A CODE STRUCTURE FOR CERTAIN COMA ENVIRONMENTS

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Summary. Certain communication systems which employ code division multiple access as a means of supporting multiple users need very long codes to guard against the threat of intentional jamming, but cannot tolerate the lengthy acquisition time which long codes usually require. As a possible solution to this problem, the use of combination sequences has been suggested, and this paper presents some new results, both analytical and numerical, on this technique.

Introduction. Systems employing code division multiple access (CDNA) as a means of servicing simultaneously many users transmitting over the same frequency spectrum are becoming increasingly popular. As such, the design of code sets which have appropriate interference rejection properties is of fundamental importance. One class of signal waveforms that was originally designed for use as a rapid acquisition ranging signal ([1]) and which has recently been suggested for use in a multiple access system ([2]-[4]) is that of combination sequences. These are long sequences wherein each individual bit of such a sequence is formed by some logical (i.e., Boolean) operation of, say, m shorter sequences. The lengths of the m shorter sequences are chosen to be relatively prime, and the length of the combination sequence equals the product of the lengths of the shorter component sequences (see [1]-[3] for a detailed description of combination sequences).

The principle advantage of such a sequence has been the ability to acquire a sequence of length $N = \prod_{i=1}^{m} n_i$, where n_i is the length of the ith component sequence, with at most $N_1 = \sum_{i=1}^{m} n_i$ correlations. Recent work ([2],[3]) has shown that these sequences can be acquired in the presence of other combination sequences (i.e., they are potentially useful in a CDMA system). This paper describes a specific system environment employing CDMA, presents certain numerical results verifying some of the analysis in [2] and [3], and summarizes some new analytical results discussed in depth in [4].

A CDMA System. It was shown in [2] that while combination sequences provide interference rejection from other users, if <u>only</u> interference rejection is of concern in the

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system, there are better sequences to use (e.g. [5]). However, if in addition to multiple accessing capability one desires rapid acquisition, combination sequences offer distinct advantages. In particular, there are situations involving remotely piloted vehicles which use long codes to guard against the threat of jamming, yet need the ability to acquire the codes in a rapid manner. If combination sequences are used as the basic signaling waveforms, these waveforms could be acquired on a component by component basis, thus realizing the rapid acquisition, and then the receiver could matched -filter detect over the entire combination sequence length (or some predetermined partial length) to provide whatever anti-jam capability it needed.

Summary of Bounds on Correlation Variances. Reference [2] and [3] presented bounds on the variances of both partial and full correlations of components correlated with combination sequences of which they were one component, and of component sequences correlated against completely independent combination sequences (i.e., interference from multiple users).

In [4], similar derivations are presented for correlations of complete combination sequences with each other (rather than with component sequences). These results are summarized below. In all cases, two component sequences were used, and these components were assumed to be independent random binary sequences with probability of ONE equaling probability of MINUS ONE. The logical combining law was chosen to be the signum function, defined as

$$\mathrm{sgn}(x) \triangleq \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases}$$

A. Variance of crosscorrelation between two independent combination sequences:
(i) Total Correlation (C)

$$\mathrm{var}(C) = \frac{3n_1 + 3n_2 + 9}{16N}$$

(ii) Partial Correlation of length $K \leq N$ (C_k)

$$\mathrm{var}(C_k) \leq \frac{1}{4k}\left(\left[\frac{k}{n_1}\right]+\left[\frac{k}{n_2}\right] - \frac{1}{4}\left(\left\langle\left[\frac{k}{n_1}\right]-1\right\rangle + \left\langle\left[\frac{k}{n_2}\right]-1\right\rangle\right)+\frac{15}{4}\right)$$

where $[x] \triangleq$ largest integer less than x

and $\langle x\rangle \triangleq \begin{cases} x & x \geq 0 \\ 0 & x < 0 \end{cases}$

B. Variance of autocorrelation of combination sequence:

(i) Total Correlation ($R(\ell)$)

ℓ	$var(R(\ell))$
0	0
kn_1 or kn_2, k an integer	$\frac{n_1 + n_2 + 1}{4N}$
other	$< \frac{7n_1 + 7n_2 + 4\max(n_1, n_2) - 19}{16N}$

(ii) Partial Correlation ($R_k(\ell)$)

ℓ	$var(R_k(\ell))$
0	0
kn_1 or kn_2, k an integer	$< \frac{1}{4k}(2[\frac{k}{n_1}]+2[\frac{k}{n_2}] - \langle [\frac{k}{n_1}]-1 \rangle - \langle [\frac{k}{n_2}]-1 \rangle + 3)$
other	$< \frac{1}{16k}\{8[\frac{k}{n_1}]+8[\frac{k}{n_2}] - \langle [\frac{k}{n_1}]-1 \rangle - \langle [\frac{k}{n_2}]-1 \rangle + 4\max(\langle [\frac{k}{n_1}]-1 \rangle, \langle [\frac{k}{n_2}]-1 \rangle) + 11\}$

From the above results, it can be seen that if $n_1 >> 1$, $n_2 >> 1$, and $n_1 \simeq n_2 = n$, then the variances for the total correlations essentially behave as α/n, where α is some constant which in general is different for each case.

The results from [2] and [3] alluded to above show similar behavior for the variances of the correlations of a component sequence with a full combination sequence. Also, as long as k is not much less than n, the variances of the partial correlations in all cases obey the same type of functional dependence.

Consequently, assuming the number of users in the system is inversely proportional to the variance of the correlations, it can be seen that for this particular signal design using two component sequences with approximately equal lengths for each sequence, the number of users should roughly increase linearly with n, or as the square-root of the total sequence length N. Actually, the numerical results obtained to date seem to indicate that the number of users increases more rapidly than a linear relation would predict.

Simulation Results. To test the tightness of some of the above results, a computer simulation was written which generated the component sequences randomly (with equally likely ONES and MINUS ONES) and independently, and then combined them using the sgn function as described above. This was repeated, say, J times so that a multiple user system with J users was created. The receiver consisted of a pair of matched filters, one matched to the first component of one of the combination sequences (say the first combination sequence) and one matched to the second component. A block diagram of the system is shown in Figure 1.

It was assumed that al1signals were at baseband and that they were all in symbol synchronization. However, the phasing of the different sequences was random with respect to one another.

The phases of the locally generated components at the receiver were randomly picked with respect to the in-phase positions of those components in the desired combination sequence. The simulation consisted of attempting to learn the correct phase position of each of the two components of combination sequence #1 in the presence of the remaining J-1 combination sequences. This was accomplished by cross-correlating the received waveform (i.e., the sum of all J users) with each of the two locally generated components in all n_1 and n_2 phase positions respectively, and choosing that phase position for each component that resulted in peak correlation. The parameter J was increased until, for a given code length, correct acquisition was not achieved.

For component sequence lengths of $n_1 = 29$ and $n_2 = 31$, resulting in a combination sequence length N=899, correct acquisition of code #1 was achieved in the presence of five other users (i.e., J was equal to 6). When the two component lengths were increased to 39 and 41 respectively, user #1's code was correctly acquired when nine other users were present (i.e., J=10).

In addition to determining the number of users this particular signalling structure can support with a given code length, it is of interest to determine with what accuracy the partial correlation for different window sizes K can approximate the total correlation (i.e., K=N). This is of interest in situations where very long codes are employed, since the time to perform a full correlation can be excessive. As such, if one is using a coding structure having the property that partial correlations yield accurate estimates of the respective full correlations, significant savings in time can be achieved.

The results of picking a code at random with component lengths $n_1 = 29$ and $n_2 = 31$ and computing the partial correlations of the combination sequence with each of the two components is shown in Figure 2. It can be seen at least in this one case that the partial correlations are excellent estimates of the full correlations.

Also shown on Figure 2 are the peak out-of-phase correlations obtained in a number of consecutive correlations (each correlation being performed over the next phase position) equal to the component length (for component one, $n_1 = 29$, so 29 consecutive correlations were performed). This is of interest since if one is trying to acquire the combination sequence on the basis of partial correlations, one would certainly want all out-of-phase correlations to be reasonably smaller than the in-phase correlation. As can be seen from Figure 2, this example appears to satisfy this requirement.

Conclusion. Results have been presented for a specific signalling structure, namely one employing combination sequences, which demonstrate the multiple accessing capability of these codes. Since these codes have long been known to have a rapid acquisition property, they appear to be a reasonable choice of signal design for systems requiring long codes but which cannot tolerate a long acquisition period.

References

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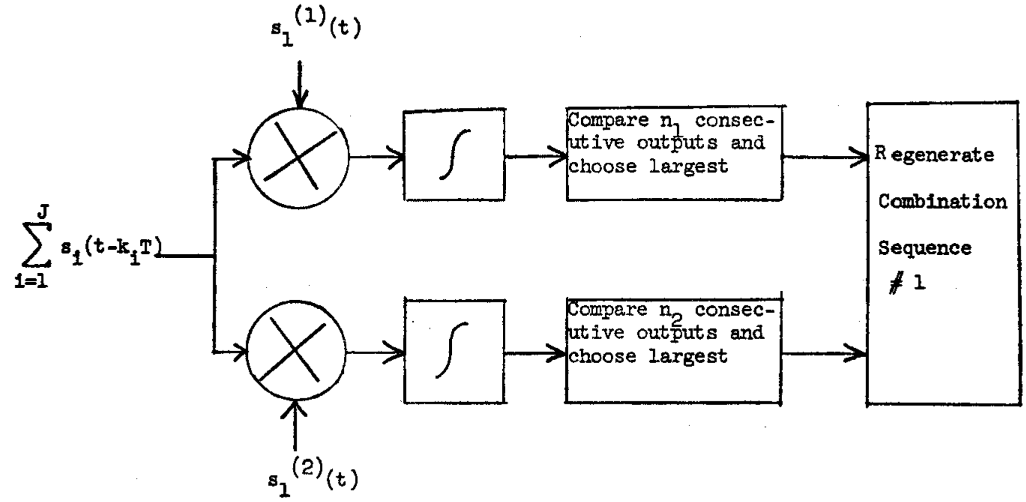

$s_1^{(j)}$ = j^{th} component of combination sequence #1, j = 1,2

$s_i(t)$ = i^{th} combination sequence

T = duration of code symbol

k_i = integer determining relative phase position of $s_i(t)$

Figure 1. Block Diagram of Simulation

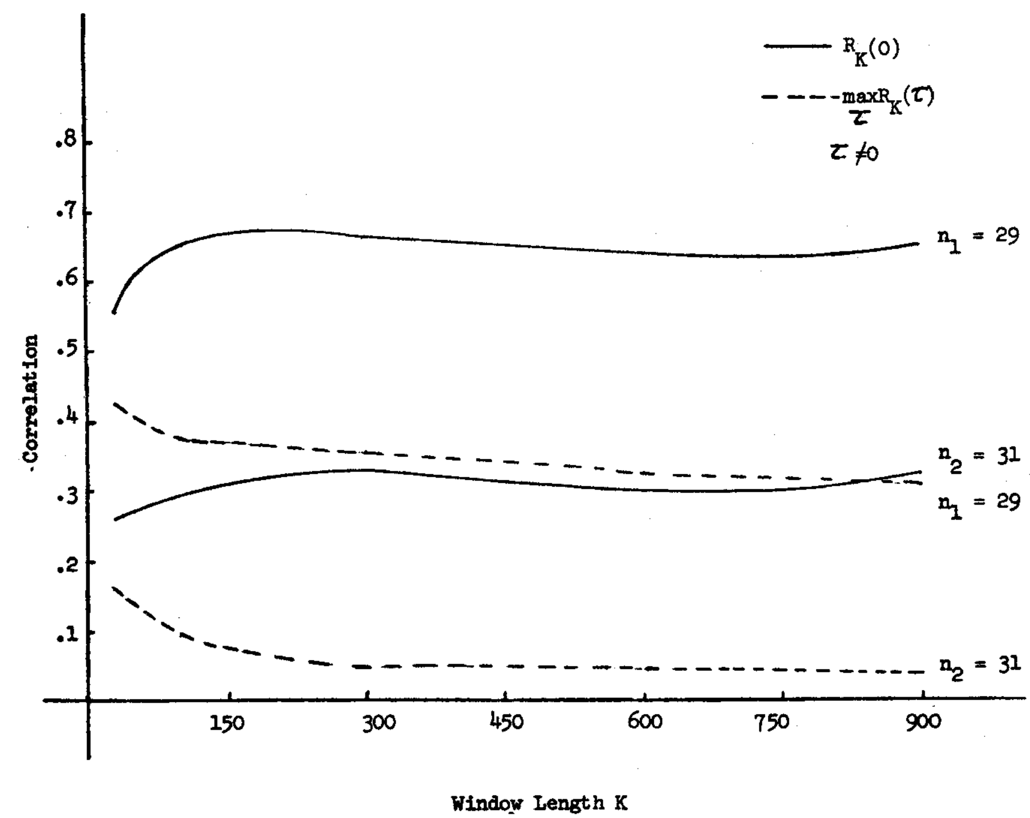

Figure 2. Partial Correlation Results for Combination Sequence of Length N=899