Optimum Subcarrier Deviation for PCM/FM+FM/FM Systems using IRIG Constant Bandwidth Channels

William P. Osborne and Donald E. Whiteman
Center for Space Telemetering and Telecommunications Systems
New Mexico State University
Las Cruces, New Mexico

Abstract

A typical PCM/FM system is designed for a peak to peak carrier deviation of 0.7 times the bit rate of the PCM data. This value of PCM data deviation has been shown to be optimum for both PCM/FM [1] and PCM/FM+FM/FM [2] systems. However, the optimum deviation of the carrier by the FM subcarrier for a PCM/FM+FM/FM system is dependent upon the specific subcarrier type and required subcarrier signal to noise ratio at the receiver output.

This paper introduces a simple method to calculate the optimum deviation of the carrier by FM subcarriers for a PCM/FM+FM/FM system. The method developed is used to calculate the optimum value of subcarrier deviation for two sample PCM/FM+FM/FM systems when IRIG constant bandwidth channels are used as FM subcarriers. The calculated optimum values of FM subcarrier deviation of the carrier, subcarrier mod index, are utilized in a companion paper to study the performance of PCM/FM+FM/FM systems [3].

The work presented herein can be extended to calculate the optimum deviation of the carrier by FM subcarriers for any PCM/FM+FM/FM system.

Key Words

PCM/FM+FM/FM, PCM/FM, Subcarrier Deviation, Subcarrier Modulation Index

Introduction

The transmission of PCM/FM is typically accomplished by setting the PCM peak deviation at 0.35 times the bit rate and the receiver intermediate frequency, IF, bandwidth equal to the bit rate, R_b. These values were determined to be optimum by Tjhung and Wittke[l] for a PCM/FM system. A variation of the PCM/FM system is to add subcarriers to the PCM data stream and modulate an FM transmitter with the composite signal. This type of system is called a PCM/FM+FM/FM system. The optimum mod index for the PCM data on the carrier is 0.35 times R_b per Law[2] and Osborne/Whiteman[3]. The optimum subcarrier mod index is dependent upon the specific PCM/FM+FM/FM system.

This paper demonstrates the calculation of the optimum subcarrier deviation, mod index, for two PCM/FM+FM/FM transmission systems. The first PCM/FM+FM/FM transmission system studied contains two IRIG Class A Constant Bandwidth Channel subcarriers. The subcarriers are at 64 and 96 kHz. These subcarriers are combined with a 32 kBPS PCM data stream prior to an FM transmitter. The spectrum of the FM transmitter input is shown in Figure 1.

The second PCM/FM+FM/FM transmission system consists of a single IRIG Class A subcarrier placed at the first null of the 32kBPS PCM data stream. The resulting spectrum, FM transmitter input, is shown in Figure 1.

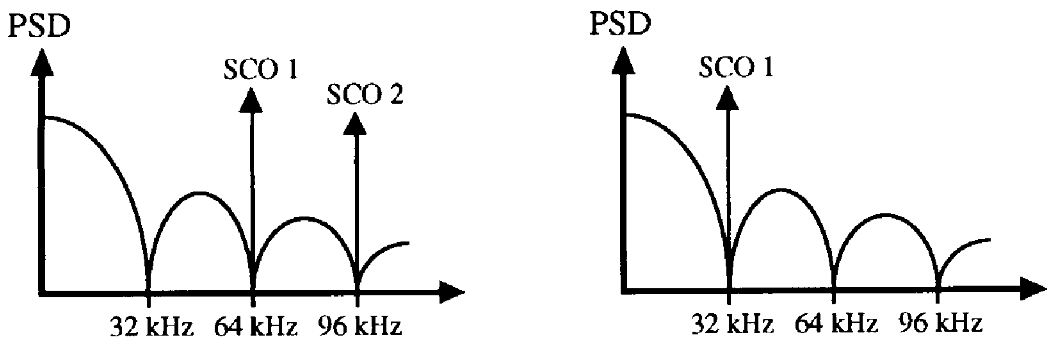

Figure 1 - Spectrum of PCM Data with Dual and Single Subcarriers

Derivation of Subcarrier Signal to Noise Ratio

The subcarrier S/N, signal to noise ratio, is the signal to noise ratio of the subcarrier, SC, data at the output of the subcarrier receiver LPF, lowpass filter. The subcarrier portion of a PCM/FM+FM/FM receiver can be modeled as shown in Figure 2.

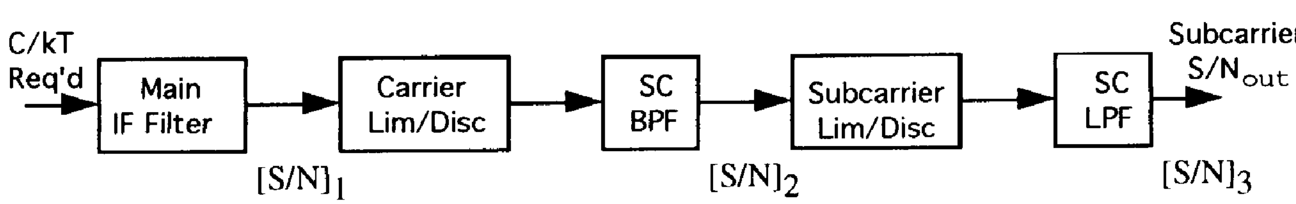

Figure 2 - Subcarrier Receiver

The required value of carrier to noise ratio, C/kT, for this system can be calculated for a given subcarrier output signal to noise ratio, S/N_{out} or $[S/N]_3$. [2,4,5,6]

There are several requirements placed on the subcarrier receiver in order to assure proper operation. First, the S/N at the input to the carrier limiter/discriminator, $[S/N]_1$, must be >12 dB. Second, the S/N at the input to the SC limiter/discriminator, $[S/N]_2$, must be $\geq$ 12 dB. Finally, the carrier limiter/discriminator should reach threshold prior to the SC limiter/discriminator. [2,4] The noise power spectral density, PSD, out of the main IF filter, Figure 3, can be used to calculate $[S/N]_1$. [4,5,6]

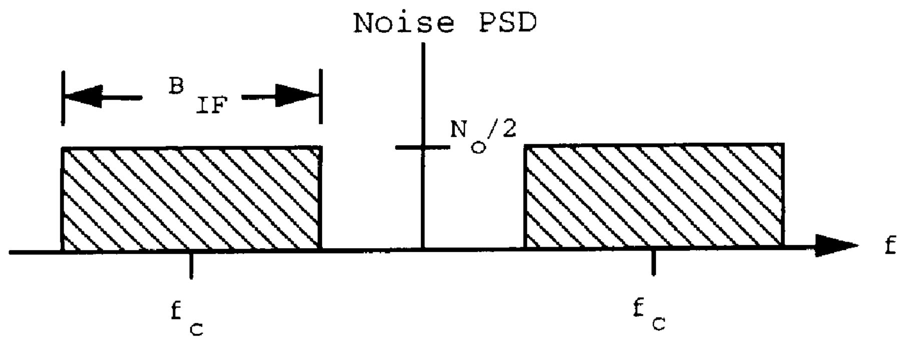

Figure 3 - IF Filter Output Noise Spectrum

The value of $[S/N]_1$ is

$$[S/N]_1 = (A^2/2)/(2(N_0/2)(B_{IF}))$$

where, B_{IF} = Main IF Filter Bandwidth
$C = A^2/2$ = Transmitted Carrier Power
$C/kT = (A^2/2)/N_o$ = Carrier to Noise Ratio.

Since we require $[S/N]_1$ to be $\geq$ 12 dB, the required value of C/kT to yield 12 dB at the carrier limiter/discriminator input is

$$[C/kT]Req'd_l = 12 \text{ dB} + 10 \log B_{IF}.$$

At the output of the SC bandpass filter, BPF, the noise power spectral density is $f^2 N_0/A^2$, Figure 4. The resulting noise power is [5,6]

$$\text{Noise Power}_2 = 2 \int_{f_{sc} - B_{sc}/^{2}}^{f_{sc} + B_{sc}/2} \text{PSD Noise} \approx 2\,[f_{sc}^{2}\, N_o/A^2]\, B_{sc} = 2\, B_{sc}\, [f_{sc}^{2}\, N_o/A^2]$$

where f_{sc} = subcarrier frequency
B_{sc} = subcarrier bandpass filter bandwidth.

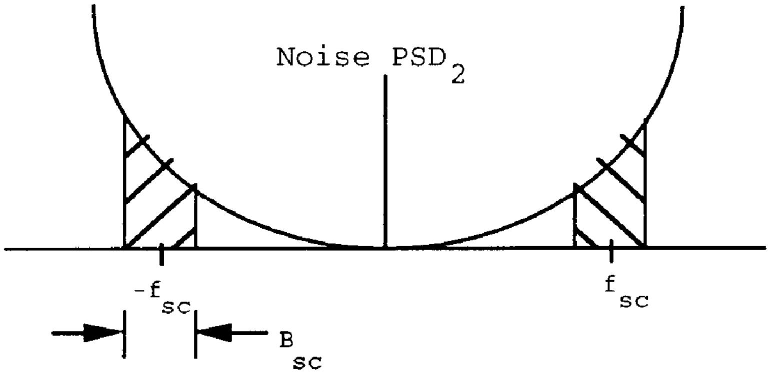

Figure 4 - Subcarrier BPF Output Noise PSD

The signal power at the output of the SC Limiter/Discriminator is

$$\text{Signal Power}_2 = \Delta f_{sc/c}^{\ 2} / 2,$$

where $\Delta f_{sc/c}$ is the peak deviation of the subcarrier on the carrier. The resulting $[S/N]_2$ value is [4]

$$[S/N]_2 = (\Delta f_{sc/c}^{\ 2} / 2) / (2B_{sc}[f_{sc}^{\ 2}N_o/A^2]) = 1/2\ (\Delta f_{sc/c}/ f_{sc})^2\ (l/B_{sc})\ (A^2/2)/N_o$$

Letting the modulation of the carrier by the subcarrier, msc, equal $(\Delta f_{sc/c}/f_{sc})$ yields

$$[S/N]_2 = 1/2\ (msc)^2\ (l/B_{sc})\ (C/kT)$$

Since the required $[S/N]_2$ at the input to the SC lim/disc is 12 dB the required value of C/kT is

$$[C/kT]Req'd_2 = 12dB - 10\log(1/2B_{sc}) - 20\log(msc)$$

Finally, the signal to noise ratio at the output of the subcarrier LPF can be calculated. The Noise PSD at the output of the SC LPF is shown in Figure 5. [4,5,6]

The noise power at the output of the subcarrier LPF is [4]

$$\text{Noise Power}_3 = \int_{-f_m}^{f_m} f^{\ 2}N_o/D^2 = 2N_0/D^2 \int_0^{f_m} f^2 = [2\ N_0/D^2 f^{\ 3}/3] * \Big|_0^{f_m} = 2/3\ N_o/D^2 f_m^{\ 3}$$

where f_m is the maximum message frequency and D^2/N_0 is the carrier to noise ratio at the input to the SC LPF. The signal power is [4]

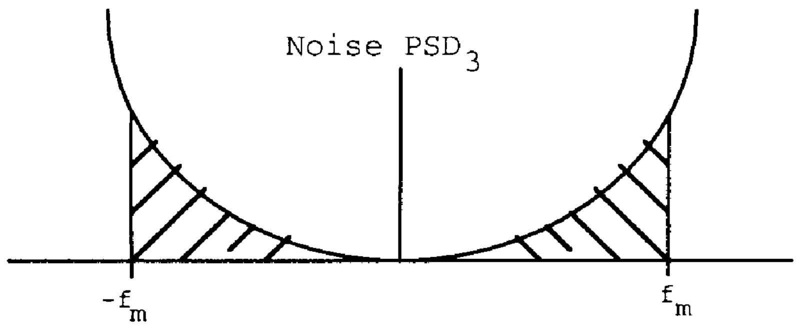

Figure 5 - Subcarrier LPF Output Noise PSD

$$\text{Signal Power}_3 = \Delta f/_{m/sc}{}^2/2\ ,$$

where $f_{m/sc}$ is thc peak deviation of the message on the subcarrier voltage controlled oscillator, VCO. The resulting S/N at the subcarrier LPF output is [4]

$$[S/N]_3 = (\Delta f_{m/sc}{}^2/2)/(2/3(N_o/D^2)f_m{}^3) = 3/2(\Delta f_{m/sc}/f_m)^2\ ((D^2/2)/N_o)\ (l/f_m)$$
$$= 3/2(\Delta f_{m/sc}/f_m)^2(l/f_m)[S/N]_2 B_{sc} = 3/2(\Delta f_{m/sc}/f_m)^2(B_{sc}/f_m)1/2(msc)^2(l/B_{sc})[C/kT]$$
$$= 3/4(\Delta f_{m/sc}/f_m)^2(msc)^2(l/f_m)[C/kT]$$

Therefore, the required C/kT to meet the subcarrier $[S/N]_3$ requirement is

$$[C/kT]Req'd_3 = [S/N]_3\text{-l0 log } [3/4(\Delta f_{m/sc}/f_m)^2(msc)^2(l/f_m)]$$

The optimum operating point, mod index, for the subcarriers should minimize the required value of C/kT for a given required subcarrier signal to noise ratio. Using the previous three equations for C/kT req'd the optimum mod index for the PCM/FM+FM/FM subcarriers can be calculated.

Calculation of Optimum Subcarrier Deviation, Dual Subcarriers

The optimum subcarrier deviation, mod index, for the dual SC system, Figure 1, can be calculated using the equations derived previously. The subcarrier receiver IF bandwidth, B_{IF}, for the two SC system is set equal to the peak Carson's rule bandwidth of the composite, PCM+SC, signal or an RMS estimate of the required bandwidth [4]. To simplify calculation the values of R_b, f_{sc1} and f_{sc2} were scaled by 32 kHz to 1 BPS, 2 Hz and 3 Hz. The peak bandwidth estimate yields

$$B_{IF} = 2[f_{sc2}+\Delta f_{peak}+f_{scl}(msc)+f_{sc2}(msc)] = 2[3+.35+2(msc)+3(msc)] = 6.7+10(msc)$$

when the FM modulator peak deviation, Δf_{peak}, is set equal to 0.35 times 1 BPS. The resulting value of $[C/kT]Req'd_1$ is

$$[C/kT]Req'd_1 = 12dB+10\ 1og[6.7+10(msc)]$$

The required value of $[C/kT]Req'd_2$ is

$$[C/kT]Req'd_2 = 12\ dB - 10\ log(1/2B_{sc}) - 20log(msc),$$

where B_{sc} is $2()f_{sc}+f_m)$ which equals 2(2000+400) Hz for IRIG class A constant bandwidth channels. When scaled the resulting B_{sc} is 0.15 Hz. Therefore

$$[C/kT]Req'd_2 = 12\ dB - 10\ log(1/2B_{sc}) - 20\ log(msc) = 12\ dB - 10\ log(1/(2(.15))-20\ log(msc)$$
$$= 12\ dB-5.2288\ dB - 20\ log(msc) = 6.7712\ dB - 20\ log(msc)$$

The value of $[C/kT]Req'd_3$ can be determined for a given required $[S/N]_{out}$ and msc:

$$[C/kT]Req'd_3 = [S/N]_{out} - 10\ log[3/4()f_{m/sc}/f_m)^2\ (msc)^2\ (l/f_m)]$$
$$= [S/N]_{out} - 10\ log[3/4()f_{m/sc}/f_m)^2(1/f_m)] - 20\ log[msc]$$
$$= [S/N]_{out} - 10\ log[3/4(5)^2(32000/400)] - 20\ log[msc]$$
$$= [S/N]_{out} - 31.761 - 20\ log[msc]$$

The RMS bandwidth estimate yields a value of IF bandwidth given by

$$B_{IF} = 2[f_{sc2}+\{)f_{peak}{}^2+[f_{sc1}(msc)]^2+[f_{sc2}(msc)]^2\}^{1/2}]$$
$$= 2[3+\{(.35)^2+(2*msc)^2+(3*msc)^2\}^{1/2}]$$

The resulting equation for $[C/kT]\ Req'd_1$ using the RMS method to calculate the required IF bandwidth is

$$[C/kT]Req'd_1 = 12dB+10\ log[6+2\{0.1225+13(msc)^2\}^{1/2}]$$

The equations for $[C/kT]Req'd_2$ and $[C/kT]Req'd_3$ are unchanged for the RMS estimate.

Calculation of Optimum Subcarrier Deviation, Single Subcarrier

The optimum deviation for the single SC system of Figure 1 was also calculated. If the IF bandwidth for the single SC system is set equal to the peak Carson's rule bandwidth of the composite PCM+SC signal. The resulting IF bandwidth is

$$B_{IF} = 2[f_{sc}+)f_{peak}+f_{sc}(msc)] = 2[1+.35+1(msc)] = 2.70+1(msc)$$

The resulting value of $[C/kT]Req'd_1$ is

$$[C/kT]Req'd_1 = 12dB + 10\ log[2.70+msc]$$

The required value of $[C/kT]Req'd_2$ is

$$[C/kT]Req'd_2 = 12 \text{ dB} - 10 \log(1/2B_{sc}) - 20 \log(msc),$$

where B_{sc} is 2( ) $f_{sc}+f_m$) which equals 2(2000+400) Hz for IRIG class A constant bandwidth channels. When scaled the resulting B_{sc} is 0.15 Hz. Therefore

$$\begin{aligned}[C/kT]Req'd_2 &= 12 \text{ dB} - 10 \log(1/2B_{sc}) - 20 \log(msc)\\ &= 12 \text{ dB} - 10 \log(1/(2(.15)) - 20 \log(msc)\\ &= 12 \text{ dB} - 5.2288 \text{ dB} - 20 \log(msc) = 6.7712 \text{ dB}-20 \log(msc)\end{aligned}$$

As in the case of the dual SC system the value of $[C/kT]Req'd_3$ can be determined for a given required $[S/N]_{out}$ and msc:

$$[C/kT]Req'd_3 = [S/N]_{out} - 31.761 - 20 \log[msc]$$

The value of $[C/kT]Req'\ d_3$ is not dependent upon the number of subcarriers or their frequency if only IRIG class A subcarriers are used. The RMS estimate yields an IF bandwidth of [4]

$$B_{IF} = 2[f_{sc}+\{) f_{peak}{}^2+[f_{sc}(msc)^2\}^{1/2}] = 2[1+\{(.35)^2+(msc)^2\}^{1/2}]$$

The resulting equation for $[C/kT]Req'd_1$ using the RMS method to calculate the IF bandwidth is

$$[C/kT]Req'd_1 = 12dB+101og[2+2\{0.1225 +(msc)^2\}^{1/2}]$$

The equations for $[C/kT]Req'd_2$ and $[C/kT]Req'd_3$ are unchanged. The equations presented for [C/kT]Req'd are next used to calculate the optimum operating point, msc value, for both the dual and single subcarrier systems.

Dual Subcarrier System, Subcarrier S/N = 45 dB

The equations derived previously are used to determine the optimum value of msc for dual SC operation. The SC's are at 64 and 96 kHz, scaled to 2 and 3 Hz. The thresholds for the SC receiver are plotted for FM modulator peak deviation,) f_{peak}=0.35, using the peak method, Figure 6, and RMS method, Figure 7, to calculate IF bandwidth.

The plots indicate that for a SC S/N of 45 dB the optimum value of msc is approximately equal to 0.35. This results in the least required transmitter power, C/kT, to meet the SC receiver threshold requirements. Since the PCM data rate is scaled to 1

BPS, the required C/kT value of 23 dB yields an E_b/N_o of 23 dB.

This value of E_b/N_o should easily yield bit error rates, BER's, less than the value of 10^{-6} typically required.

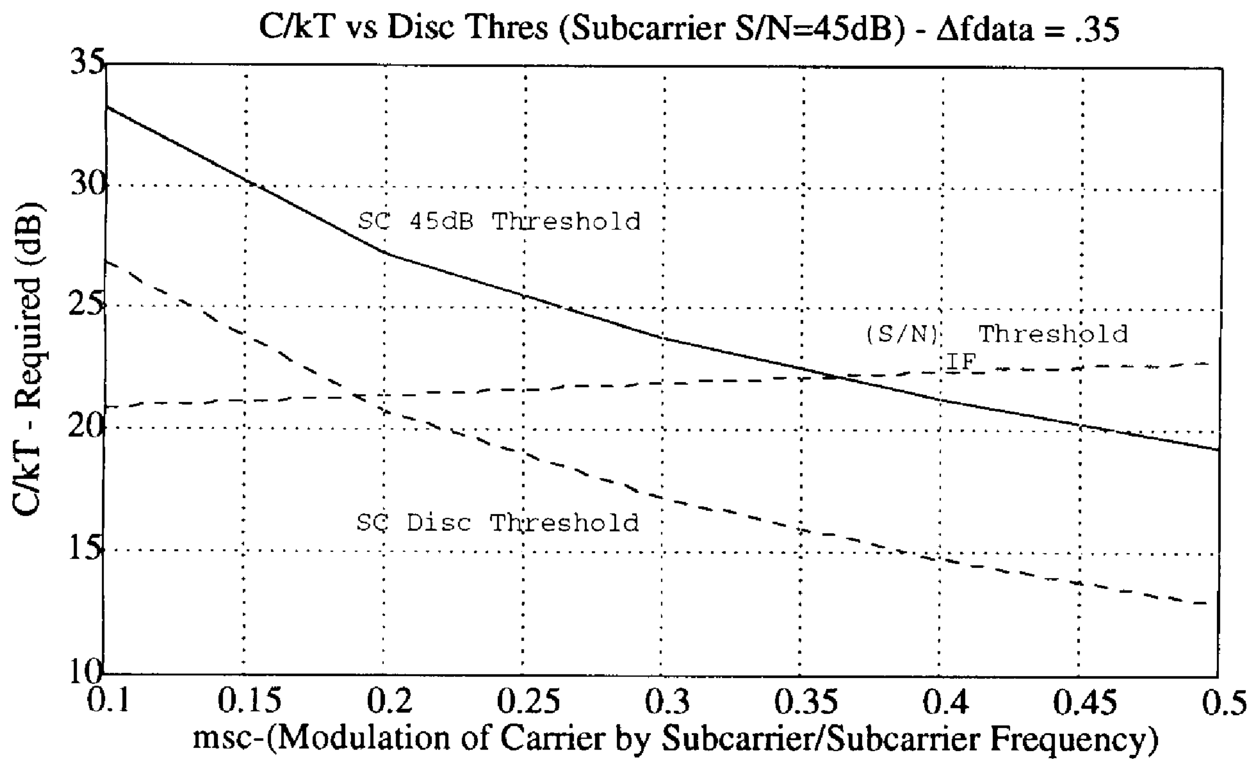

Figure 6 - Two Subcarriers, Peak Method

Dual Subcarrier System, Subcarrier S/N = 39 dB

The equations derived for a dual subcarrier system are next used to derive the optimum value of msc for a dual SC system with required subcarrier S/N of 39 dB. The SC's are again at 64 and 96 kHz, scaled to 2 and 3 Hz. The calculated thresholds, using the method demonstrated for the dual SC with 45 dB S/N, indicate that the optimum value of msc is approximately equal to 0.20. This results in the least required transmitter power, C/kT, to meet the SC receiver threshold requirements. Since the PCM data rate is scaled to 1 BPS, the required C/kT of 22 dB yields a value of E_b/N_o of 22 dB. This value of E_b/N_o should also yield BER's less than 10^{-6}.

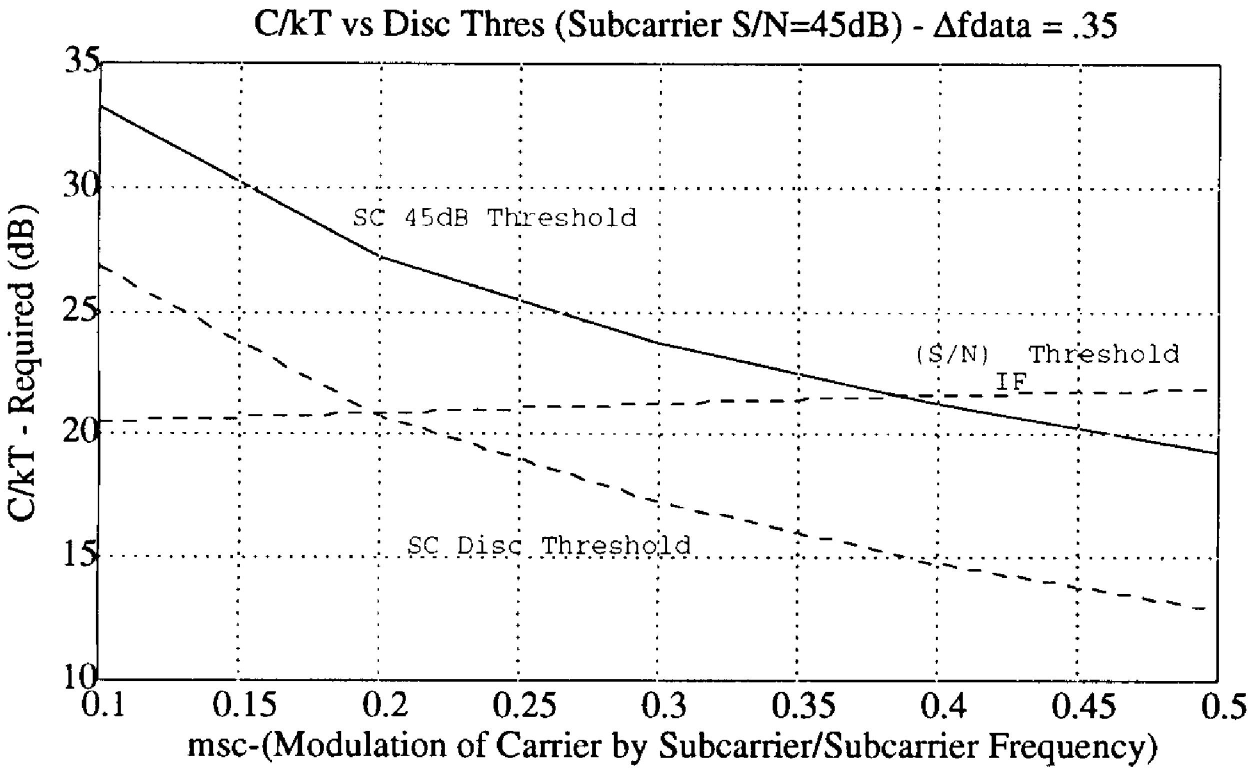

Figure 7 - Two Subcarriers, RMS Method

Single Subcarrier System, Subcarrier S/N = 45 dB

The SC is at 32 kHz, Figure 2, and scaled to 1 Hz. The calculated thresholds, again using the method demonstrated for the dual SC with 45 dB S/N, indicate that the optimum value of msc is approximately equal to 0.575. This results in the least required transmitter power, C/kT, to meet the SC receiver threshold requirements. The required C/kT of 18 dB yields an E_b/N_o of 18 dB. This value of E_b/N_o should also yield BER's less than 10^{-6}.

Conclusions

The results presented indicate that the optimum subcarrier deviation can be simply calculated for any PCM/FM+FM/FM system. Whether the Peak or RMS estimate for required IF bandwidth is more accurate would depend upon the specific system. However, the actual optimum deviation should fall somewhere between the optimum deviations calculated using each method.

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