Digital Baseband Signal Combining

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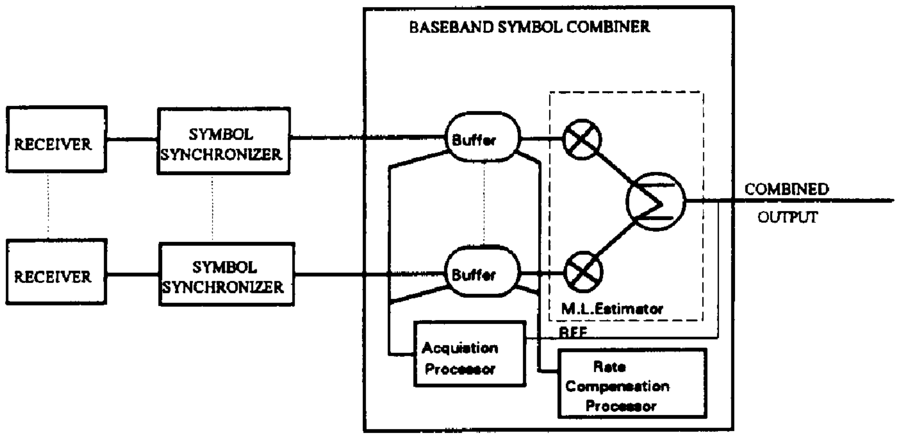
Abstract

Unlike conventional pre- or postdetection digital signal combining approaches, the Digital Baseband Symbol Combiner (DBSC) utilizes detected baseband symbol metrics from the individual antenna-receiver system symbol synchronizers as inputs to the combining process. Additionally, symbol timing from the individual synchronizers is used to aid the DBSC perform closed-loop tracking and doppler rate compensation. The DBSC can be shown to provide an equivalent signal-to-noise ratio (SNR) improvement as the conventional approaches when the individual antenna-receiver system noise is characterized as gaussian and resulting symbol metrics are considered statistically independent. This paper discusses the theoretical approach to a proposed symbol combining technique which would provide near optimum real-time data in support of the NASA Space Shuttle Orbiter ascent-phase operations at the Merritt Island complex, Kennedy Space Center, Florida.

Discussion

It can be shown that space, frequency, polarization, and other forms of diversity combining can offer increased SNRs, which result in improved bit error probabilities. However, diversity combining requires complete knowledge of the overall system delays and noise characterization to be effective. Implementation of exact phase and/or bit delays can be quite challenging with existing analog technologies. The following paragraphs suggest a digital implementation of spatial diversity which can be shown to be equivalent in terms of obtainable SNR performance with existing combining approaches. Spatial or space diversity uses M antennas sufficiently separated spatially, thus observables from each antenna can be considered as independent random processes. The M independent random variables obtained from these processes can then be combined in such a way as to produce a single decision with improved bit error performance.



Digital Baseband Signal Combining Process

Maximum-Likelihood Estimator

Let us assume we have as an input to our decision process signals originating from M spatially spaced antenna-receiver subsystems. In addition, each signal has been previously *conditioned* by a symbol synchronizer before our combining process. We also assume the k^{th} transmitted symbol is to be presented to the combiner by all subsystems simultaneously. This constraint removes any path delays and or doppler effects from consideration, at least for the moment. The question before us is knowing the statistics of the noise and the nature of the signals, how should one choose an estimation strategy among all competing strategies so as to render decisions with a minimum probability of error.

Here we choose the maximum-likelihood estimator (MLE) which not only provides us with an estimation strategy, but has the properties of being unbiased, sufficient and efficient. The MLE for our purposed signal combiner can be shown to have the form

$$L = \sum_{i=1}^{m} \frac{x_i}{\sigma_i^2}$$

In the above expression, L is the combiner output statistic, σ_i is the rms noise variance, and x_i is the symbol synchronizer's *matched filter*; i.e., the correlator output signal estimate. In order to emulate this estimator, each symbol synchronizer must provide not only an estimate of the contaminated signal x_i, but also an estimate of the power of the noise contaminate. Together these estimates provide not only the binary decision on the current symbol (the sign of the magnitude of the signal estimate), but an indication of the SNR at the output of the synchronizer. The SNR from each synchronizer provides the required weighting; i.e., measure of confidence with each binary decision. It is constructive to note that the statistic L could have been written as

$$L=\sum_{i=1}^{m} sign(x_i)\frac{|x_i|}{\sigma_i^{\,2}} \qquad \text{where } sign(x_i)=\left\{\begin{array}{ll} +1 & \text{if } x_i \ge 0 \\ -1 & \text{otherwise} \end{array}\right\}$$

Here the binary decision is expressed as the $sign(x_i)$. This form of the statistic L is more representative of the combining process. For example, using the above representation of L, consider what happens when each of the M inputs have equal SNRs. Thus the SNR can be factored out of the expression, and the decision based solely on the sum of the $sign(X_i)$. Here, our decision is simply a *majority rule,* where each synchronizer vote is simply the $sign(X_i)$.

For *an additive white gaussian noise* (AWGN) channel the logarithm of the likelihood function utilized to partition the decision regions for binary symbol recovery can be shown to be of the form

$$G=\ln\frac{p(\mathbf{y}|\mathrm{H}_1)}{p(\mathbf{y}|\mathrm{H}_0)}=\ln\prod_{i=1}^{n}\frac{p(y_i|\mathrm{H}_1)}{p(y_i|\mathrm{H}_0)}=\sum_{i=1}^{n}\frac{2x_{1i}y_i}{\sigma^2}\ge 0 \quad \text{Choose } \mathrm{H}_1$$

$$\text{where} \quad \mathrm{y_i}=\mathrm{x_{ki}}+\mathrm{n_i} \;, \quad \mathrm{Var}(\mathrm{n_i})=\sigma^2=\frac{\mathrm{N}}{2} \;, \quad \mathrm{x_{ki}}=\pm\sqrt{E_s} \;, \quad \mathrm{k}=0,1 \;\text{ and }\; \mathrm{i}=0,1,2,\ldots n$$

As defined above, the statistic G, commonly called the *metric,* is a gaussian random variable with mean, $4E_s/N$, and variance, $8E_s/N$. The *matched filter* (i.e.; the correlator output of a symbol synchronizer) can be characterized with the following parameters

$$S=\frac{\sigma^2}{2}G=\sum_{i=1}^{n}x_1 y_i \qquad \text{mean } \pm E_s \text{ and variance } \sigma^2 E_s$$

Where S represents the correlator output, x_i and y_i are as defined above. Additionally, the relationship that exists between the likelihood function and that of the *matched filter* utilized by our symbol synchronizer substantiates its optimum performance in the sense of minimizing the probability of error. If we were to form a new variable Z representing the ratio of expected value to variance of either random variable S or G, the resulting expected value would be the same, namely,

$$E(Z)=\frac{\mu}{Var(S)}=\frac{\pm E_s}{\sigma^2 E_s}=\frac{sign(S)}{\sigma^2} \qquad \text{where } \mu=\mathrm{E(S)} \text{ and E( ) is the expectation operator.}$$

From this expression, we can see that the value provided by the symbol synchronizer is just the signed decision weighted by the inverse of the noise power. The probability of error for the combiner MLE, given a AWGN channel with m inputs, is

$$PE = P\left(z > \sqrt{\frac{\left(\sum_{i=1}^{m}\frac{1}{\sigma_i^2}\right)^2}{\frac{1}{E_s}\sum_{i=1}^{m}\frac{1}{\sigma_i^2}}} \right) = P\left(z > \sqrt{\frac{E_s}{\sigma^2}\sum_{i=1}^{m}\frac{1}{\alpha_i^2}} \right) = P\left(z > \sqrt{\frac{2E_s}{N}\sum_{i=1}^{m}\beta_i} \right)$$

$$\sigma = \min\{\sigma_1, \sigma_2, \ldots\ldots, \sigma_m\} \quad \text{where } \sigma_k = \alpha_k \sigma \text{, and } \beta_k = \left(\frac{1}{\alpha_k}\right)^2$$

In this expression, the sum of the β_k's is the signal-to-noise improvement since this will always be greater than one. The maximum improvement attainable is noted with equal noise powers, since the sum of the β_k's equals its maximum value, m. However, we note that no matter how small the SNRs are for the input channels, the combiner output can be no worse than the largest SNR input.

Note, in both formulations of the statistic L, we assumed an AWGN channel. By using an optimum set of transition probabilities, one can map an AWGN channel to a *discrete memoryless channel,* DMC. In general, decisions based on these mappings are not optimum. However, when enough discrete levels are used, decisions based on the DMC can be shown to approach that of the optimum in a probabilistic sense. The measure of confidence displayed with each decision is accomplished by *soft quantization* provided by each symbol synchronizer. Traditionally, symbol synchronizers developed with *soft quantizing* (soft decision) outputs are used on *coded* channels to improve the error correctability of the decoder.

Acquisition and Tracking

In this combining approach we use an active correlation process to perform alignment for symbol acquisition and signature validation. When processing more than two symbol synchronizer inputs, a majority rule process can be employed to insure the signatures for the most part are from the same vehicle or target. However, before the individual signatures can be compared with one another, they must be aligned to account for system propagation delays and the effects due to doppler. Each individual symbol synchronizer, autonomously recovers the symbol values transmitted by the vehicle of interest as well as a timing estimate. After a pre-assigned number of symbols are obtained, the recovered symbol sequence enters a variable length buffer and the active correlation process begins. This pre-assigned number is a function of the maximum propagation delay given all combiner input sequences. The correlation process compares the combined output sequence with each newly acquired input. Once the sequence is aligned and its signature validated, the newly acquired sequence can be employed by the combiner to assist with subsequent symbol decision.

The probability of acquisition in some fixed number of time units is a function of the designed *false alarm and detection* probabilities. Clearly, these probabilities are directly related to the individual antenna-receiver subsystem SNR, which is responsible for the correlation loss experienced. Our goal is to design a procedure that provides opium acquisition and re-acquisition tunes given symbol sequences for each combiner input. The *cost* or *risk* involved, is a degraded combiner performance which results when the input symbol error rates are increased. This in turn influences our ability to predict with some degree of confidence, that sequences are in fact aligned and belong to the same source. The probability of detecting an aligned pair of sequences of length N can be written as

$$P_{ACQ} = P_D + P_D(1-P_D) + P_D(1-P_D)^2 + \cdots, + P_D(1-P_D)^{N-1}$$

where P_D represents the probability of detection.

The probability of detection is accepting the hypothesis that the correlation value is greater than some fixed value L. We will attempt to address the effect of correlation loss due to symbol errors in the discussion that follows.

The autocorrelation value $\lambda(j)$, given a symbol sequence a_n, is defined as

$$\lambda(j) = \frac{1}{N}\sum_{i=1}^{N} b_i b_{i+j}$$

where the sequence a_n is a binary sequence with b_n defined by:

$$b_n = 2a_n - 1 \text{ , thus } a_n = \begin{cases} 1 \Rightarrow b_n = 1 \\ 0 \Rightarrow b_n = -1 \end{cases}$$

The value $\lambda(j)$ can be shown to be equivalent to

$$\lambda(j) = \frac{(N-k)-k}{N} = 1 - \frac{2k}{N}$$

where N-k = the number of symbols in agreement

k = the number of symbols in disagreement.

Let's assume that one of the input sequences to the correlation process has a symbol error probability of R and the combiner reference symbol error probability is S. The resultant symbol error probability which effects the correlation value, is R+S-2RS. RS is the probability of simultaneous symbol errors effecting the same symbol position. These errors do not change the correlation value. Given the input sources are modeled as AWGN channels, the following expressions describe R and S.

$$R = \Pr\left(n > \frac{\mu}{\sqrt{\sigma^2}} = \sqrt{\frac{2E_s}{N}}\right) \quad \text{while} \quad S = \Pr\left(n > \frac{\mu}{\sqrt{\sigma^2}} = \sqrt{\frac{2E_s}{N}}\beta\right)$$

where β is defined as the combiner improvement coefficient

The implementation of the combiner correlation process is described by some value D(j)=N-k; i.e.; the correlation value D(j) is the number of symbols in agreement. When operating in a noiseless environment and j=0, D(0)=N. We are now able to define detection and false alarm probabilities.

$$P_D = \Pr\left(D(0) > N - L \middle| j = 0; p\right) \quad \text{and} \quad P_{FA} = \Pr\left(D(j) > N - L \middle| j \neq 0; p\right)$$

where p is the combined channel error probability, and L is our design theshold.

The detection probability given the sequences are aligned (i.e.; j=0), and the combined channel error probability p, is just the probability that the number of combined errors is less than L. This representation can be modeled as a binomial distribution

$$P_D = \Pr\left(D(0) > N - L \middle| j = 0; p\right) = \Pr(y \leq L) = \sum_{y=0}^{L} \binom{N}{y} p^y (1-p)^{N-y}$$

However, in order to minimizes the false alarm probability, we need to choose L such that N-L > N-k. If this were not the case, we would always declare our sequences aligned regardless of the value j. Thus, N-L > N-k implies L < k. Additionally, when the total number of errors effecting the correlation process is less than k-L (i.e., Np < k-L) then Pr[D(j) > N-L]=0. In this case, no false alarms are ever generated. Before considering the case when Np > k-L, we need to introduce a probability distribution which will help analyze this case. Consider the problem of computing the probability of finding x type k's in a sample size n, from a population N, where the number of type k's in the population is k, and the number of non-type k's is N-k. To compute this probability we define the discrete Hypergeometric distribution

$$\Pr(x = m) = \frac{\binom{k}{m}\binom{N-k}{n-m}}{\binom{N}{n}}$$

Recall from the discussion above that errors that effect one channel, which occur in positions that disagree (type k), are modified and increase the value of D(j) accordingly while errors which occur in positions that agree (type N-k), decrease the value of D(j). The Hypergeometric distribution can be used to select the n=Np error locations where m are of type k and Np-m are of type N-k. We now wish to find that value of x for which the smallest number of type k's will result in a positive false alarm probability, i.e.;

$$N - L \le (N - k) - (Np - x) + x$$

$$2x \ge N[1 - (1 - p)] + k - L$$

$$x \ge \frac{Np + k - L}{2}$$

However, the hypergeometric distribution requires $x \le k$,

$$\therefore \quad \frac{Np + k - L}{2} \le x \le k$$

The Hypergeometric distribution can be approximated by the Poisson distribution for large k and small Np.

$$\Pr(x \ge m) \approx \sum_{x=m}^{Np} e^{-kp} \frac{(kp)^x}{x!} \qquad \text{where } m = \frac{Np + k - L}{2}.$$

By simultaneously solving for L, we will be able to choose an optimizing value such that we permit detection of alignment and reject the detection hypothesis when operating in both high correlation and low SNR environments.

Rate Compensation

As mentioned in the above paragraphs, the derived timing from each of the symbol synchronizers provides the source timing for capturing symbol estimates. These estimates are placed in individual variable length buffers to compensate for vehicle doppler and propagation delays between receiver subsystems. During an active track, the vehicle doppler is non-zero, thus timing estimates from the symbols synchronizers are not equal. An initial delay provided during signal acquisition and a means of doppler rate compensation must be provided by the combiner to prevent the variable buffers from *underflowing* or *overflowing*. We search for an optimum means, in the sense of minimizing buffer size, by which to manage the rate at which symbols are processed by the combiner, given M non-coherent inputs. Let N_i represent the number of symbols which the i^{th} synchronizer places in the variable length buffer during some fixed tune interval while N_o denotes the number of symbols obtained by the combining process from each buffer during that same interval. Mathematically we can represent this process by

$$\Delta_T = N1 + N_2 + \cdots, + N_M$$ where Δ_T denotes the total size of the M buffer lengths.

We note that the number of symbols per unit of time is frequency and the derivative of the total size can be obtained by subtracting MN_o from the right side of the expression.

This new expression can be interpreted as a change in total size with respect to time;i.e.,

$$\frac{\partial \Delta_T}{\partial T} = f_1 + f_2 +, \cdots, + f_M - Mf_o$$

This expression can be set equal to zero and solved to provided the optimum output rate. Thus

$$\frac{\partial \Delta_T}{\partial T} = 0 \qquad \Rightarrow \qquad f_0 = \frac{f_1 + f_2 + \cdots, + f_M}{M}$$

where the optimum output is the arithmetic mean of the input rates.

Conclusion

We noted at the end of the development of the MLE, that regardless of the input SNRs, the combiner output would be no worse than the best input. We add this word of caution concerning the use of traditional symbol synchronizer quantized metrics. Traditional synchronizer metrics are formed with the rms noise power substituted for noise variance in symbol descisions weighted by the signal-to-noise estimates. The resultant combiner performance using this sub-optimum statistic only approaches that which we described when operating in a high SNR environment.

During our discussion concerning acquisition, we addressed correlation loss due to random errors resulting from the processing of a noise corrupted signal. However, due to physical constraints when implementing the correlation algorithm, the length of the sequences which can be processed for a given sample will undoubtedly be much less than the actual data sequence periodicity, This will result in the accumulation or averaging of partial sequence correlation values. This problem is identical to that treated in the analysis of spread spectrum correlation loss which occurs when the integration time is less than the pseudorandom spreading code length.

Having a *priori* knowledge of the target location and antenna propagation delay, permits the alignment process to begin searching in the most probable symbol positions first. Adding this modification to the proposed acquisition scenario decreases the mean acquisition time.

Our theoretical development of the MLE indicates a realization of up to 10 log M dB SNR improvement, when combining M antennas. Experimental results obtained in a laboratory environment have validated the arguments presented. However, the acquisition results obtained when the false alarm probability is not identically zero, are not as conclusive. Further work is required to obtain operational correlation parameters, which will support *least favorable configuration* combiner evaluation,

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