

ON THE PERFORMANCE OF PCM/FM + FM/FM SYSTEMS

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ABSTRACT

Much modem telemetry is transmitted in a digital format and to be compatible with existing range equipment the digital data is impressed on the carrier using FM modulation. The receiving system in common use employs an FM limiter/discriminator as a detector followed by an integrate and dump matched filter for bit detection. This system has been studied by previous authors [1] and it is well known that in the absence of frequency uncertainty the optimum transmission parameters consist of a modulation index of .7 (peak-to-peak deviation divided by the bit rate) and an IF filter bandwidth equal to the bit rate followed by a limiter discriminator.

In many cases, there is a need for some small amount of analog telemetry transmission in addition to the digital data discussed above. In these cases it is common practice to include analog subcarriers on the main carrier with the digital data modulating the carrier at baseband, the resulting system is called PCM/FM + FM/FM. These hybrid analog/digital systems are the subject of this paper. In particular this paper addresses the performance of these systems through simulation using the Block Oriented System Simulator (BOSS) from Comdisco and with analytical techniques to obtain the BER versus SNR curves for these systems. The simulation is used over a wide range of parameters to find the optimum values of modulation index and IF bandwidth for these systems.

KEYWORDS

PCM, FM, Demodulation and Power Spectrum

INTRODUCTION

Much modem telemetry is transmitted in a digital format and to be compatible with existing range equipment the digital data is impressed on the carrier using FM modulation. The receiving system in common use employs an FM limiter/discriminator as a detector followed by an integrate and dump matched filter for bit detection. This system has been studied by previous authors [1] and it is well known that in the absence of frequency uncertainty the optimum transmission parameters consist of a modulation index of .7 (peak-to-peak deviation divided by the bit rate) and an IF filter bandwidth equal to the bit rate followed by a limiter discriminator.

In many cases, there is a need for some small amount of analog telemetry transmission in addition to the digital data discussed above. In these cases it is common practice to include analog subcarriers on the main carrier with the digital data modulating the carrier at baseband, the resulting system is called PCM/FM + FM/FM. These hybrid analog/digital systems are the subject of this paper. In particular this paper addresses the performance of these systems through simulation using the Block Oriented System Simulator (BOSS) from Comdisco and with analytical techniques to obtain the BER versus SNR curves for these systems. The simulation is used over a wide range of parameters to find the optimum values of modulation index and IF bandwidth for these systems.

The receiving system studied, for the demodulation of these compound signals, which was first suggested by Law [2], employs a narrow IF filter and discriminator for the digital data and a second wider (Carson's Rule) IF filter and discriminator for the analog signals. The results show that for small subcarrier mod indexes that the receiver of Law produces low BER's for a given value of C/KT and surprisingly that this receiver is optimized by the same parameter set that was optimum for PCM/FM. It is also shown that for large subcarrier mod indexes that this receiver breaks down due to the nonlinear interaction of the subcarriers and the digital data in the modulation process. In a companion paper the optimum values of subcarrier mod indexes are examined [3].

DEVELOPMENT OF BOSS MODEL

The PCM/FM+FM/FM System adds subcarriers to the NRZ-L data stream prior to the modulation of the FM modulator. The resulting composite signal is used to FM modulate a carrier. The modulator peak deviation is typically set at $0.35 \cdot R_b$ for a PCM/FM system. The value of $0.35 \cdot R_b$ was determined to be optimum for a PCM/FM

system [1]. However, little study has been done to verify that this value holds as optimum for a PCM/FM+FM/FM system.

In order to determine if a value of $0.35 \cdot R_b$ is optimum a BOSS system was created to simulate the operation of a PCM/FM+FM/FM system. The first system developed models the system which Law[2] presents in his telemetry handbook. Law's system consists of a 256 kbps NRZ-L data stream and IRIG class C subcarriers at 256 and 288 kHz. Law's subcarrier oscillator, SCO, peak deviation was set at 70 kHz.

The simulation results for the system developed were compared to Law's results for PCM receiver IF bandwidths of 300 and 500 kHz and peak FM modulator deviation of $0.35 \cdot R_b$ to verify that the BOSS system developed operates properly.

A similar BOSS model was developed for the PCM/FM+FM/FM system except that an SCO module is added, as shown in Figure 1 and 2, the IF filter is sixth order.

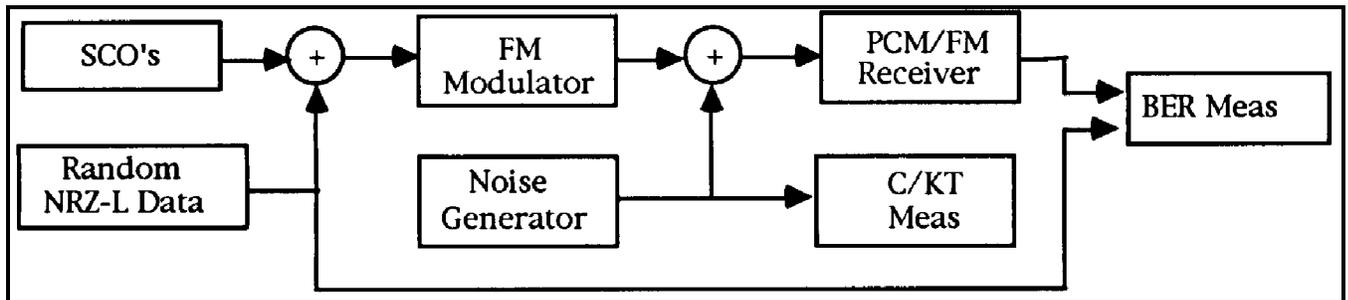


Figure 1. Basic BOSS PCM/FM+FM/FM System

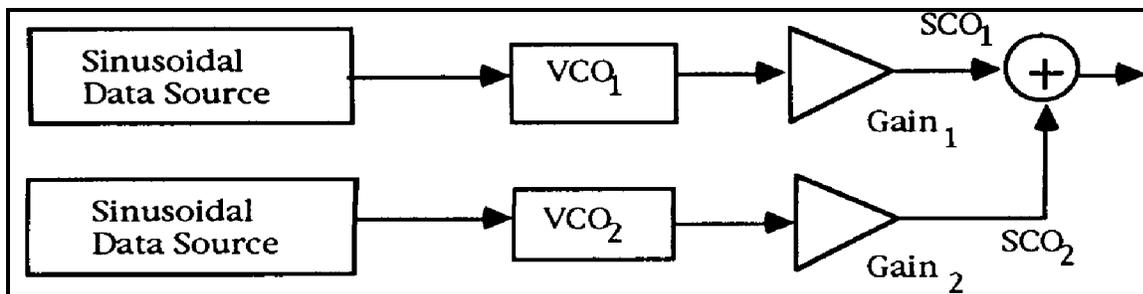


Figure 2. BOSS SCO Generator

The SCO generator, Figure 2, consists of two sinusoidal data sources feeding two subcarrier voltage controlled oscillators, VCO's.

To simulate Law's tests the 256 kbps data stream and 256 and 288 kHz VCO's used by Law were scaled to a NRZ-L data rate of 1 BPS. This results in the SCO VCO's being set at 1.0 and 1.25 Hz. Law specifies the SCO peak deviation at 70 kHz which when scaled results in an SCO_1 peak deviation of .2734 Hz/Volt and SCO_2 peak

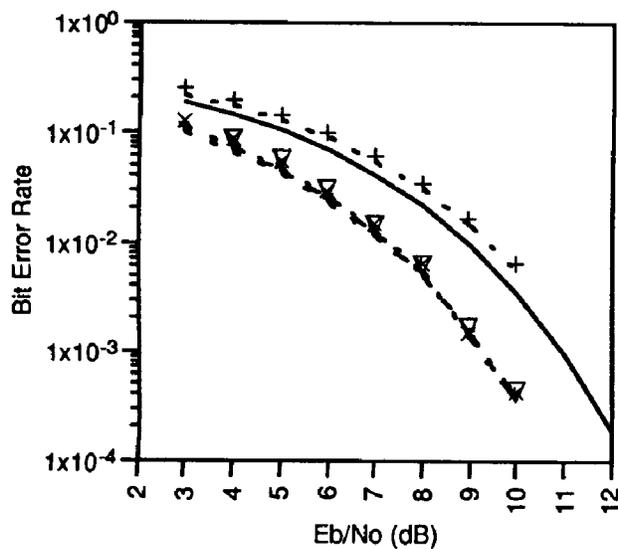
deviation of .2431 Hz/Volt. Since the FM modulator deviation, NRZ-L data deviation, is set to $0.35 \cdot R_b$, the gain required after each SCO can be calculated to yield the required SCO deviations.

$$\text{Gain } \text{SCO}_1 = 0.2734/0.35 = 0.78125$$

$$\text{Gain } \text{SCO}_2 = 0.2431/0.35 = 0.69440$$

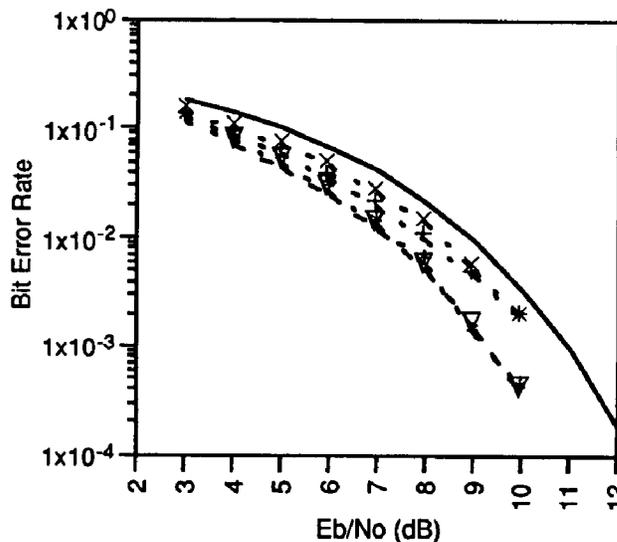
The peak deviation of the subcarrier VCO's set to $8 \text{ kHz}/256 \text{ kHz} = 0.03125 \text{ Hz/Volt}$, for Class C IRIG constant bandwidth channels. The frequency of the sinusoidal data sources are set at $2 \text{ kHz}/256 \text{ kHz} = 0.00781 \text{ Hz}$.

The BOSS model was first used to check the original Tjhung and Wittke results and to test the model. For these tests the SCO's are turned off and a standard PCM/FM system results. The results are shown in figures 3 and 4. Figure 3 confirms the results in [1] for the selection of IF bandwidth and figure 4 confirms the results for the selection of peak deviation.



- upper bound = $.5 \text{ Exp}(E_b/2N_o)$
- ▽ Tjhung and Wittke, $BT=1.0$, $\Delta f \text{ peak}=0.35$
- * BOSS, $BT=0.8$, $\Delta f \text{ peak}= 0.35$
- × BOSS, $BT=1.0$, $\Delta f \text{ peak}= 0.35$
- + BOSS, $BT=2.0$, $\Delta f \text{ peak}= 0.35$

Figure 3.



- upper bound = $.5 \text{ Exp}(E_b/2N_o)$
- ▽ Tjhung and Wittke, $BT=1.0$, $\Delta f \text{ peak}=0.35$
- + BOSS, $BT=1.0$, $\Delta f \text{ peak}= 0.25$
- * BOSS, $BT=1.0$, $\Delta f \text{ peak}= 0.35$
- × BOSS, $BT=1.0$, $\Delta f \text{ peak}= 0.50$

Figure 4.

Next the BOSS model created was used to simulate Law's tests for $BT = 300$ and 500 kHz , $BT = 1.172$ and 1.953 when scaled. The BOSS simulation results agree closely with Law's results.

The BOSS simulation results are plotted vs. Law's results in figure 5. Wittke's results for $BT = 1.0$ are also plotted for comparison. It can be seen that for a given C/kT the addition of SCO's results in a small increase in bit error rate, in this case.

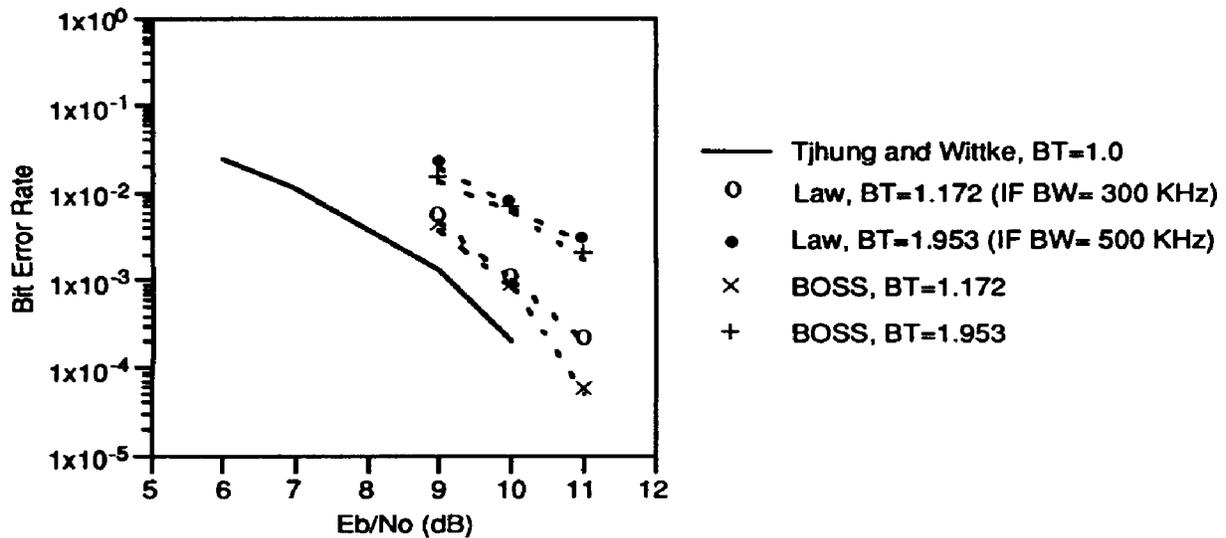


Figure 5.

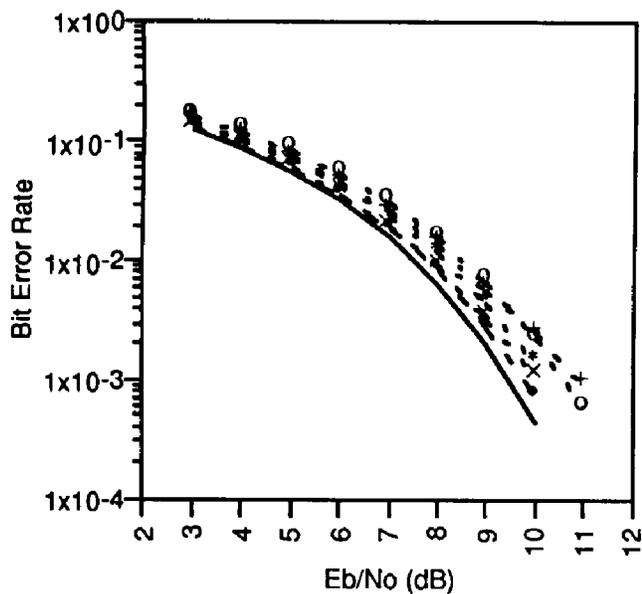
The plots indicate that results with the system model developed agree with Law's results and therefore yields accurate results for simulation of a PCM/FM + FM/FM system.

PCM/FM + FM/FM TESTS

Having verified that the PCM/FM+FM/FM BOSS model is correct, we are now in a position to study these systems. The results will depend upon the center frequency and the deviation of the analog subcarriers. In a comparison paper the optimum value of subcarrier deviation with respect to minimizing total transmitted power [3] is determined. Using the values of deviation given in this paper for two cases described below the BER versus E_b/N_0 were investigated.

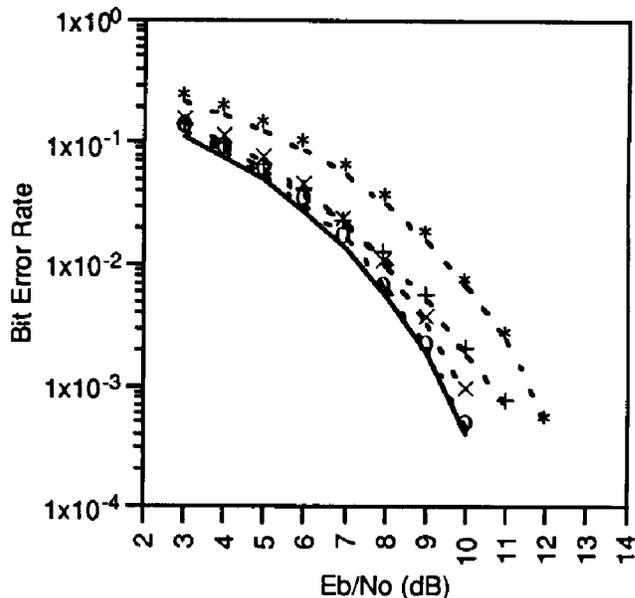
- Case 1. Two IRIG class A Constant Bandwidth Channel subcarriers at 64 and 56 kHz combined with a 32 kbps PCM data stream.
- Case 2. Single IRIG Class A subcarrier at 32 kHz combined with a 32 kbps PCM data stream.

The results of the simulations for these two cases using a data only receiver are shown in figures 6, 7 (for case 1) and 8, 9 (for case 2).



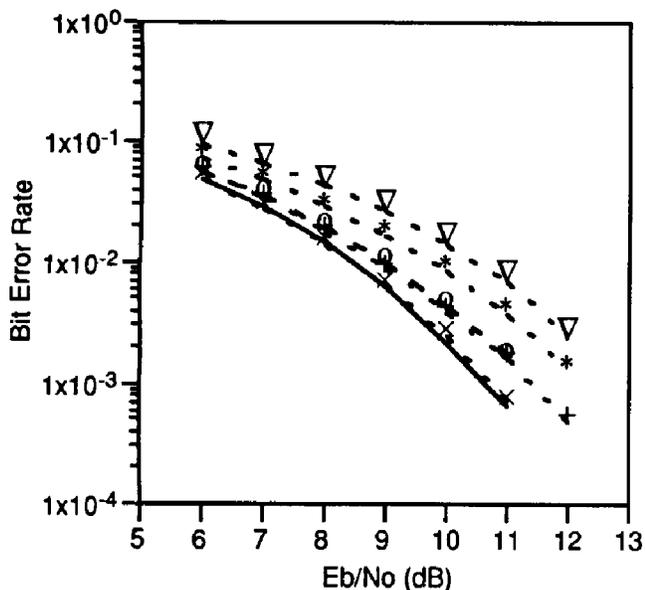
- + BT=1.0, Δf peak= 0.25
- × BT=1.0, Δf peak= 0.30
- BT=1.0, Δf peak= 0.35
- BT=1.0, Δf peak= 0.40
- * BT=1.0, Δf peak= 0.45
- BT=1.0, Δf peak= 0.50

Figure 6



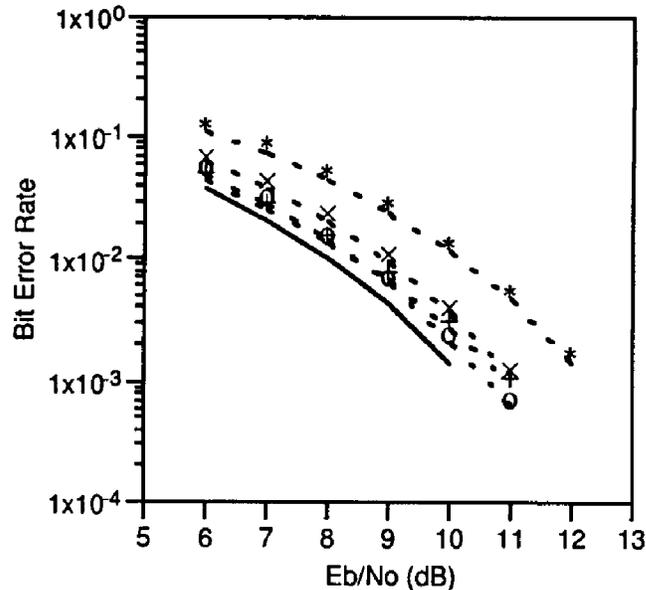
- + BT=0.6, Δf peak= 0.35
- BT=0.8, Δf peak= 0.35
- BT=1.0, Δf peak= 0.35
- × BT=1.2, Δf peak= 0.35
- * BT=2.0, Δf peak= 0.35

Figure 7



- + BT=1.0, Δf peak= 0.25
- × BT=1.0, Δf peak= 0.30
- BT=1.0, Δf peak= 0.35
- BT=1.0, Δf peak= 0.40
- * BT=1.0, Δf peak= 0.45
- ▽ BT=1.0, Δf peak= 0.50

Figure 8



- + BT=0.6, Δf peak= 0.35
- BT=0.8, Δf peak= 0.35
- BT=1.0, Δf peak= 0.35
- × BT=1.2, Δf peak= 0.35
- * BT=2.0, Δf peak= 0.35

Figure 9

From these figures one can see that the optimum values of PCM carrier deviation and IF bandwidth for a data only receiver are the $.35R_b$ and $BT = 1.0$ that were found for

the PCM/FM only case. This is a surprising result considering that the FM modulation process is nonlinear yet the results indicate that the presence of the subcarrier has no effect upon the data when a data only receiver is used.

To understand the reason for this result consider the following analysis. Let $x(t)$ be the FM subcarrier

$$\begin{aligned} x(t) &= \text{Gain} \times \sin[2\pi f_{sc}t + D_s \sin(2\pi f_m t)] \\ &\approx \text{Gain} \times \sin(2\pi f_{sc}t) \end{aligned}$$

where f_{sc} is the frequency of the subcarrier

f_m is the frequency of the message

D_s is the modulation index of the subcarrier and the message.

After the carrier modulator, let $s(t)$ be the complex composite signal

$$s(t) = \exp(j2\pi\Delta f \int d(t)dt) + j2\pi \int x(t)dt$$

The fourier transform of $s(t)$ can be expressed as a convolution in the frequency domain as follows,

$$F[s(t)] = F[e^{j2\pi\Delta f \int d(t)dt}] * F[e^{j(msc)\sin(2\pi f_m t)}],$$

where $d(t) = \pm 1$ for $-nT \leq t \leq (n+1)T$, $n=0,1,2,\dots$

and msc is the modulation index of the carrier by the subcarrier.

The first term is the transform of the data and the second term is the transform of a carrier FM modulated by a sinewave. The power spectrum of the data is given in [4] as,

$$\begin{aligned} |F[u(t)]|^2 &= W(f) \\ &= \frac{2A^2 \sin^2[(\omega - \omega_1)/2]T \sin^2[(\omega - \omega_2)/2]T}{T[1 - 2\cos(\omega - \alpha)T \cos\beta T + \cos^2\beta T]} \left[\frac{1}{\omega - \omega_1} - \frac{1}{\omega - \omega_2} \right]^2 \\ &+ \frac{2A^2 \sin^2[(\omega + \omega_1)/2]T \sin^2[(\omega + \omega_2)/2]T}{T[1 - 2\cos(\omega + \alpha)T \cos\beta T + \cos^2\beta T]} \left[\frac{1}{\omega + \omega_1} - \frac{1}{\omega + \omega_2} \right]^2 \end{aligned}$$

The power spectrum of sinewave FM modulated carrier is given by,

$$|F[e^{j(msc)\sin(2\pi f_c t)}]|^2 = \sum_{n=-\infty}^{+\infty} J_n^2(msc)\delta(f - nf_{sc})$$

Hence,

$$\begin{aligned} |F[s(t)]|^2 &= \sum_{n=-\infty}^{+\infty} J_n^2(msc)W(f - nf_{sc}) * 0.5 \\ &= 0.5[J_0^2(msc)W(f) + J_1^2(msc)W(f - f_{sc}) + J_{-1}^2(msc)W(f + f_{sc}) + \text{higher frequency terms}] \end{aligned}$$

This expression is for one FM modulated subcarrier. Figure 10a and figure 10b shows spectral density of modulated signal for A=1, msc=0.2 and 1.0 and $f_{sc}=1$ hz, neglecting higher frequency terms as they are outside the IF bandwidth. In the figure response of the IF filter (which is Bessel filter of 6th order, 3 dB bandwidth=1 hz) is shown to give an idea how much interference the subcarriers produce inside the IF bandwidth.

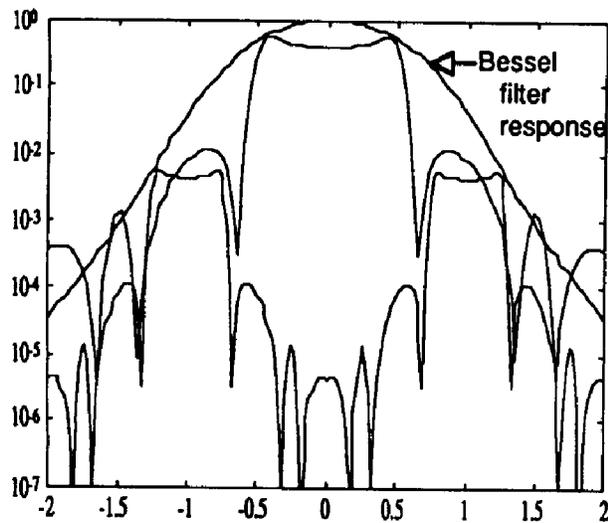


Figure 10 a. msc=0.2

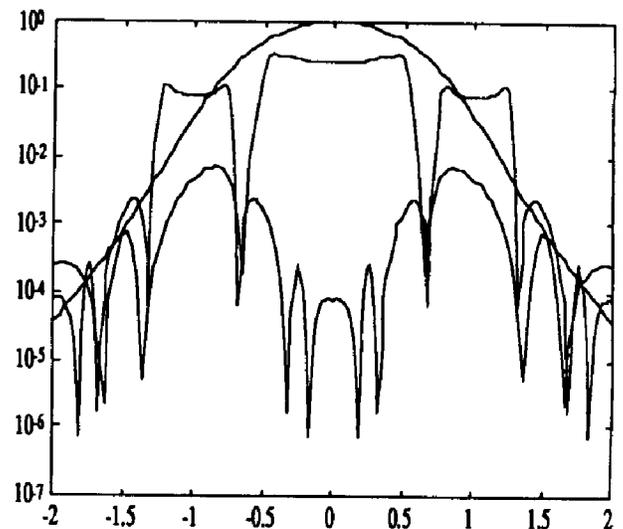


Figure 10 b. msc=1.0

For two subcarriers,

$$|F[s(t)]|^2 = \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} J_n^2(msc)J_m^2(msc)W(f - nf_{sc1} - mf_{sc2}) * 0.5$$

$$\begin{aligned} |F[s(t)]|^2 &= 0.5[J_0^2(msc)J_0^2(msc)W(f) + J_1^2(msc)J_{-1}^2(msc)W(f - f_{sc1} + f_{sc2}) \\ &\quad + J_{-1}^2(msc)J_1^2(msc)W(f + f_{sc1} - f_{sc2}) + \text{higher frequency terms}] \end{aligned}$$

Figure 11a and figure 11b shows spectral density of modulated signal for A=1, msc=0.2 and 1.0 and $f_{sc1}=2$ hz and $f_{sc2}=3$ hz, neglecting higher frequency terms as they are outside the IF bandwidth.

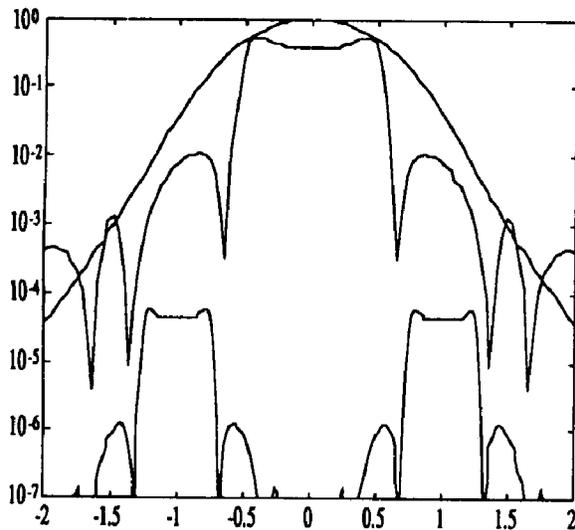


Figure 11a. msc=0.2

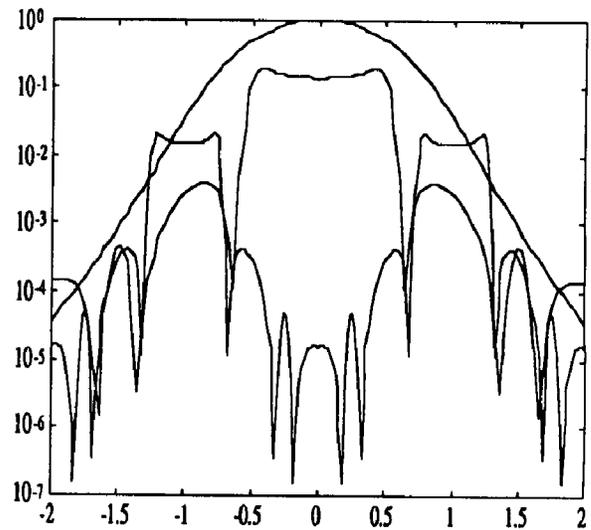


Figure 11b. msc=1.0

SUMMARY

The performance of a dual receiver system for the demodulation of PCM/FM+ FM/FM has been studied. The modulation parameters for such a system have been optimized to minimize BER as a function of C/KT . The set of parameters that was shown to be optimum for data detection are $\Delta_{Data} = .35 R_b$ and IF bandwidth equal to R_b with the analog subcarrier mod index less than 0.5.

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