j \geq l, \text{ and} \langle y_{j,k}, y_{l,m} \rangle = d_{j,l} d_{k,m}, \tag{1}$$

where $j,k,l,m \in \mathrm{Z}$, where Z is the set of integers. In wavelet transmission, we use these two basic functions to construct our digital signals. Let $f_T(t) = \frac{1}{\sqrt{T}} f(t/T)$ and $y_T(t) = \frac{1}{\sqrt{T}} y(t/T)$. These scaled functions can be used to transmit one symbol of information every T seconds according to the baseband signal model

$$s_d(t) = \sqrt{2S}\, g(t), \text{ where } g(t) = \sum_{i \in \mathbb{Z}} a_i f_T(t - iT), \tag{2}$$

S is the signal energy, $a_i \in$ ☝, and ☝ is some real or complex signal constellation with 2^{n_b} signal points. T is said to be the symbol time, the time interval between transmission of adjacent symbols, even though the duration of the symbol is qT seconds. Each scaling function transmits a single symbol. By the shift orthogonality property, the outputs of a matched filter provide sufficient information to detect the received signal independently of any other symbols. By properties (1), a signal spanning the same space can be written using basis functions f_{2T} and y_{2T}, each with independent data streams:

$$g(t) = \sum_{i \in \mathbb{Z}} a_{y,i} y_{2T}(t - 2iT) + a_{f,i} f_{2T}(t - 2iT). \tag{3}$$

The signal amplitudes $a_{y,i}$ and $a_{f,i}$ are drawn from constellations containing 2^{n_b} signals. The data rate of (3) is the same that of (2). The transmission using N_s scales can be written as

$$g(t) = \sum_{s=1}^{N_s} \sum_{i \in \mathbb{Z}} a_{y,s,i} y_{2^s T}(t - i2^s T) + \sum_{i \in \mathbb{Z}} a_{f,i} f_{2T}(t - 2iT). \tag{4}$$

For the sake of the computations below, it will be convenient to introduce more compact notation. Let

$$p_s(t) = \begin{cases} y_{2^s T}(t) & 1 \le s \le N_s \\ f_{2^{N_s} T}(t) & s = N_s + 1, \end{cases} \tag{5}$$

represent the scaling function and the wavelets on the different scales, depending on the value of s. Throughout the paper, let $s' = \min(s, N_s)$ and $s' = \min(s, N_s)$ denote the actual scale number and let $T_{s'} = 2^{s'} T$. The signal amplitudes on scale s at the i^{th} symbol be indexed by multiples of $2^{s'}$, with

$$b_{s,2^{s'}i} = \begin{cases} a_{y,s,i} & 1 \le s \le N_s \\ a_{f,i} & s = N_s + 1, \end{cases} \tag{6}$$

It is also convenient to let $q_s = 2^s q$ (the length of the support of the signal on scale s), with $q_s = q_{s'}$ when $s = N_s + 1$. Using these notational conventions, the signal (4) can be written as

$$g(t) = \sum_{s=1}^{N_s+1} \sum_{i \in C} b_{s,2^{s'}i} p_s(t - iT_{s'}). \tag{7}$$

It is interesting to examine the spectral efficiency of multiscale signaling. Due to the multiresolution property of the scaling functions, it can be shown [4] that if the signal constellation on each scale is the same, the power spectrum of the multiscale signal (7) is the same as the power spectrum of the single-scale signal (2). The spectral efficiency of the signal may be determined by means of the fractional out-of-band power (FOOBP), defined by

$$h(B) = 1 - \frac{\int_{-B/2}^{B/2} G(f)df}{\int_{-\infty}^{\infty} G(f)df}, \tag{8}$$

$G(f)$ is the power spectral density of the signal. Figure 1 illustrates the FOOBP for transmission using scaling functions from the D_4, D_6, D_8 and D_{10} families. Also shown, for comparison, is the FOOBP for flat-topped pulse signaling, MSK, SFSK, and GMSK with $h = 0.36$ [8]. For each waveform, 4-PSK signaling (2 bits/symbol) is used. Being a continuous signal, the D_4 scaling function is more spectrally localized than the flat-topped pulsed, but not as much as MSK. Scaling functions having more coefficients, hence better regularity, have better localization. The D_6 scaling function has spectral localization similar to that of MSK, D_8 and D_{10} do better than MSK. The longer scaling functions do as well as GMSK or SFSK or other spectrally efficient techniques. Even longer scaling functions are possible, resulting in even better spectral localization. The only performance penalty for longer signals is a slight increase in latency at the matched filter, and the need for filters matched at every scale.

For the purposes of analysis through the fading channel, the signal of (7) is assumed to be binary, $b_{s,i} \in \pm 1$. The model for the fast fading Rician channel is [6],

$$\bar{r}(t) = \bar{s}_d(t) + \bar{s}_r(t) + \bar{n}_0(t), \text{ where } \bar{s}_d(t) = \mathrm{Re}\{\sqrt{2S} g(t) \exp[j2pf_c t]\} \tag{9}$$

and $g(t)$ is given by (7). The signal $\bar{s}_r(t)$ is the fading component, given by

$$\bar{s}_r(t) = \mathrm{Re}\{x(t)g(t-t_D)\exp[j2p(f_c+f_D)(t-t_D)]\} \tag{10}$$

where t_D is the delay of the fading component, f_D is the Doppler shift of the signal, and $x(t)$ is a complex, stationary, zero-mean Gaussian random process with autocorrelation function

$$R_x(t) = \tfrac{1}{2}E[x^*(t)x(t+t)] = Dr_x(t). \tag{11}$$

D a constant and $r_x(t)$ is normalized so that $r_x(0)=1$. The baseband equivalent fading component is

$$s_r(t) = x(t)\sum_{s=1}^{N_s+1}\sum_{i\in\Gamma} b_{s,2^{s'}i}\,p_s(t-t_D-iT_{s'})\exp[j2p(f_Dt-(f_c+f_D)t_D)]. \tag{12}$$

The channel also introduces the AWGN baseband noise process $n_0(t)$ with correlation

$$R_n(t) = N_0 d(t).$$

The baseband equivalent received signal is $r(t) = s_d(t) + s_r(t) + n_0(t)$.

The receiver uses filters matched to the signal on each scale and employs differential detection. The receiver for scale s correlates the received signal with a signal $p_s(t-kT)$--- the matched waveform starting at the k^{th} signal interval for $k\in 2^{s'}\Gamma$ --- and integrates over $[kT,(k+q_s)T]$. The output of the matched filter on scale s at the sample instant is

$$z_s((k+q_s)T) \equiv z_{s,1}(k) = c_s((k+q_s)T) + r_s((k+q_s)T) + n_s((k+q_s)T) \tag{13}$$

where c_s is the direct signal part, r_s is the fading signal part, and n_s is the noise part. The signal part can be written as

$$c_s((k+q_s)T) \equiv c_{s,1}(k) = \int_{kT}^{(k+q_s)T} p_s(t-kT)s_d(t)dt = b_{s,k}\sqrt{2S}\,. \tag{14}$$

The noise part of the output,

$$n_s((k+q_s)T) \equiv n_{s,1} = \int_{kT}^{(k+q_s)T} p_s(t-kT)n_0(t)dt = N_0, \tag{15}$$

due to the unit-energy of the waveform $p_s(t)$. The fading component output of the matched filter is

$$r_s((k+q_s)T) \equiv f_{s,1}(k) = \int_{kT}^{(k+q_s)T} p_s(t-kT)s_r(t)dt\,. \tag{16}$$

For differential detection, the matched filter output at the previous sample interval is also used, which is

$$z_s((k-2^{s'}+q_s)T) = c_s((k-2^{s'}+q_s)T) + r_s((k-2^{s'}+q_s)T) + n_s((k-2^{s'}+q_s)T) \tag{17}$$

PROBABILITY OF ERROR

Although the symbols at other scales and shifts are orthogonal to the symbol transmitted at scale s, and interval k transmitted in the direct portion of the received signal, the random multiplicative factor and delay of the fading component $s_r(t)$ makes it so that the elements of $s_r(t)$ are not necessarily orthogonal to signal at scale s or to each other. The probability of error must account for this interference for each possible bit pattern in the interfering signals and for each alignment of the matched filter relative to the other scales.

Let $_{s,k}$ be the set of bits on other scales that overlap with the signal on scale s and starting position k and let $P_{s,k}(e \mid I_{s,k})$ denote the probability of (bit) error for the k^{th} position symbol at scale s, given all the interfering bits. Note that it is necessary to express the probability as a function of the symbol interval k, since amount of overlap of the matched filter on scale s with other symbols depends upon the position of the matched filter relative to the symbols on the other scales. The conditioning on the bits may be eliminated by

$$P_{s,k}(e) = \frac{1}{\left|I_{s,k}\right|} \sum_{\mathrm{I}_{s,k}} P_{s,k}(e \mid I_{s,k}), \tag{19}$$

where the sum is taken over all possible interfering bits. The probability of error for symbols at scale s is obtained by averaging the probability of errors over the different matched filter alignments,

$$P_s(e) = \frac{1}{2^{N_s - s'}} \sum_{i=0}^{2^{N_s - s'} - 1} P_{s,2^{s'} i}(e). \tag{20}$$

Finally, the overall probability of error is a combination of the probabilities on each scale, weighted by the fraction of bits in the overall transmission that are sent on each scale,

$$P(e) = \sum_{s=1}^{N_s+1} \frac{1}{2^{s'}} P_s(e). \tag{21}$$

Conditioned upon the interfering bits, the fading components of the matched filter outputs are Gaussian and the vector

$$\mathbf{v}(k \mid I_{s,k}) \equiv \begin{bmatrix} z_{s,1}(k) - c_{s,1}(k) \\ z_{s,2}(k) - c_{s,2}(k) \end{bmatrix} = \begin{bmatrix} f_{s,1}(k) - n_{s,1} \\ f_{s,2}(k) - n_{s,2} \end{bmatrix} \tag{22}$$

is Gaussian with zero mean and covariance

$$\mathrm{K_s}(k \mid \mathrm{I}_{s,k}) = \frac{1}{2} E[\mathbf{v}^{\mathrm{H}}(k \mid \mathrm{I}_{s,k}) \mathbf{v}(k \mid \mathrm{I}_{s,k})] = \begin{bmatrix} \mathrm{K}_{s,11}(k) & \mathrm{K}_{s,12}(k) \\ \mathrm{K}_{s,21}(k) & \mathrm{K}_{s,22}(k) \end{bmatrix}. \tag{23}$$

Assuming that $n_0(t)$ and $x(t)$ are statistically independent, then

$$\mathrm{K_s}(k \mid I_{s,k}) = \begin{bmatrix} n_{s,11} & n_{s,12} \\ n_{s,21} & n_{s,22} \end{bmatrix} + \begin{bmatrix} r_s(k;1,1 \mid I_{s,k}) & r_s(k;1,2 \mid I_{s,k}) \\ r_s(k;2,1 \mid I_{s,k}) & r_s(k;2,2 \mid I_{s,k}) \end{bmatrix}, \tag{24}$$

where the correlation of the fading components is

$$r_s(k;n,m \mid I_{s,k}) = \tfrac{1}{2} E[r_{s,n}(k) r^*_{s,m}(k) \mid I_{s,k}]. \tag{25}$$

In the appendix, expressions for the correlation of the fading components are developed.

Let $q_{s,k}$ be the difference in angle between $z_{s,1}(k)$ and $z_{s,2}(k)$. If the bits at interval k and $k - 2^s$ are the same, then a correct decision is made if $-p/2 < q_{s,k} < p/2$. The probability of this event not occurring is [6]

$$P_{s,k}(e \mid I_{s,k}) = F_{s,k}(-p/2) - F_{s,k}(p/2), \tag{26}$$

where $F_{s,k}$ is given below. If the bits at interval k and $k-2^s$ are of opposite sign, then a correct decision is made if $p/2 < q_{s,k} < 3p/2$ and the probability of error is

$$P_{s,k}(e \mid I_{s,k}) = F_{s,k}(p/2) - F_{s,k}(3p/2) = F_{s,k}(p/2) - F_{s,k}(-p/2) \; . \tag{27}$$

The function $F_{s,k}$ is defined by [9]

$$F_{s,k}(y) = \int_{-p/2}^{p/2} \frac{e^{-E_{s,k}}}{4p} \left[\frac{W_{s,k} \sin(\Delta\Phi_{s,k} - y)}{U_{s,k} - V_{s,k} \sin t - W_{s,k} \cos(\Delta\Phi_{s,k} - y) \cos t} + \frac{h_{s,k} \sin y - l_{s,k} \cos y}{1 - (h_{s,k} \cos y - l_{s,k} \sin y) \cos t} \right] dt \tag{28}$$

where

$$E_{s,k} = \frac{U_{s,k} - V_{s,k} \sin t - W_{s,k} \cos(\Delta\Phi_{s,k} - y) \cos t}{1 - (h_{s,k} \cos y - l_{s,k} \sin y) \cos t} \, ,$$

$$W_{s,k} = \sqrt{r_{1,s,k} r 2,s,k}, \; \Delta\Phi_{s,k} = \arg[c_{s,2}(k)] - \arg[c_{s,1}(k)], \; U_{s,k} = \frac{1}{2}(r_{1,s,k} + r_{2,s,k}), \; V_{s,k} = \frac{1}{2}(r_{2,s,k} - r_{1,s,k})$$

$$h_{s,k} = \frac{\mathrm{Re}\{K_{s,12}(k)\}}{\sqrt{K_{s,11}(k) K_{s,22}(k)}}, \; l_{s,k} = \frac{\mathrm{Im}\{K_{s,12}(k)\}}{\sqrt{K_{s,11}(k) K_{s,22}(k)}}, \; r_{1,s,k} = \frac{|c_{s,1}(k)|^2}{2K_{s,11}(k)}, \; r_{2,s,k} = \frac{|c_{s,2}(k)|^2}{2K_{s,11}(k)}$$

For the binary differential signaling described in [6], substantial simplifications result in these equations. However, for multiscale signaling, such simplification is not possible so the results presented below are obtained by numerical integration of (28).

To compute $F_{s,k}$ it is necessary to know the correlation matrix $\mathrm{K}_s(k \mid I_{s,k})$ which requires knowing the correlations of the fading components. Expressions for the fading components can be found in [10]. Due to the complexity of the expression for the correlation of the fading component and the number of potential interfering bits $I_{s,k}$, it is not computationally feasible to compute the correlation for every possible interfering bit pattern. An approximation is therefore made to the probability of error by averaging the value of $F_{s,k}$ obtained over 500 randomly generated bit patterns. (We determined by simulation that the probability of error calculated by averaging $F_{s,k}$ obtained from 500 and 10000 randomly generated bit patterns are basically the same.)

Plots of probability of error have been generated to demonstrate the performance of the wavelets. In these plots, the correlation function is for a land-mobile fading channel, $R_x(t) = DJ_0(2pB_D t)$. The particular case of $f_d = 0$, $t_D = 0$, and $S/D = 10\,\mathrm{dB}$ is shown in the probability of error plots. As a summary, there are four general observations. First, for both small and large $B_D T$, single scale wavelet signalings (signalings only with the scaling functions) generally perform better than conventional (single-scale flat-topped pulse, as reported in [6]) signal. Second, there is very little change in the probability of error among single scale wavelet signals with different support. Third, the penalty for using multiscale signaling, in term of increase in probability of error, reduces for using wavelets with longer support. And fourth, the penalty from one scale to next diminishes with the increases of the number of scales.

We observe from Figure 2 that the performance variation as the number of scales increases the smallest for the D_8 signaling. Figure 3 illustrates the probability of error for $B_D T = 0.01$ for conventional and wavelet signalings. In Figure 3(a), wavelet signalings are with one scale. We can see that there is approximately a 1dB improvement of wavelets over conventional signaling. There is very little change in the probability of error among D_2, D_4, D_6 and D_8. Figure 3(b) illustrates the probability of error for four scales of multiscale signalings. From the plots, we can see that D_4, D_6 and D_8 perform just as well as conventional signaling. However, there is a huge penalty for D_2 due to the increase in correlation among signals from different scales. Figure 4 illustrates the probability of error for $B_D T = 0.2$. The basic observation is the same for Figure 4(a) as for Figure 3(a). In Figure 4(b), we can observe that the penalty is larger when $B_D T = 0.2$ then that is when $B_D T = 0.01$ for all the families of wavelet except for D_2. Actually, D_2 with four scales is performing better in channels with $B_D T = 0.2$ then in channel $B_D T = 0.01$ while all other wavelets are performing not as good. Figure 5 illustrates the probability of error for $B_D T = 0.5$. We observe a diminishing increment of penalty as the number of scales increases. In Figure 5(a), while D_2 with single scale has around 1 dB improvement over conventional signaling through out the range shown, all signals with one scale for other families of wavelets start out with a better performance when E_b / N_0 is low and become having not as good a performance then conventional signaling as E_b / N_0 getting bigger. The crossing begins at around $E_b / N_0 = 12$ dB, first for D_8, then for D_6, and then for D_4 at around $E_b / N_0 = 14$ dB. The above order also shows the increasing performance through different wavelet families. As for two scale signals, Figure 5(b) illustrates that there is a penalty for all wavelet signalings. D_2 is performing as conventional signaling, while all other wavelets do not perform as well. D_8 being a wavelet having a longer support then D_6 suffers less performance penalty. When the number of scale increases to three, Figure 5(c) shows that as the increment of penalty is the highest, D_2 becomes the worst performer. Now, the order of lower probability of error becomes conventional signaling, follows by D_8, D_4, D_6, then D_2. As the increment of penalty diminishes, the plots of probability of error for four scales signaling in Figure 5(d) is nearly the same as that in Figure 5(c).

CONCLUSIONS

As this paper has demonstrated, wavelet signals can be used for bandwidth efficient communication. Most of the benefits are obtained when single scale scaling functions are used for transmission. We shown that the longer the support of the wavelet, the more bandwidth efficient. For signals over the fast Rician fading channel, the performance does not seem to depend significantly upon the type of wavelet used, with shorter wavelets providing only slightly more improvement than longer wavelets. However, if we are considering multiscale wavelet communication, there is no change in bandwidth efficiency over single scale wavelet communication. But for the fading channel, there is a penalty in terms of increasing probability of error for multiscale signaling. This penalty decreases as we are using wavelets with longer support. At the same time, there is also diminishes with the increment of the penalty when the number of scales is higher.

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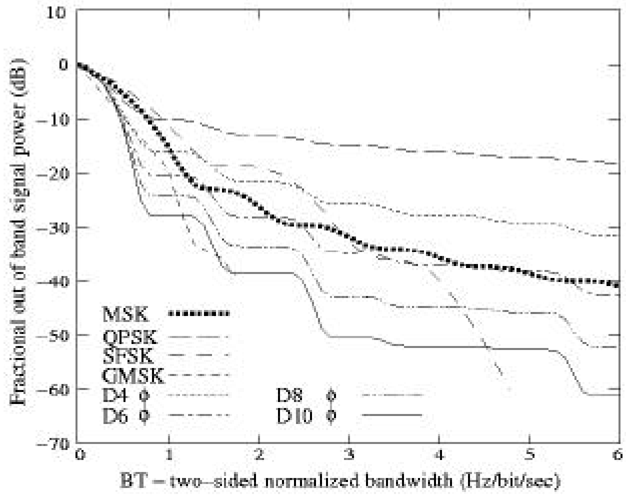


Figure 1: Fractional out-of-band power (FOOBP) for QPSK, MSK, GMSK, and scaling functions with 4,6,8, and 10 coefficients.

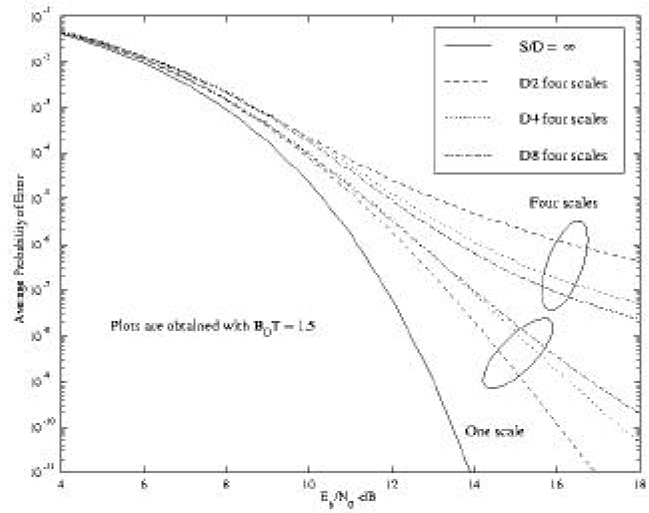
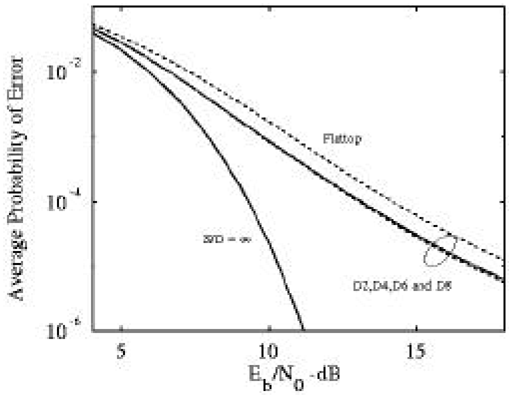
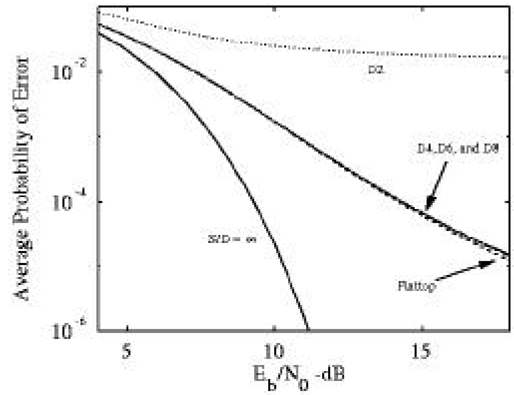


Figure 2: Illustration of probability of error penalty for multiscale wavelet signalings.

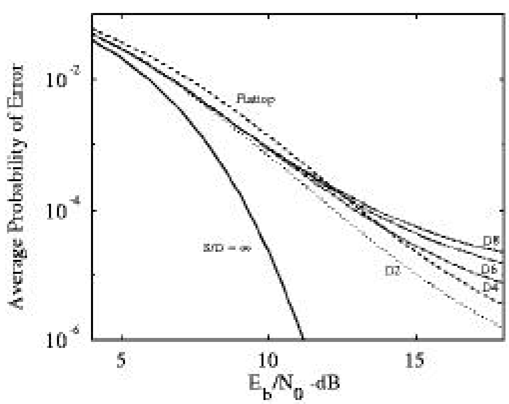


(a) Comparison of probability of error for conventional signaling and single scale wavelet signalings.



(b) Comparison of probability of error for conventional and wavelet signalings with four scales.

Figure 3: Probability of error comparisons for $B_D T = 0.01$.



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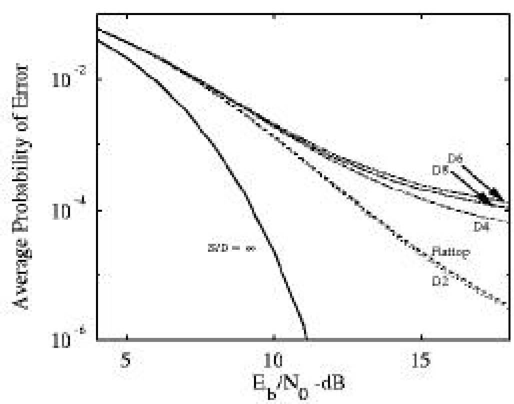
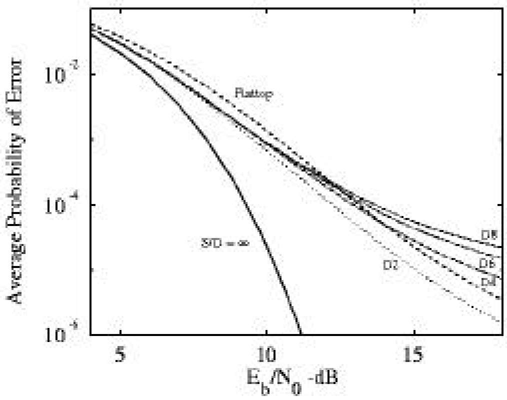
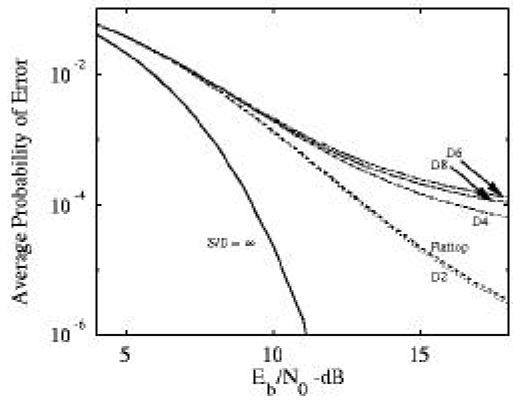


(b) Comparison of probability of error for conventional and wavelet signalings with four scales.

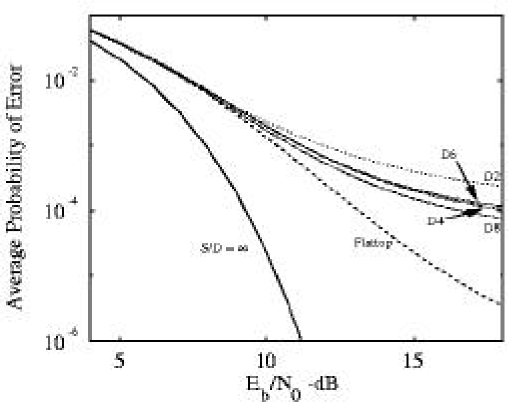
Figure 4: Probability of error comparisons for $B_D T = 0.2$.



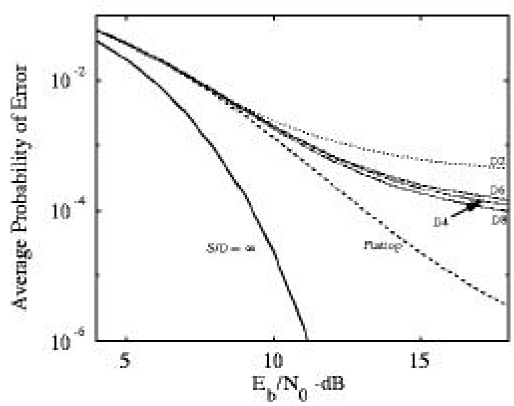
(a) Comparison of probability of error for conventional signaling and single scale wavelet signalings.



(b) Comparison of probability of error for conventional and wavelet signalings with two scales.



(c) Comparison of probability of error for conventional and wavelet signalings with three scales.



(d) Comparison of probability of error for conventional and wavelet signalings with four scales.

Figure 5: Probability of error comparisons for $B_D T = 0.5$.