PERFORMANCE OF NONCOHERENT MFSK CHANNELS WITH CODING[1]

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Summary. Computer simulation of data transmission over a noncoherent channel with predetection signal-to-noise ratio $ST/N_0 = 1$ shows that convolutional coding can reduce the energy requirement by 4.5 dB at a bit error rate of 0.001. The effects of receiver quantization and choice of number of tones are analyzed; nearly optimum performance is attained with eight quantization levels and sixteen tones at $ST/N_0 = 1$. The effects of changing ST/N_0 are also analyzed; for lower ST/N_0, accurate extrapolations can be made from the data, but for higher ST/N_0 the results are more complicated. These analyses will be useful in designing telemetry systems when coherence is limited by turbulence in the signal propagation medium or oscillator instability.

Introduction. A simulation program has been written to evaluate the performance of a particular class of codes for transmission of data over a noisy noncoherent channel, with a peak power constraint.

The MFSK code uses k bits to select one of $M = 2^k$ tones (channel symbols); the power of each tone is fixed at a value S, but the duration of the tone may be varied to achieve acceptable communication. The channel performance is analyzed both for the MFSK with no further coding, and for MFSK preceded by a convolutional coder (i. e., the codes are concatenated). The convolutional codes are rate $1/\nu$, where for simplicity of decoding we consider only the cases $k = 1$ or $k = \nu$ for $2 \leq \nu \leq 4$.

The receiver utilizes M envelope detectors, one for each possible tone, integrating over the channel coherence time T. The duration of each tone is restricted to be a multiple of T, and the tones are orthogonal on the interval T. The M detector outputs are quantized to eight levels; the level numbers are approximately log-likelihood metrics, as needed for optimum decoding. If the tone duration is N_rT, then each of M postdetection filters sums N_r of these

[1] This paper presents the results of one phase of research carried out at the Jet Propulsion Laboratory, California Institute of Technology, under Contract NAS-100, sponsored by the National Aeronautics and Space Administration.

metrics for the final branch metrics, which are sent to an optimum (Viterbi) convolutional decoder. The encoding-channel-decoding system is diagrammed in Figure 1.

The object of this investigation is to determine the effects of choice of M, and of convolutional coding for a channel specified in terms of the predetection signal-to-noise ratio ST/N_0, where signal power S, coherence time T, and one-sided noise spectral density N_0 are all fixed. The value $ST/N_0 = 1.0$ was used for all simulations discussed in this article. The effect of this choice will also be discussed.

Curves of error probability P_E versus bit-energy-to-noise ratio E_b/N_0 characterize the channel performance for various M and code choices. The E_b/N_0 corresponding to $P_E = 0.001$ is compared to a theoretical minimum, $(E_b/N_0)_{min}$ for a simple figure of merit. The minimum is simply S/N_0C, where C is the theoretical capacity of the channel, in bits per second.

Effect of Quantization. It has been shown by Butman and Levitt [2] that the performance degradation caused by using eight quantization levels, instead of an infinite number, is less than the degradation caused by decreasing from eight to four levels. That is, using eight levels may be justified, but using more is not. This conclusion was borne out in simulations with two, four, eight, and sixteen levels. All further simulations used eight-level quantization, with uniform level spacing equal to one half the most likely value of the envelope detector output in the presence of noise, but no signal. Since the spacing does not increase with the signal level, it is near optimum only for $ST/N_0 < 4$. [3] Above this value, error probability does not continue to decrease with ST/N_0, but levels off. In our case, $ST/N_0 = 1$, the quantization scheme is not a limitation.

Choice of M-Uncoded Case. The convolutional code is omitted to give a reference case. The parameters which can be varied to give reliable communication are the number of tones, M, and the duration of the tones, N_rT. Only increasing N_r will increase Eb/N_0, in order to decrease P_E.

The receiver decodes the k bits corresponding to the tone with the highest log-likelihood metric after postdetection filtering. If an error is made in this decision, (k/2)(M/M - 1) bit errors will result on the average.

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[2] Butman, S., and Levitt, B. K., "Capacity of Noncoherent, Soft Decision MFSK Signaling, " The Deep Space Network Progress Report, Technical Report 32-1526, Vol. XV, pp. 146-155, Jet Propulsion Laboratory, Pasadena, California, March 1973.

[3] Butman, S., and Levitt, op. cit.

Figure 2 shows the resulting bit error probabilities, P_E versus E_b/N_0 for M = 2, 4, 8 and 16. As expected, performance improves significantly as M increases from 2 to 4 to 8, but not so much when M goes to 16, because the MFSK capacity is approaching the $M = \infty$ limit.

Effect of Coding. When convolutional coding is used, each input bit gives rise to ν bits out of the coder. Using k = ν, i.e., $M = 2^\nu$ tones, one tone will be selected for each single bit input. Alternatively, using k = 1, ν choices of two tones can be made (BFSK). Other combinations are possible, but if the selection of a tone involves coder bits from more than a single input, the Viterbi decoder in the receiver becomes more complicated.

A typical characteristic of convolutional coding is increased error rate at low E_b/N_0, followed by sharply falling error rate, crossing below the uncoded reference. Figure 2b shows the error performance of the noncoherent channel using short (constraint length four) codes for ν = 2, 3, 4, and M = 2, 4, 8, 16. The codes for the various values of ν perform almost identically at M = 2, the only place they can be compared. The codes used are all optimal in the sense of free distance. [4]

Figure 3 compares the coded and uncoded performance for each value of M. The difference is a saving of 1 to 3 dB at moderate error rates ($P_E = 0.01$), and more at lower error rates.

Longer codes result in further energy savings at low P_E. Figure 4 compares for M = 4 the cases uncoded, constraint length 4, constraint length 8, and constraint length 12.

Figure of Merit, or Code Efficiency. From the figures we have: (E_b/N_0) (@$P_E = 0.001$). To get (E_b/N_0)min, we first need the capacity of the channel. The capacity of a wideband coherent channel is $C_\infty = S/N_0 \ell n2$ bits /sec. Butman and Klass[5] show that the noncoherent channel capacity C is approximated very closely by:

$$C = C_\infty \frac{ST/N_0}{2 + ST/N_0} .$$

[4] Larsen, K. J., "Short Convolutional Codes with Maximal Free Distance for Rates 1/2, 1/3, and 1/4, " IEEE Transactions on Information Theory, IT-19, pp. 371-372, May 1973.

[5] Butman, S., and Klass, M. J., "Capacity of Noncoherent Channels, " The Deep Space Network Progress Report, Technical Report 32-1526, Vol. XVII, pp. 85-93, Jet Propulsion Laboratory, Pasadena, California, September 1973.

For our example $ST/N_0 = 1.0$, this is a degradation by the factor 1/3, or -4.8 dB. The MFSK capacity, C_M, when M is fixed, is approximately [6] (for low ST/N_0):

$$C_M = C(1 - \frac{1}{M}) \quad .$$

Using two theoretical capacities, C and C_M, we can compute two figures of merit, one based on unrestricted M, the other based on the fixed M chosen; then the difference between them (in dB) will reflect the degradation due to the finite choice of M. Evidently, this degradation will be $10 \log_{10} (1 - 1/M)$. The figure of merit:

$$\frac{\left(E_b/N_0\right)_{min,\, M}}{(E_b/N_0)(@\, P_E = 0.001)}$$

is then just the efficiency of coding. These efficiencies are tabulated in Table I. For M = 4, the difference between the uncoded and the constraint length 12 code is 4.5 dB.

Dependence on Predetection Signal-to-Noise Ratio. All the data presented so far could be calculated and tabulated for other values of ST/N_0. This has been started for ST/N_0 = 0.25, 0.5, 1.0, 2.0, and 4. What are the expected results? Assume the code efficiency remains about constant. Then E_b/N_0 is proportional to $(E_b/N_0)_{min}$, and

$$\left(\frac{E_b}{N_0}\right)_{min} = \frac{S}{N_0 C_M} = \ell n2 \frac{C_\infty}{C_M} = \frac{\ell n2}{(1 - 1/M)} \frac{2 + ST/N_0}{ST/N_0} \quad ,$$

which increases as ST/N_0 decreases, proportionally when $ST/N_0 \ll 2$. In order to attain this necessary increase in E_b/N_0, the tone repetitions N_r must be increased inversely as the square of St/N_0.

In the region $ST/N_0 > 2$, the approximation

$$C_M = C(1 - \frac{1}{M})$$

no longer holds. In this region $C = C_\infty$ and

$$C_M \approx \frac{\log_2 M}{T} = \frac{k}{T} \quad .$$

Now transmission without coding at rate k/T (which will yield low P_E if ST/N_0 is high) will result in a coding efficiency of unity with reference to the MFSK channel, but the degradation due to small M will be severe. That is

[6] Butman, S., and Klass, M. J., op. cit.

$$\frac{\left(E_b/N_0\right)_{min,\,M}}{(E_b/N_0)(P_E\ low)} = 1$$

but

$$\frac{\left(E_b/N_0\right)_{min}}{(E_b/N_0)(P_E\ low)} = \frac{\left(E_b/N_0\right)_{min}}{\left(E_b/N_0\right)_{min,\ M}} = \frac{C_M}{C} = \frac{\log_2 M}{(ST/N_0)\ \ell n2} << 1$$

which is very low unless M is unmanageably large.

This waste of energy in the high ST/N_0 channel can be reduced by using a tone duration T' shorter than the channel coherence time T. Further reduction is possible by techniques of "partial coherence," using data in each subinterval T' to estimate the phase in the others.

Figure 5 shows the effect of changing ST/N_0 on the performance curve of a channel using a constraint length 4 rate 1/2 code with M = 4. The inverse proportionality between E_b/N_0 and ST/N_0 is evident at the low values of ST/N_0, and breaks down as expected for $ST/N_0 > 2$.

Conclusions. We have simulated MFSK transmission through a noncoherent channel, as described by the coherence time model. Preliminary results show that coding (beyond the simple repeat code) can increase efficiency by 4.5 dB at moderate error rates ($P_E = 0.001$). Further savings result at lower error rates.

The effect of coding on the very noncoherent channel ($ST/N_0 << 2$) is now fairly well understood; accurate extrapolations of performance can be made from the data presented. Note that ideal conditions are approached very rapidly as the number of tones increases to 16 and the receiver quantization increases to 8 levels.

In designing such a noncoherent system, power cost and energy cost must be considered as separate parameters, since increasing power, and hence ST/N_0, can allow a proportional decrease in total energy requirements.

More work is needed to understand the behavior of the high ST/N_0 channel, and to develop a good coding scheme for it. The problem of tone synchronization has also been left for future work.

Table 1. Code Efficiencies

$$\frac{ST}{N_0} = 1.0, \quad P_E = 0.001$$

Constraint Length	ν	M	$N_r(P_E = 0.001)$	$\frac{E_b}{N_0}(P_E = 0.001)$	$\left(\frac{E_b}{N_0}\right)_{min, M}$	Coding Efficiency
4	2	2	9			
4	3	2	6	18	4.160	0.230
4	4	2	4			
4	2	4	12	12	2.773	0.232
4	3	8	10	10	2.277	0.228
4	4	16	10	10	2.219	0.222
4	2	4	12	12	2.773	0.232
8	2	4	9	9	2.773	0.310
12	2	4	8	8	2.773	0.349
Uncoded		2	36	36	4.160	0.115
Uncoded		4	44	22	2.773	0.126
Uncoded		8	45	15	2.277	0.152
Uncoded		16	56	14	2.219	0.158

Appendix A

All the data presented are from simulations using a Xerox Sigma 5 Computer. The subroutine simulating the receiver can be changed to represent various schemes ($k = \nu$, $k = 2$, uncoded, or hard decision); its result is an array of metrics (a posteriori probabilities) which is sent to the Viterbi decoder subroutine. The program simulates transmission until 100 errors are received, then estimates P_E as the number of errors over the number of bits transmitted. The relative standard deviation of this estimate is $1/\sqrt{100} = 10\%$.

The convolutional codes used can be represented by their code generating polynomials, [7] or by another equivalent representation of the coder connection matrix. For example, the short-rate 1/2 code has coefficients 1101 and 1111 in the code generating polynomial, expressed in octal as 15, 17. Writing these in a 2 X 4 matrix, we can also reduce the columns to four base four numbers as follows:

[7] Larsen, K. J., op. cit.

15 17	1101 1111
	3313

Larsen lists codes in the first form, whereas the simulation program takes the second. The codes used in the simulations are listed both ways in Table I-A.

Table I-A. Convolutional Codes Used

1 /Rate, ν	Constraint Length	Row Form (Octal)	Column Form (Base 2^{ν})
2	4	15, 17	3313
3	4	17, 12, 15	7567
4	4	17, 13, 15, 15	FBCF
2	8	247,371	31311223
2	12	4335, 5723	301133032213

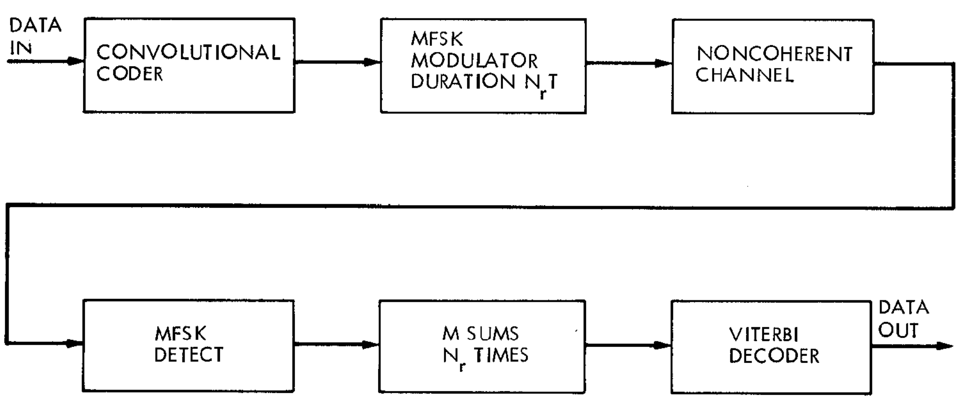

Fig. 1. Coding-Channel-Decoding System

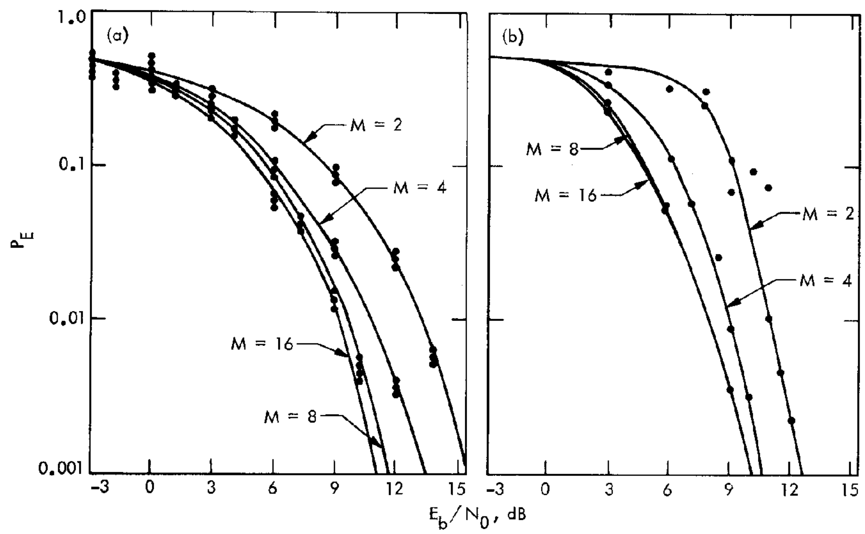

Fig. 2. Performance for Various M

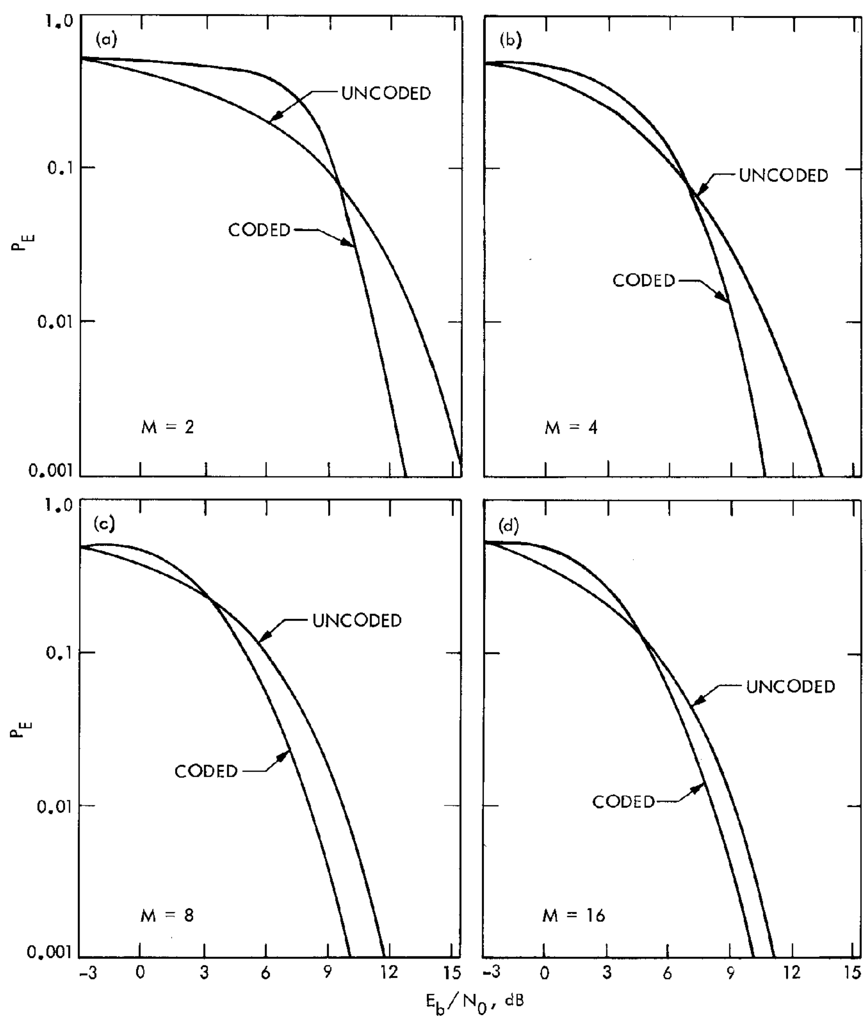

Fig. 3. Coded Versus Uncoded Performance

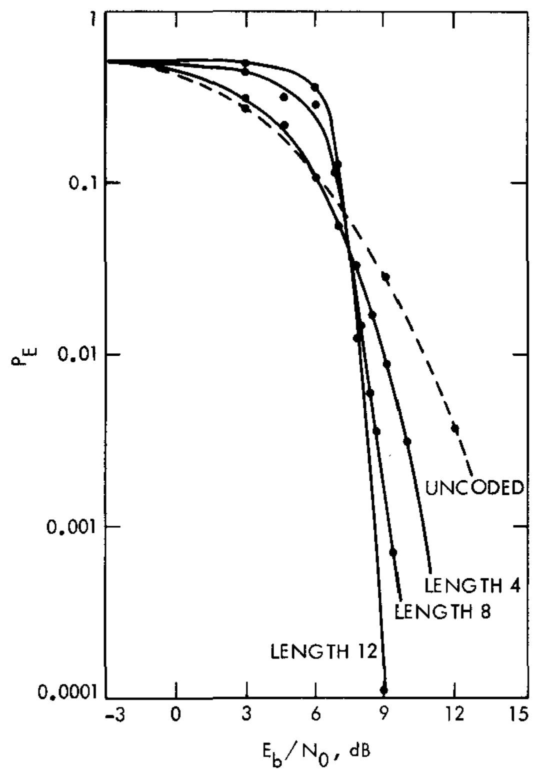

Fig. 4. Performance With Longer Codes

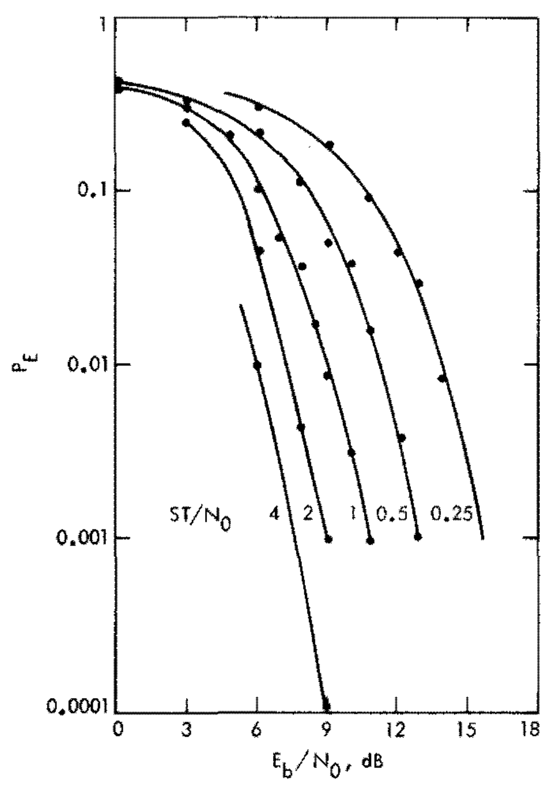

Fig. 5. Effect of Predetection Signal-to-Noise Ratio